Buckling of Prestressed Steel Girders

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ABSTRACT

Prestressing of steel girders, in order to gain economy of material, is starting to become popular in the United States. An inherent danger in the stressing operation is loss of stability of the girder between the points of attachment of the tendon. The paper presents design charts for the elastic buckling load induced by stressing an eccentric tendon, and uses this to obtain a design buckling strength in accordance with the LRFD Specification.

INTRODUCTION

Prestressing of steel plate girders may lead to substantial economies of material. Densford et al.[1] quote savings of 30% in steel tonnage and 27% in concrete tonnage gained by prestressing a short composite steel-concrete bridge that was designed by the Oklahoma Department of Transportation. Similar savings may be obtained for steel girders. Although the use of prestressed steel girders is relatively new in the United States, the technology is well-established in eastern Europe.[2,3]

Perhaps the easiest way to prestress a steel girder is to use straight high-strength rods, which are anchored at the ends of the beam as shown in Fig. 1, and stressed in a manner analogous to that for prestressing concrete beams. These "hard" anchorages may have a number of "soft" anchorages between them, allowing relative movement of the tendons. Other methods of prestressing steel girders are discussed in Ref. 1. Stressing the high-strength rods induces substantial compressive stresses in the bottom flange of the beam before the external loads are applied, and raises the question of the stability of the girder under this loading. If the attachment of the tendon to the web is at large spacing intervals, then the girder may buckle between the points of attachment in an overall or lateral mode.[4] The designer must be certain that the prestressing force is not large enough to cause buckling of the girder. The use of LRFD design methods and the method in this paper may be used to calculate the buckling strength.

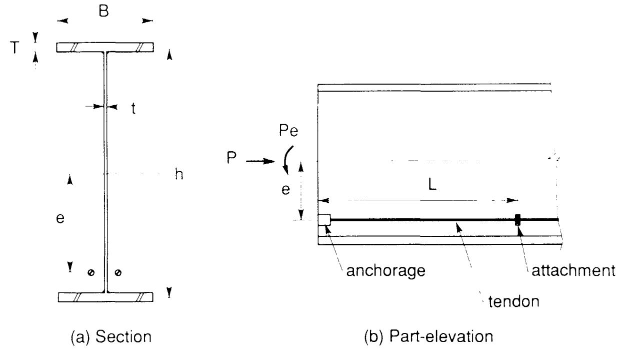
The advantages of prestressing are reflected most for plate girders, and these are most economically fabricated from thin plate elements. Studies of the stability of plate girders with thin webs have shown that the overall buckling mode is distortional,[5,6] rather than flexural-torsional, characterized by bending or distortion in the plane of the cross-section, as shown in Fig. 2. Distortional buckling loads have been shown to be significantly lower than flexural-torsional buckling loads if the web of the girder is slender.[5]

This paper uses a method of analysis that was developed by the author[6] to produce charts for the prestressing force required to cause elastic distortional buckling of slender plate girders. An example is presented to illustrate the use of the design charts to calculate the maximum prestressing force in accordance with the LRFD Specification.[7] The use of LRFD methods is becoming more widespread in the United States, and the paper illustrates how LRFD design may be used in a design situation.

ANALYSIS

A straight steel girder prestressed with two tendons at an eccentricity e to the centroid of the section is shown in Fig. 1. Provided that the girder is simply supported, the cross-section is subjected to an axial force P and moment Pe applied at the centroid, where P is the prestressing force. Of course, this does not account for the amplification of the moment due to in-plane bending, but this is taken account of by the amplification term in the LRFD provisions.[7]

A computer method designed to calculate the distortional buckling load factor λ_d for the elastic buckling of these types of beam-columns is given in Ref. 6. By assuming that the beam-column buckles as a sine curve, the buckling displacements u_T, u_B, ϕ_T, ϕ_B representing the displacement u and twists ϕ of the top (T) and bottom (B) flanges may be obtained from the matrix expression.



(a) Section (b) Part-elevation

Fig. 1. Prestressed steel beam.

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$$([k] - \lambda_d [g]) \times \{\delta\} = (0) \qquad (1)$$

where

$$\{\delta\} = \{u_T,\ u_B, \phi_T, \phi_B\}^T \qquad (2)$$

and *[k]* and *[g]* are elastic 4×4 matrices. The buckling load factor λ_d is the characteristic value of Eq. 1, while the values of $\{\delta\}$ represent the buckled shape. The computer method may also be used for nonsymmetric and composite beams.

