Fast Design of Beams with C_b Greater Than 1.0

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The allowable flexural stress for steel beams with less than full lateral support is given by AISC[1,2] Eqs. (1.5-6a), (1.5-6b), or (1.5-7). Each of these equations contain the term C_b, a coefficient dependent upon the moment gradient. In Part 2 of the AISC Manual[3,4] are curves for beams with allowable moment plotted for varying unsupported lengths. The curves are for $C_b = 1.0$. The following procedure affords a fast selection of a beam using the beam curves when the value of C_b is other than 1.0. A considerable saving of steel can result.

In each of the equations, there are two variables, l and C_b, for a particular beam. In Eq. (1.5-6a), for instance:

With $C_b = 1.0$:

$$F_b = \left[2/3 \frac{-F_y(l_e/r_T)^2}{1530(10)^3 C_{be}}\right] F_y$$

With $C_b > 1.0$:

$$F_b = \left[2/3 \frac{-F_y(l_x/r_T)^2}{1530(10)^3 C_{bx}}\right] F_y$$

where

l_e = effective length with $C_b = 1.0$, in.
l_x = length with C_b having a value other than 1.0, in.
C_{be} = C_b with value of 1.0
C_{bx} = C_b with a value greater than 1.0
r_T = radius of gyration as defined by AISC, in.
F_y = specified minimum yield stress of steel, ksi

At any particular level of stress (F_b is constant) the above two relationships can be equated and the effective length, l_e, determined for any value of C_b, or:

$$l_e = l_x/\sqrt{C_{bx}}$$

Similarly for Eq. (1.5-6b),

$$F_b = \frac{170 \times 10^3 C_b}{(l/r_T)^2}; \quad l_e = \frac{l_x}{\sqrt{C_{bx}}}$$

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and for Eq. (1.5-7),

$$F_b = \frac{12 \times 10^3 C_b}{ld/A_f}; \quad l_e = \frac{l_x}{C_{bx}}$$

where

d = beam depth, in.
A_f = area of the compression flange, in.^{2}

Since l_e is affected most by C_b in Eq. (1.5-7), the equation $l_e = l_x/C_{bx}$, in., or $L_e = L_x/C_{bx}$, ft, will be used to select the most economical trial section. It is important to recognize that the Specification[1,2] states that we are to use the *largest* value of F_b given by either Eq. (1.5-6a), (1.5-6b), *or* (1.5-7). As a matter of fact, all the equations give acceptable values and any one may be used. The largest value of F_b is used as a matter of economics; however, F_b may not exceed $0.60F_y$ when the unbraced length is greater than L_u. Equation (1.5-6b) applies when $F_b \leq F_y/3$ and, for economical reasons, should not be used for selecting a member.

The significance of the effective length, L_e, will be examined from two points of view. The first provides a practical application for use with the charts in the Manual[3,4] and follows directly. The second consists of additional theoretical considerations providing back up to the first and is contained in the Appendix following the examples.

DESIGN PROCEDURE

Step 1—Select trial section:

Using the given moment and effective length ($L_e = L_x/C_{bx}$), select a beam in the normal manner, as described in the Manual. This is a trial section since the beam curve is a composite of the three equations and the portion of the curve representing Eq. (1.5-7) may not be apparent (see Fig. 1).

Step 2—Check limiting stress:

Since the Specification limits the bending stress to $0.60F_y$, the initial choice of section should satisfy this requirement. This is done by observing the value of the allowable moment of the beam when $F_b = 0.60F_y$, indicated as the horizontal segment of the beam curve terminating at an open circle "**o**" which is L_u (see Fig. 1). Examples

1 and 2 illustrate this limit. If the trial section does not satisfy this requirement, try the next stronger beam indicated by the curves and repeat Step 2.

Step 3—Check allowable stress:

Find the allowable bending stress by Eq. (1.5-7), multiply by the section modulus, and compare the computed allowable moment with that required (see Example 1). For convenience in checking, the following equation may be used:

$$M = \frac{S_x F_b}{12} = \frac{S_x}{12}\frac{12000\ C_b}{ld/A_f},\ \text{kip-in.}$$

With $l = 12L$,

$$M = \frac{83.3\ S_x C_b}{Ld/A_f},\ \text{kip-ft}$$

If the moment computed by this equation is greater than the design moment, the section is adequate and no additional checking is necessary.

