Understanding Composite Beam Design Methods Using LRFD

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A recommended source of information for an understanding of Load and Resistance Factor Design concepts is through the composite design sections of the LRFD Specification and LRFD Manual. Specifically, the structural engineer designing a shear-connected composite beam has a new tool in the Composite Beam Selection Tables published in Part 4 of the LRFD Manual. These tables allow the designer much more flexibility in the alternative selection process that leads to better reliability and design economy. To properly exploit these options, however, an understanding of the fundamental composite strength limit theory and the model from which the design tables were created is needed. A particularly attractive benefit is that preliminary design manipulation can proceed with no required information regarding the strength properties of the concrete. When discrete cost parameters of the variable material requirements are known, cost optimization is also feasible.

LIMIT CONDITIONS—FULL COMPOSITE BEAM DESIGN

The limiting strengths of a composite flexural member composed of a steel beam attached to a concrete floor slab is shown in Fig. 1. Under the usual conditions of loads and layout geometry, experience indicates the ultimate tensile strength of the steel beam is less than the ultimate compressive strength of the concrete slab. In other words, the steel controls and the concrete is under-utilized, with the result the plastic neutral axis (PNA) is located within the depth of the slab. This condition is represented conservatively in the LRFD Manual as the full composite action situation with the PNA located at the top of the steel flange (Position 1, TFL).

There are important exceptions to these usual design conditions where the ultimate compressive strength of the concrete is less than the ultimate tensile strength of the steel beam. Although this means the steel is under-utilized (a condition normally avoided for economical reasons), physical realities may force the composite design to be controlled by the concrete strength. This occurs often where steel metal deck is perpendicular to the beam, effectively reducing the concrete area, or where the slab is relatively thin, or for edge beams and spandrel members where the concrete effective width is reduced. For conditions where concrete controls, the design conditions require the plastic neutral axis be located somewhere within the steel beam, as in Fig. 2, to permit a top portion of the steel to assist the concrete slab in resisting part of the compression force. From a design point of view, analytical techniques are different, depending on whether the PNA is located in the top-flange or the upper part of the web.

It is important for designers to understand the LRFD Manual, in its composite beam selection tables, does *not* cover the condition of concrete control (LRFD Manual, p. 4-6). Further, since the concrete control condition fully utilizes the concrete slab strength, it should be considered a *full* composite design situation, where enough shear con-

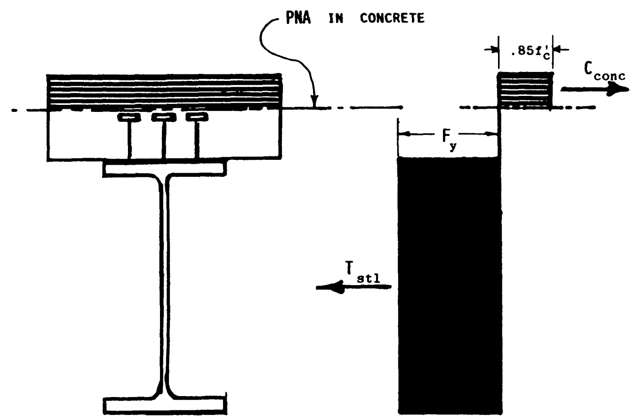

Fig. 1. Full composite limit condition—steel controls

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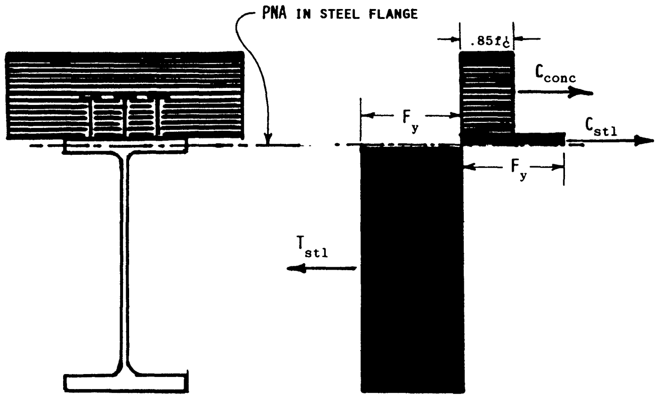