In using the computer program, values of the axial force P and moment Pe were input, along with the section geometry in Fig. 1. Values of λ_d were obtained, giving the elastic distortional buckling load P_d as

$$P_d = \lambda_d P \qquad (3)$$

Figures 3a to 3d give values of the buckling load P_d for a range of geometries typical of plate girders. The values of P_d are normalized with respect to the Euler buckling load P_e, where

$$P_e = \pi^2 E I_y / L^2 \qquad (4)$$

and

$$I_y = B^3 T / 6 \qquad (5)$$

with E being Young's modulus (29,000 ksi).

Figure 2 shows the buckling mode, obtained from $\{\delta\}$ in Eq. 1, for $B/h = 0.2$, $h/t = 200$, $T/t = 4$ and $L/h = 6$. The marked nature of the distortion of the web of the girder is evident in this case.

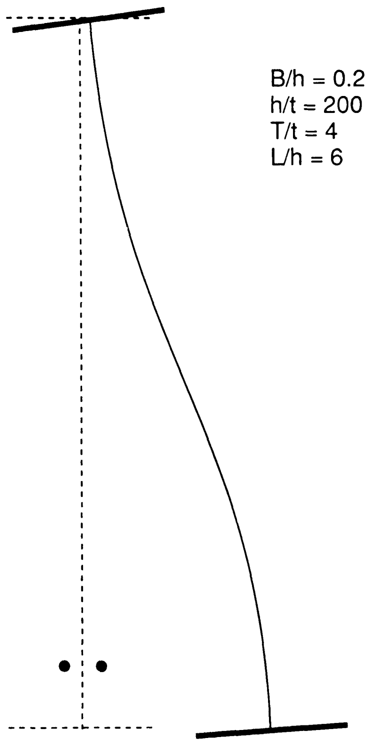

Fig. 2. Buckled shape.

APPLICATION

The application of the design graphs in Figs. 3a–3d is best illustrated by an example. It is required to calculate the design prestressing force that would cause a 100 ft long end anchored plate girder with a "soft" tendon connection at mid-span to buckle. The buckling length is thus $L = 50$ ft. The girder is fabricated from two 12-in.×1.0-in. flange plates and a 50-in.×0.3-in. web plate. The tendon is located 23.0 in. from the centroid. Take $F_y = 50$ ksi and $E = 29{,}000$ ksi. For this problem

$h = 50.0 + 1.0 = 51.0$ in.
$B = 12.0$ in.
$T = 1.0$ in.
$t = 0.3$ in.
$L = 1200 / 2 = 600$ in.
$e = 23.0$ in.
$h/t = 51.0 / 0.3 = 170$
$B/h = 12.0 / 51.0 = 0.24$
$T/t = 1.0 / 0.3 = 3.33$
$L/h = 600 / 51.0 = 11.8$
$e/h = 23.0 / 51.0 = 0.45$

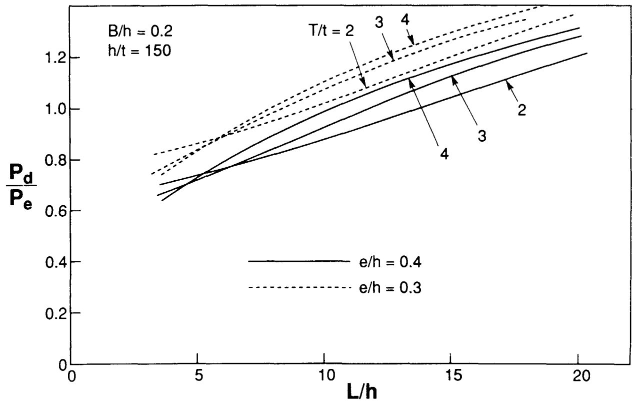

Fig. 3a. Buckling curves.

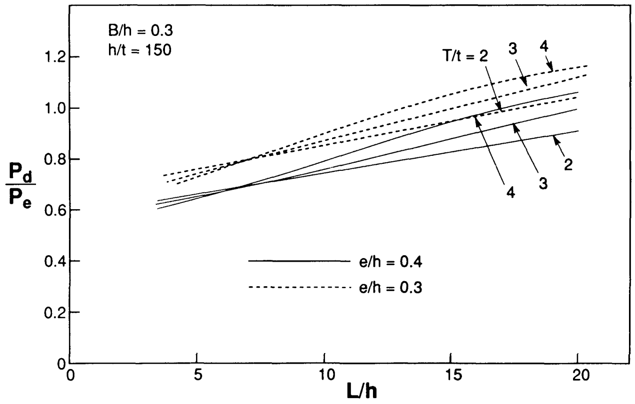

Fig. 3b. Buckling curves.

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