If insufficient, as in Example 3, try the next stronger beam indicated by the curves and repeat Steps 2 and 3, or follow Step 4.

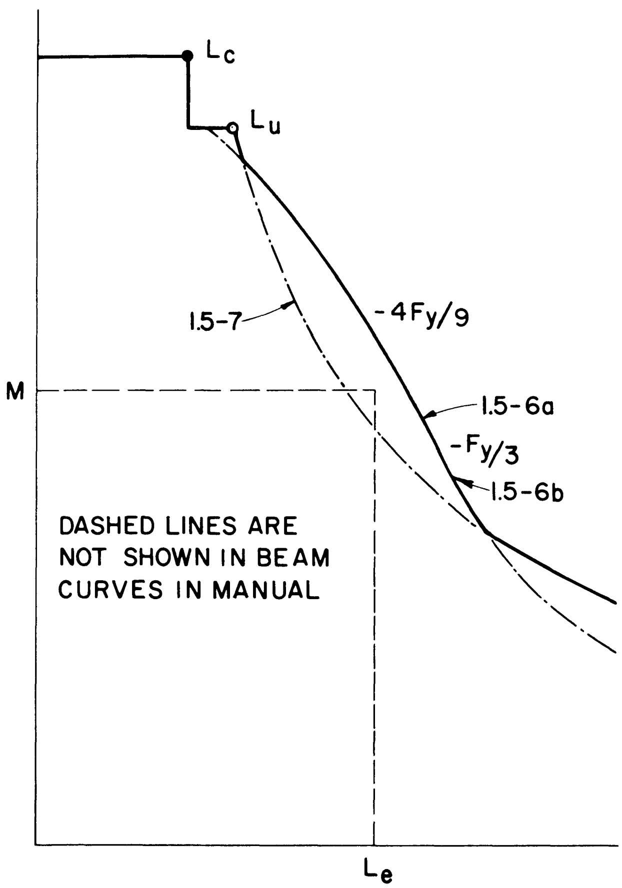

Fig. 1. Typical beam curve

Step 4—Investigate Eq. (1.5-6a):

The above check, Step 3, may indicate that the section is insufficient by Eq. (1.5-7); however, Eq. (1.5-6a) may prove satisfactory. This second check is easily and quickly done by determining the L_e value for Eq. (1.5-6a), $L_e = L_b/\sqrt{C_b}$, and with the design moment compare the result with the beam curves. If the section is satisfactory by inspection, it is unnecessary to compute F_b and M as in Step 3. This procedure is illustrated in Example 4. If this check indicates the trial section to be insufficient, choose the next larger section and repeat the procedure beginning at Step 2.

With a few exceptions, Step 4 will apply only when C_b is slightly larger than 1.0 and/or when the trial section is one of the lighter foot-weights in each nominal depth grouping. This condition is explained in greater detail in the Appendix, since for general values of C_b, F_b, and L there are no specific conditions which indicate when Eq. (1.5-6a) controls.

Finally, whenever Eq. (1.5-6a) controls and C_b is greater than one, the beam moment capacity can only be estimated from the charts using the effective length L_e. The exact beam moment capacity can be obtained only from Eq. (1.5-6a).

Example 1

Given: A framed girder with an unbraced segment of 30 ft and a C_b value of 2.0 is subjected to a maximum moment of 130 kip-ft. Using the beam charts and F_y = 50 ksi, design this segment of the beam.

Solution:

Step 1—Select trial section:

The effective length to be used in the chart is

$$L_e = 30/2.0 = 15.0\ \text{ft}$$

With M = 130 kip-ft and L_e = 15 ft, from curves select W14x48.

Step 2—Check limiting stress ($F_b \leq 0.60F_y$):

From chart read:

$$M(\text{at } L_u) = 176\ \text{kip-ft} > 130\ \text{kip-ft} \quad \textbf{o.k.}$$

Step 3—Check trial section:

Section properties of W14x48 are:

$$S_x = 70.3\ \text{in.}^3;\quad d/A_f = 2.89\ \text{in.}^{-1}$$

Allowable moment by modified Eq. (1.5-7) is

$$M = \frac{83.3\ C_b S_x}{Ld/A_f} = \frac{83.3(2.0)(70.3)}{30(2.89)}$$

$$= 135\ \text{kip-ft} > 130\ \text{kip-ft} \qquad \textbf{o.k.}$$

Use W14x48.