Fig. 2a. Full composite limit condition—concrete controls

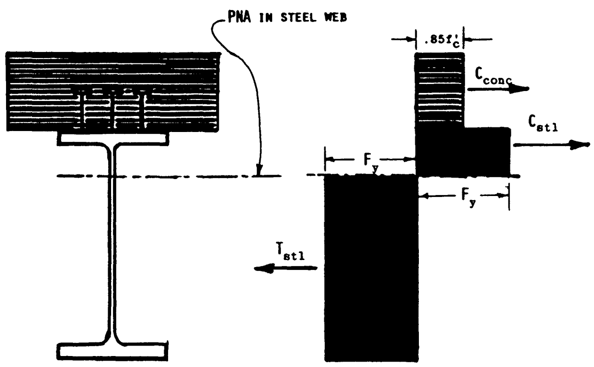

Fig. 2b. Full composite limit condition—concrete controls

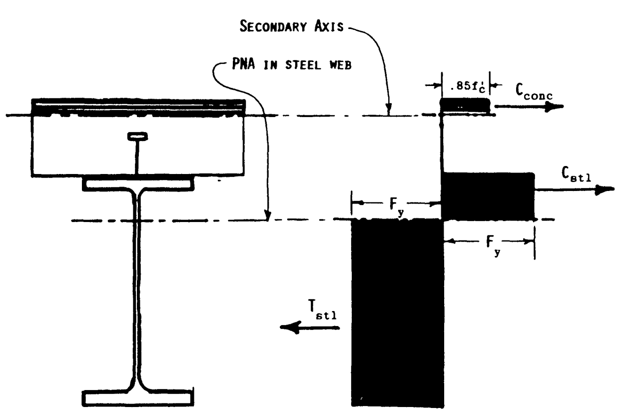

Fig. 3. Partial composite limit condition—studs control

nectors are required to develop concrete strength. Although not covered in the Manual, design procedures for the concrete control condition is not difficult and can be found in most design textbooks.

LIMIT CONDITIONS—PARTIAL COMPOSITE BEAM DESIGN

Although not stated explicitly, the LRFD Manual composite-beam selection tables provide choices of limit strengths based on "partial-composite" design philosophy. This is achieved by varying the value of horizontal shear, representing a limited amount of slip between the steel-concrete interface. As the shear is reduced, so is the number of required shear connectors, which exchanges a slight reduction in stiffness for improved economy. The practical effect of this is very similar to, but different from, the condition of concrete control. By reducing the number of studs to a value less than required for full composite (Position 1, TFL in the LRFD Manual), the PNA is lowered to a position somewhere in the steel beam, while, at the same time, fewer studs reduce the concrete stress block proportionately, moving the concrete compressive force slightly higher toward the top of the slab, as in Fig. 3.

The LRFD composite-beam selection tables actually predesign seven locations of the PNA from the top of the slab to quarter points within the top steel beam flange and two positions in the web, exploiting a full range of design possibilities of partial-composite conditions.

MINIMUM WEIGHT VS. MINIMUM COST

Traditionally, using allowable stress design (ASD) procedures, most designers use the criteria of *minimum weight* of the steel beam as the economic objective of the composite flexural design procedure. This is always achieved by selecting a *full-composite*, steel control situation. Using LRFD methods, this is accomplished by ascertaining the PNA is *not* in the steel beam, as mentioned earlier.

However, most designers are aware the economic objective should really be a *minimum cost* criteria, a more difficult condition to evaluate. Figure 4 gives an indication of how a given design condition can generate a series of alternative solutions using the partial composite design advantage of the LRFD Composite-Beam Selection Tables. Hence, the true advantage of the LRFD method is the opportunity to select the minimum cost alternative, since often steel beam weights heavier than the minimum weight possible may actually have the least cost. In other words, the LRFD Manual procedure in reality provides an optimization model that should be exploited fully by the structural designer. In essence, a trade-off between steel weight and stud quantity for a given design condition and material cost provides the essential parameters for such an optimization model.

OPTIMIZATION MODEL FOR COMPOSITE BEAMS

To develop a reasonable and useful model to quickly compare alternative composite design solutions, several usual conditions are stipulated. Here they would include limiting the steel to ASTM A-36, regular-weight concrete at a 3 ksi compressive strength, ¾-in. headed studs and a uniformly loaded simple beam.

