# Flexural-torsional Buckling for Pairs of Angles Used as Columns

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While the end result is correct, Ex. 7 on page 2-47 of AISC *Load and Resistance Factor Design Manual*[1] requires some modification and amplification to be fully illustrative of correct design procedure. The following revised and expanded version attempts to accomplish this.

**Example**

Member ABC in the illustrated roof truss (Fig. 1) is braced in the plane of the truss at A, B and C, but is braced out-of-plane only at the eaves and peak (A and C). Using a double-angle member of 5 x 3 x ½ angles (short legs back-to-back with ⅜ in. spacers), and 36 ksi steel and a factored load of 70 kips, determine the number of connectors required. Assume $K = 1.0$.

*Solution:*

(see p. 2-72 of the Column Tables in the Manual[1])

$KL_x = 10$ ft, $\phi\ P_n = 76$ kips

$KL_y = 20$ ft, $\phi\ P_n = 128$ kips

Both of these tabulated design strengths presume adequate connectors. The Y-axis value is given for three equally spaced connectors and includes flexural-torsional buckling effects, if any. In this example, three equally spaced connectors means connectors spaced at 5 ft.

For the X-axis, the maximum slenderness ratio of an individual angle, $a/r_z$, must not exceed the member slenderness ratio $KL_x/r_x$.

*Calculate:*

$a/r_z \leq KL_x/r_x = (10 \times 12)/\ 0.829 = 145$

$a = 145 \times 0.648 = 94$ in.

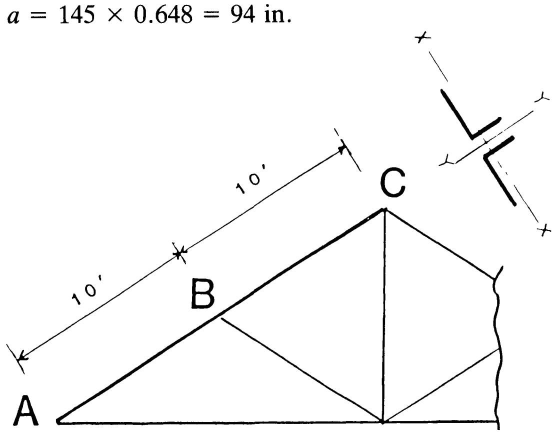


*Figure 1*

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One connector is required; $L_z = (10 \times 12)/2 = 60$ in. $= 5$ ft.

Therefore, gussets at A, B and C, plus one intermediate connector in length AB and one in length BC are adequate. (This concludes the LRFD Manual example.)

**COMMENTARY**

It should be observed that no comparison of $KL_x/r_x$ with $KL_y/r_y$ has been made, nor should any be done. The occurrence of flexural-torsional buckling for sections having one axis of symmetry results in quite different steps than in determining critical stress only from maximum $KL/r$. A second complication results from the fact the connectors must function in shear for buckling about the Y-axis, but not about the X-axis. This second phenomenon can be dealt with rather simply by the formulas in Sect. E4 of the LRFD Manual which give a modified slenderness ratio about the Y-axis. But this modified slenderness ratio is *not* used in the usual column formulas (E2-2) and (E2-3), which are intended for members that buckle in flexure. Instead, the formulas given in Appendix E must be used first to compute an effective member slenderness $\lambda e$ due to flexural-torsional buckling, which is finally used in the regular column formulas.

For example, consider the same double angle member ($5 \times 3 \times ½$), with $F_y = 36$ and $KL_y = 8$ ft. The column table indicates the use of three connectors; therefore $a = 8/(3+1) = 2$ ft $= 24$ in. Then, $a/r_z = 24/0.648 = 37.04$. Since this is less than 50, the modified slenderness ratio is the same as $KL_y/r_y = (8 \times 12)/2.5 = 38.40$. If this value were (incorrectly) used in the usual column formulas, the design strength would be calculated thus:

$$\lambda_c = \frac{KL}{\pi r}\sqrt{\frac{F_y}{E}} = \frac{38.40}{\pi}\sqrt{\frac{36}{29000}} = 0.4307$$

$$F_{cr} = (0.658)^{\lambda^2_c}\ F_y = (0.658)^{(0.4307)^2} \times 36 = 33.31$$

$$\phi P_n = \phi A_g\ F_{cr} = 0.85 \times 7.5 \times 33.31 = 212.4 \text{ kips}$$

The correct procedure, including torsion, is this:

From p. 1-159: $r_o = 2.69$, $H = 0.965$

From p. 1-144: for one angle $J = 0.3220$

*Calculate:*

$$F_{ez} \text{ (very nearly)} = \frac{G\ J}{Ar_o^2} = \frac{11{,}200 \times (2 \times 0.3220)}{7.50 \times (2.69)^2}$$

$$= 132.90$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 \times 29{,}000}{(38.40)^2} = 194.10$$

$$F_e = \frac{F_{ey} + F_{ez}}{2H}\left(1 - \sqrt{1 - \frac{4 F_{ey} F_{ez} H}{(F_{ey} + Fez)^2}}\right)$$

$$= \frac{194.10 + 132.90}{2 \times 0.965} \times$$

$$\left(1 - \sqrt{1 - \frac{4 \times 194.10 \times 132.90 \times 0.965}{(194.10 + 132.90)^2}}\right)$$

$$= 124.99$$

$$\lambda_e = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{36}{124.99}} = 0.5367$$

$$F_{cr} = 0.658^{\lambda^2_e} F_y = (0.658)^{(0.5367)^2} \times 36 = 31.91$$

$$\phi P_n = \phi A_g F_{cr} = 0.85 \times 7.5 \times 31.91 = 203.4 \text{ kips}$$

This computed design strength agrees with the value in the LRFD Manual, 204 kips. There is about 4% reduction because of torsion.