Example 2

Given: A framed girder with an unbraced segment of 20 ft and a C_b value of 2.1 is subjected to a maximum moment of 680 kip-ft. Using the beam charts and F_y = 36 ksi, design this segment of the beam.

Solution:

Step 1—Select trial section:

The effective length to be used in the chart is:

$$L_e = 20/2.1 = 9.5 \text{ ft}$$

With M = 680 kip-ft and L_e = 9.5 ft, from curves select W33x118.

Step 2—Check limiting stress ($F_b \leq 0.60F_y$)

From chart read:

$$M(\text{at } L_u) = 658 \text{ kip-ft} < 680 \text{ kip-ft} \quad \textbf{n.g.}$$

Select from curves the next larger section, W33x130.

Step 2—(Repeat):

From chart read:

$$M(\text{at } L_u) = 744 \text{ kip-ft} > 680 \text{ kip-ft} \quad \textbf{o.k.}$$

Step 3—Check trial section:

Section properties of W33x130 are:

$$S_x = 406 \text{ in.}^3; \quad d/A_f = 3.36 \text{ in.}^{-1}$$

Allowable moment by modified Eq. (1.5-7) is:

$$M = \frac{83.3\ C_b S_x}{Ld/A_f} = \frac{83.3(2.1)(406)}{20(3.36)}$$

$$= 1057 \text{ kip-ft} > 680 \text{ kip-ft} \qquad \textbf{o.k.}$$

Use W33x130.

Example 3

Given: A framed girder with an unbraced segment of 24 ft and a C_b value of 2.1 is subjected to a maximum moment of 375 kip-ft. Using the beam charts and F_y = 36 ksi, design this segment of the beam.

Solution:

Step 1—Select trial section:

The effective length to be used in the chart is:

$$L_e = 24/2.1 = 11.4 \text{ ft}$$

With M = 375 kip-ft and L_e = 11.4 ft, from curves select W27x84.

Step 2—Check limiting stress ($F_y \leq 0.60F_y$):

From chart read:

$$M(\text{at } L_u) = 391 \text{ kip-ft} > 375 \text{ kip-ft} \quad \textbf{o.k.}$$

Step 3—Check trial section:

Section properties of W27x84 are:

$$S_x = 213 \text{ in.}^3; \quad d/A_f = 4.19 \text{ in.}^{-1}$$

Allowable moment by modified Eq. (1.5-7) is:

$$M = \frac{83.3\ C_b S_x}{Ld/A_f} = \frac{83.3(2.1)(213)}{24(4.19)}$$

$$= 470.5 \text{ kip-ft} < 375 \text{ kip-ft} \qquad \textbf{n.g.}$$

This indicates that that portion of the curve used is for Eq. (1.5-6a). Also, this illustrates the need to check the trial section by Eq. (1.5-7). Since C_b is considerably greater than 1.0, choose a larger section (note the difference in this approach and the one used in Example 4). Try W24x94.

Step 2—(Repeat):

From chart read

$$M(\text{at } L_u) = 407 \text{ kip-ft} > 375 \text{ kip-ft} \quad \textbf{o.k.}$$

Step 3—(Repeat):

Section properties of W24x94 are:

$$S_x = 222 \text{ in.}^3; \quad d/A_f = 3.06 \text{ in.}^{-1}$$

Allowable moment by modified Eq. (1.5-7) is:

$$M = \frac{83.3\ C_b S_x}{Ld/A_f} = \frac{83.3(2.1)(222)}{24(3.06)}$$

$$= 528 \text{ kip-ft} > 375 \text{ kip-ft} \qquad \textbf{o.k.}$$

Use W24x94.

Example 4

Given: A framed girder with an unbraced segment of 15.5 ft and a C_b value of 1.1 is subjected to a maximum moment of 345 kip-ft. Using the beam charts and F_y = 36 ksi, design this segment of the beam.

Solution:

Step 1—Select trial section:

The effective length to be used in the chart is

$$L_e = \frac{15.5}{1.1} = 14.1 \text{ ft}$$

With M = 345 kip-ft and L_e = 14.1 ft, from the curves select W27x84.