From these conditions, an economic cost evaluation can be made by minimizing an equivalent cost-rated beam weight. Since the beam cost and stud cost are the variables of interest, their total cost Q would be the sum of each of their respective costs, such as

$$Q = W_s \cdot L \cdot C_m + N_s \cdot C_s$$

where W_s = steel beam weight (lb./ft)
L = beam span (ft)
C_m = fabricator's cost of mill steel ($/lb.)
C_s = cost of a field-installed stud ($/stud)
N_s = stud quantity per beam

This can be restated as an equivalent cost-rated beam weight W_s' by

$$W_s' = W_s + (N_s/L)\,(C_s/C_m)$$

For preliminary design evaluation, the alternative selections would have variables in W_s and N_s. The span L is given, and for a given job-size and geographical region, C_s/C_m could range between 6 to 12 using a trial value of 10 as a starting point.

In applying the optimization model to the design comparison of Fig. 4, assuming a beam span of 20 ft and a C_s/C_m ratio of 10, we have

Beam	N_s	W_s'
W18×35	38	54
W18×40	20	50
W18×46	12	52
W18×55	0	55

From this comparison, the W18×40 using 20 studs should be selected. An additional benefit of using heavier beam weights is the reduction of construction load deflections when employing unshored steel beam construction methods. For the material design conditions stated, further simplification of the optimization model is possible.

From the development of the procedures in the LRFD Manual on p. 4-5 for partial composite (stud control) methods

$$nQ_n \geq \Sigma\, Q_n = vF_y A_s$$

where v = percent of full-composite action and $n = \frac{1}{2} N_s$ resulting in

$$N_s \geq 2vF_y A_s/Q_n$$

where F_y = 36 ksi
A_s = $W_s/3.4$
Q_n = 21 kips

substituting

$$Ns > 1.01\, vW_s = vW_s$$

Hence

$$W_s' = W_s\,[1 + (v/L) \cdot (C_s/C_m)]$$

Applying the same criteria, we obtain essentially the same results as before

Beam	v	W_s'
W18×35	1.000	52.5
W18×40	.526	50.5
W18×46	.316	53.2
W18×55	0	55.0

A study of the minimum cost variable shows it is relatively sensitive to span length. By increasing the span lengths to 30 ft, 36 ft and 40 ft, Fig. 5 notes a condition of parity for

