Stiffness Reduction Factor for LRFD of Columns

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The stiffness reduction factors (*SRF*'s) given in Table A (page 2-8) of the LRFD Manual[1] has an error that needs to be corrected. The error occurs in adopting the technique originally proposed by Yura[2] in the context of ASD to LRFD.

It is well known that when using the standard alignment charts to determine the effective length factor K of a column, the restraint factors G should reflect the inelastic behavior of the column. For elastic behavior, the G factor at either end of the column is defined as

$$G_{elastic} = \frac{\Sigma(EI/L)_{col}}{\Sigma(EI/L)_{beam}} \tag{1}$$

The inelastic G factor is then related to this through

$$G_{inelastic} = \frac{E_t}{E} G_{elastic} = SRF \times G_{elastic} \tag{2}$$

Since the critical buckling stress is directly proportional to the modulus, the *SRF* may also be expressed as

$$SRF = \frac{F_{cr,\ inelastic}}{F_{cr,\ elastic}} \tag{3}$$

In estimating the *SRF* in a design problem, Yura[2] assumed that the column is designed very close to its capacity. In the context of LRFD this is equivalent to the assumption that

$$P_u \approx \phi_c P_n = \phi_c F_{cr,\ inelastic} A \tag{4}$$

in which $\phi_c = 0.85$ is the resistance factor for compression members, and A is the cross sectional area of the column. Based on Equation 4 we obtain

$$F_{cr,\ inelastic} \approx \frac{P_u}{\phi_c A} \tag{5}$$

The LRFD expressions for the inelastic and elastic critical stresses are

$$F_{cr,\ inelastic} = (0.658^{\lambda_c^2}) F_y, \quad \text{when } \lambda_c < 1.5 \tag{6}$$

and

$$F_{cr,\ elastic} = \frac{0.877}{\lambda_c^2} F_y, \quad \text{when } \lambda_c \geq 1.5 \tag{7}$$

in which λ_c is a slenderness parameter.

Two points are worth noting at this stage. First, the transition from inelastic to elastic behavior takes place at $\lambda_c = 1.5$, and therefore the $SRF = 1$ when $\lambda_c \geq 1.5$. For $\lambda_c = 1.5$, from Equations 6 and 7, we obtain $F_{cr,\ inelastic} = F_{cr,\ elastic} = 0.390F_y$, and from Equation 5 we conclude that $P_u/(\phi_c A)$ should be 14.0 ksi for 36 ksi steel and 19.4 ksi for 50 ksi steel. According to Table A of the LRFD Manual, however, the $SRF = 1$ when P_u/A is 14.0 ksi for 36 ksi steel and 19.4 ksi for 50 ksi steel. The error is immediately obvious. The table has been prepared for $P_u/(\phi_c A)$ and not for P_u/A. Secondly, the error can also be noticed by the fact that Table A of the LRFD Manual gives the *SRF* for values of P_u/A up to F_y. For a column designed to capacity, it is $P_n/A = P_u/(\phi_c A)$ that can reach F_y, while P_u/A must always be less than or equal to $\phi_c F_y$.

To reemphasize, in estimating the *SRF* the designer should enter Table A of the LRFD Manual with $P_u/(\phi_c A)$ and not P_u/A.

For steels with yield strengths other than 36 or 50 ksi, the *SRF* can be estimated by the following steps:

1. Solve for λ_c using Eq. 5 and 6 to obtain

$$\lambda_c = \left[\frac{\log\left(\dfrac{P_u}{\phi_c A F_y}\right)}{\log 0.658} \right]^{1/2} \tag{8}$$

2. Compute the *SRF* using Eq. 3, 6 and 7 to yield

$$SRF = \begin{cases} \dfrac{\lambda_c^2 (0.658^{\lambda_c^2})}{0.877}, & \text{when } \lambda_c < 1.5 \\ 1 & \text{when } \lambda_c \geq 1.5 \end{cases} \tag{9}$$

In their latest text on LRFD, Salmon and Johnson[3] give a table of the *SRF* (which they denote by β_s) for 36 ksi steel. This table gives the *SRF* as a function of λ_c that is consistent with Equation 9.

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REFERENCES

1. American Institute of Steel Construction, Inc., *Load and Resistance Factor Design, Manual of Steel Construction*, (Chicago: AISC, 1986).
2. Yura, Joseph A., "The Effective Length of Columns in

Unbraced Frames," *Engineering Journal,* 8:No.2 (2nd Quarter 1971) 37–42.
3. Salmon, Charles G. and Johnson, John E., *Steel Structures: Design and Behavior,* 3rd ed., (New York: Harper and Row, 1990) p. 334.

AISC NOTE

Future printings of the 1st Edition of the *LRFD Manual of Steel Construction* will be corrected to reflect this error. The heading of Table A on pages 2-8 and 2-9 will be changed to $P_u/\phi A$.