Step 2—Check limiting stress ($F_b \leq 0.60F_y$):

From chart read:

$$M(\text{at } L_u) = 391 \text{ kip-ft} > 365 \text{ kip-ft} \quad \textbf{o.k.}$$

Step 3—Check trial section:

Section properties of W27x84 are:

$$S_x = 213 \text{ in.}^3; \quad d/A_f = 4.19 \text{ in.}^{-1}$$

Allowable moment by modified Eq. (1.5-7) is:

$$M = \frac{83.3\, C_b S_x}{Ld/A_f} = \frac{83.3(1.1)(213)}{15.5(4.19)}$$

$$= 301 \text{ kip-ft} < 345 \text{ kip-ft} \qquad \textbf{n.g.}$$

This indicates that the portion of the curve used is for Eq. (1.5-6a). Since the C_b value of 1.1 is near 1.0 and the W27x84 is one of the lighter 27-in. sections, it is likely that the section will suffice. Go to Step 4.

Step 4—Investigate Eq. (1.5-6a):

Use $L_e = 15.5/\sqrt{1.1} = 14.8$ ft.

With $M = 345$ kip-ft and $L_e = 14.8$ ft, from the curves it is seen that W27x84 is satisfactory. No further checking is required.

Use W27x84.

REFERENCES

1. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *Nov. 1, 1978, American Institute of Steel Construction, Inc., New York, N.Y.*
2. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *Feb. 12, 1969, American Institute of Steel Construction, Inc., New York, N.Y.*
3. Manual of Steel Construction *Seventh Edition, 1970 and 1973. American Institute of Steel Construction, Inc., New York, N.Y.*
4. Manual of Steel Construction *Eighth Edition, 1980. American Institute of Steel Construction, Inc., Chicago, Ill.*

APPENDIX

In the main body of the paper the authors have demonstrated a procedure which allows a quick adjustment of the beam unbraced length, when C_b is greater than one, to an effective length facilitating use of the AISC Manual design charts which are plotted for C_b equal to one. This Appendix is intended to provide additional material explaining the validity and scope of the procedure not required in general design use.

The most significant factor to evaluate is the magnitude of the influence that C_b has on Eqs. (1.5-6a), (1.5-6), and (1.5-7). This may be accomplished by determining the relationship between C_b and F_b. Since the unbraced length L is a common term when this occurs, it will be used as the equating variable.

The three equations become:

1. For Eq. (1.5-6a):

$$L^2 = \frac{1530000\, r_T{}^2}{144 F_y}\, [2/3 - F_b/F_y] C_b$$

2. For Eq. (1.5-6b):

$$L^2 = \frac{170000\, r_T{}^2}{144 F_b}\, C_b$$

3. For Eq. (1.5-7):

$$L = \frac{1000}{F_b d/A_f}\, C_b$$

The condition for most economical design is determined by the first and third equation. By squaring the third and equating to the first, the following relationship for C_b is obtained:

$$C_b = \frac{1.53}{144}\, [d/A_f\, r_T]^2\, [2/3 - F_b/F_y]\, F_b{}^2/F_y$$

This is the general equation for the intersection of Eqs. (1.5-6a) and (1.5-7).

It may be noted that, for any particular shape and grade of steel, C_b is influenced only by the magnitude of the stress F_b. The point of maximum effect can be found by taking the first derivative of C_b with respect to F_b and setting it equal to zero. There are two possible solutions. The first, when $F_b = 0$, is obviously a trivia solution, and the second occurs when $F_b = 4F_y/9$. The second derivative is always positive, verifying that this is a maximum condition. The maximum value of C_b is obtained by substituting $F_b = 4F_y/9$ into the equation for C_b, or:

$$C_b = \frac{0.34}{729}\, [d/A_f\, r_T]^2\, F_y$$

At this value of C_b, curves representing Eqs. (1.5-6a) and (1.5-7) are tangent. For values of C_b greater than this, Eq. (1.5-7) will govern.

Of less interest, from a design point of view, is when the bending stress is less than $F_y/3$ or when the bending stress is given by either Eq. (1.5-6b) or Eq. (1.5-7). Again eliminating the unbraced length L by squaring equation 3 and equating it to equation 2 will provide the following relationship:

$$C_b = \frac{0.170}{144}\, [d/A_f\, r_T]^2\, F_b$$

The first derivative of C_b with respect to F_b is a positive constant, indicating that it is a maximum value at each stress level throughout the applicable range of F_b ($F_y/3$ to 0).