Thus there are two quite different procedures going from slenderness ratio to design strength, as described in the Manual notes, p. 2-46. It could be depicted graphically as one column curve for bending only and another, below it, for bending plus torsion (Fig. 2).

It rarely should be necessary for the designer to use the rather complicated procedure just shown. On those occasions when the design strength must be calculated including flexural-torsional buckling, it will be simpler to determine the modified slenderness ratio for several values of $KL_y$, plot these and the corresponding values of design strength from the Manual's Column Tables and then interpolate.

For example: Find the design strength of a member made of double angles 8 × 6 × 1, long legs back-to-back with ⅜ space, and one connector at the middle of a 22-ft length. Assume $K = 1.0$ for both axes and $F_y = 36$ ksi.

*Solution:*

For X-axis, $\phi P_n = 440$ kips

$$KL/r = \frac{22 \times 12}{2.49} = 106.02$$

$$a/r_z = \frac{11 \times 12}{1.28} = 103.13 \quad \textbf{o.k.}$$

For Y-axis, $\phi P_n$ is tabulated as 429 kips, but only if *two* intermediate connectors are used. With only one connector, calculate modified $KL/r = \sqrt{(KL/r)^2 + (a/r - 50)^2}$

$$= \sqrt{\left(\frac{22 \times 12}{2.52}\right)^2 + (103.13 - 50)^2} = 117.46$$

Using data from the column tables and the procedure above:

| $(KL)_y$ | $\phi P_n$ | $n$ | $a$ | *modified* $(KL/r)_y$ |
|---|---|---|---|---|
| 22 | 429 | 2 | 7.33 | 106.42 |
| 24 | 379 | 2 | 8.00 | 116.99 |
| 26 | 330 | 2 | 8.67 | 127.70 |

Interpolating for $KL/r = 117.46$ yields $\phi P_n = \phi$ 377 kips, which is the design strength.

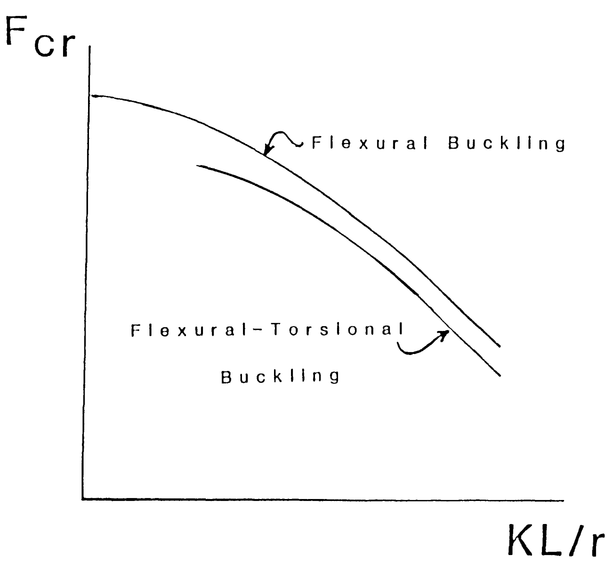


*Figure 2*

## SUGGESTED READING AND CONCLUSIONS

Textbooks on stability[2] and cold-formed steel design[3] discuss three differential equations for elastic buckling about axes X, Y and Z. Double angles have one axis of symmetry (the Y-axis) and the equation for buckling about the X-axis is uncoupled from the other two. It is appropriate to use ordinary column formulas for that axis. The *combination* of torsion and flexure about the Y-axis always results in elastic buckling loads lower than for either pure torsional (Z-axis) buckling or simple flexural (Y-axis) buckling.

Using comparisons of X and Y axis slenderness ratios as indications of load-carrying capacity for double angles is fraught with traps for the unwary. While it is true that Y-axis strength will control the design if $(KL/r)_y$ is larger than $(KL/r)_x$, the reverse is not always true. When $(KL/r)_x$ exceeds $(KL/r)_y$, both axes should be checked, since the provisions of Sect. E and Appendix E may increase the effective member slenderness for the Y-axis. The design aids of the LRFD Manual and the interpolation method suggested herein should assist in correctly applying the new Specifications.

## REFERENCES


1. *American Institute of Steel Construction, Inc.* Load and Resistance Factor Design (LRFD) Manual of Steel Construction *1st Ed., 1986, Chicago, Ill.*
2. *Galambos, Theodore V.* Structural Members and Frames *Prentice-Hall, 1968, Englewood Cliffs, N.J.*
3. *Yu, Wei-Wen* Cold Formed Steel Design *Wiley-Interscience, 1985.*