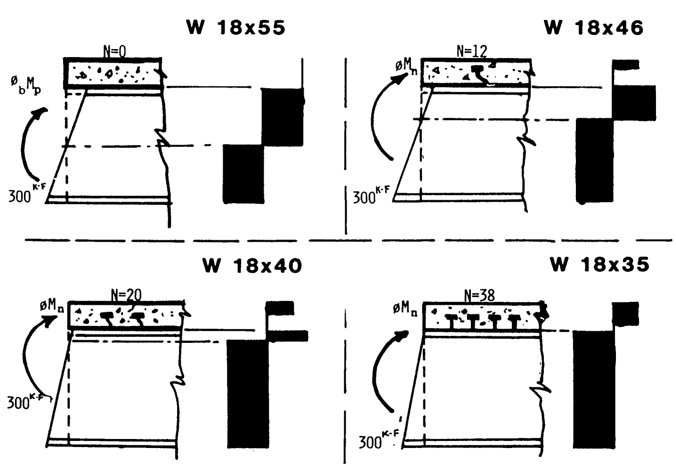

Fig. 4. Alternative solutions for given bending moment

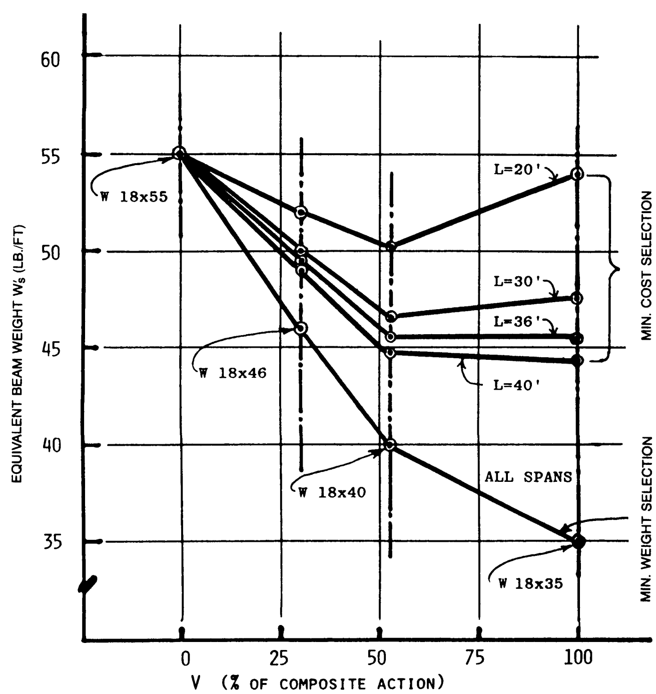

Fig. 5. Minimum cost vs. minimum weight economy

all spans above 30 ft and above 50% composite action. In other words, the *combined* material costs of beam and stud installation do not vary much in those instances. Hence, for such conditions, the designer should opt for the heavier steel weight to obtain the benefit of more stiffness during construction, a reduced dead-load sag and the need for shores or even eliminating a beam camber requirement.

PRACTICAL IMPLICATIONS OF LRFD COMPOSITE BEAM METHODS

A review of the LRFD composite-beam selection tables reveals the best economy will most always be found when the PNA is located in the top flange of the steel beam. This is so, since it represents approximately the partial composite conditions ranging from 50% to 90%.

Taking the general case for the condition of the PNA in the steel beam and referring to the LRFD Manual nomenclature

$$M_p = F_y A_s d/2 + \Sigma\, Q_n \cdot Y2 - C_{st} \cdot Y1$$

since

$$Q_n = v F_y A_s$$
$$C_{st} = (Y1/t_f) \cdot (A_f/A_s) \cdot (F_y A_s) = (p) \cdot (f) \cdot (F_y A_s)$$

then

$$(M_p/F_y A_s) = (\tfrac{1}{2})\,(d) + (v \cdot Y2) - (p \cdot f \cdot Y1) = (\tfrac{1}{2})\,(d) + Y2 - pf(Y1 + 2\,Y2)$$

since

$$v = 1 - 2pf$$

For most steel beams, $f \approx 0.25$; and since $p = Y1/t_f$, therefore

$$(Mp/F_y A_s) = (\tfrac{1}{2})\,(d) + Y2 - (pY2/2) - (Y1^2/4t_f)$$

Ignoring the squared term, since $Y1$ is often less than unity, a quick method for evaluating composite strength is

$$\phi M_n = M_p = \phi F_y A_s\,[d/2 + Y2\,(1 - p/2)\,]$$

For PNA at top of beam flange

$$\phi M_{n\text{-}1} = .85\,(\Sigma\, Q_{n\text{-}1}) \cdot (d/2 + Y2)/12 \text{ (kip–ft)}$$

For PNA at bottom of beam flange

$$\phi M_{n\text{-}5} = .85\,(\Sigma\, Q_{n\text{-}1}) \cdot (d/2 + Y2/2)/12 \text{ (kip–ft)}$$

To compare the quick method with the LRFD Manual composite tables, select a W18×35, assuming $Y2$=4.0 in. and $Y1$=0 (Position 1) and $Y1 = t_f$ (position 5)

$$\phi M_{n\text{-}1} = .85\,[(371) \cdot (8.85 + 4.00)]/12 = 337.7^{\text{kip-ft}}$$
$$\phi M_{n\text{-}1} \text{ (LRFD Manual)} = 338^{\text{kip-ft}}$$
$$\phi M_{n\text{-}5} = .85\,[(371) \cdot (8.85 + 2.00)]/12 = 285.1^{\text{kip-ft}}$$
$$\phi M_{n\text{-}5} \text{ (LRFD Manual)} = 283^{\text{kip-ft}}$$

Such quick procedures are most appropriate during preliminary analysis to obtain reasonable material and geometry layout options as well as material estimates for feasibility studies.

SUMMARY

This paper presents a very brief review of practical procedures toward a better understanding of the LRFD Composite-Beam Design method. Particular mention was placed on those conditions where concrete strength might control and would not be covered by the Manual techniques.

A second important aspect is development of an optimization model that provides an equivalent-beam weight technique for selection of minimum cost, composite-design alternatives. Sensitivity checks of this model suggest little variation in cost-oriented choices for spans above 30 ft where 50% or more composite action is utilized.

Finally, based on the above cost-sensitivity information, a quick-method strength formula was developed for the desired condition of a stud-controlled design where the PNA was located in the top flange of the steel beam. This method appears to be reasonably accurate for preliminary work during feasibility and alternative design studies. For final designs, it must be emphasized that complete analytical procedures following the more exact equations or tables should be used.