Once the value of C_b is determined at any desired stress level, the unbraced length L can be determined by substituting into the appropriate equation (1, 2, or 3) given earlier. Knowing these two values is of theoretical interest, but is of little practical value in a design situation.

In theory, the four variables C_b, L, M, and F_y give virtually unlimited combinations, making it impractical to develop specific rules governing the effect of C_b as a design parameter. The AISC Manual charts provide a ready solution to situations involving the latter three, while the main body of this paper demonstrates a technique for handling a variable C_b value. The equations that are derived for C_b in this Appendix provide a theoretical and general solution which permits evaluation of moment (or stress) levels at

Table 1. Distribution of Equation Control with C_b = 1.0

Equations	F_y = 36 ksi		F_y = 50 ksi	
	W-shape	M-shape	W-shape	M-shape
$F_b = 0.60F_y$				
Eq. (1.5-6a)	9	3	27	4
Eq. (1.5-7)	136	5	118	4
$F_b = 0.45F_y$				
Eq. (1.5-6a)	46	5	81	5
Eq. (1.5-7)	99	3	64	3
$F_b = F_y/3$				
Eq. (1.5-6a)	34	4	65	5
Eq. (1.5-7)	111	4	80	3
Total Beam Shapes	145	8	145	8

varying values of C_b for each of the 153 shapes that are considered to be beams (145 W and eight M shapes). These comparisons are presented in two tables, with each table having results for both F_y = 36 ksi and F_y = 50 ksi steel.

Table 1 represents the relative influence of Eqs. (1.5-6a) and (1.5-7) at three key stress levels: $0.60F_y$, $0.45F_y$ and $F_y/3$ for a constant value (1.0) of C_b. Equation (1.5-7) obviously dominates a greater part of most beams for the useful design range. Equation (1.5-6a) only dominates at the $0.45F_y$ level for F_y = 50 ksi material. Equation (1.5-6a) obviously has a greater effect on beams made with steel having F_y = 50 ksi.

Table 2 indicates the relative effect of increasing C_b from 1.0 to the upper limit of 2.30. The tabular values indicate which equation controls (or both) throughout the whole range of stress $0.60F_y$ to $F_y/3$ that is of interest. It is apparent that in going from C_b = 1.0 to C_b = 1.25 the influence of Eq. (1.5-6a) is significantly reduced. It may not appear as obvious as stated, but recall that, when considering Eq. (1.5-6a) and computing the effective length, the square root is involved. The following paragraph will clarify this point.

The beams that are most affected by Eq. (1.5-6a) are those that are nominally the lighter footweights in each nominal depth grouping. These shapes, however, are

Table 2. Distribution of Equation Control with Varying C_b

	F_y = 36 ksi		F_y = 50 ksi	
	W-shape	M-shape	W-shape	M-shape
C_b = 1.0				
Eq. (1.5-6a)	9	3	27	4
Eq. (1.5-7)	99	3	64	3
Mixed	37	2	54	1
C_b = 1.25				
Eq. (1.5-6a)	1	2	13	3
Eq. (1.5-7)	115	4	84	3
Mixed	29	2	48	2
C_b = 1.50				
Eq. (1.5-6a)	0	0	9	3
Eq. (1.5-7)	126	4	104	3
Mixed	19	4	32	2
C_b = 1.75				
Eq. (1.5-6a)	0	0	1	2
Eq. (1.5-7)	133	5	115	4
Mixed	12	3	29	2
C_b = 2.30				
Eq. (1.5-6a)	0	0	0	0
Eq. (1.5-7)	144	6	132	5
Mixed	1	2	13	3
Total Beam Shapes	145	8	145	8

generally considered to be the most economical by weight, especially at stress levels near $0.60F_y$. At this higher stress, Eq. (1.5-6a) controls by relatively small margins and, therefore, with a C_b only somewhat larger than one, the control in most cases shifts to Eq. (1.5-7). However, for a particular design length and C_b, the difference between the two effective length values (L_x/C_{bx} vs. $L_x/\sqrt{C_{bx}}$) may be just enough to make a slightly lighter section satisfactory for the design moment.

The two tables and associated discussion illustrate the influence of C_b on the design equations and, therefore, on the beam charts presented in the AISC Manual. The Appendix reinforces the contention of the authors that the design procedure recommended in the main body of the text can be used to obtain accurate design results from the beam charts when C_b is not equal to unity.