Ponding of Two-Way Roof Systems

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SEVERAL PAPERS HAVE been published[1, 2] analyzing roof beams subject to ponding. However, the scope of these papers has been limited to the one-directional action of beams. That is, the flexural members are considered supported by unyielding knife edges with no consideration given to surface deflection transverse to the beam span under study. The effect of interaction between members in a roof framing system can be considerable and should not be neglected. The present AISC Specification[3] is cognizant of the ponding problem. Chinn[1] points out that the Specification provision is arbitrary in nature and could be overly conservative. It is interesting to note here that in all the cases of collapse attributed to the ponding phenomenon that the author has reviewed, the members involved did violate the present Specification provision. However, the provision may actually be unconservative for very large spans.

The purpose of this paper is to analyze the ponding of a roof system, accounting for the interaction of members and to develop a design aid suitable for office use. A restriction imposed on the analysis is that the structural system must consist essentially of two-way framing (i.e., main girders or primary members and secondary sub-members spanning perpendicular to the girders) with the deck contributing negligible deflection to the system. However, if in the absence of other sub-members the deck spans a substantial distance between main members, it should be treated as the secondary system.

ANALYSIS

Ponding may be defined as a situation caused by the flexible nature of a structural assembly, created when a flat roof retains rain water that causes deflection of the roof system, which in turn increases its volumetric capacity. This process is iterative in nature and continues until convergence, which is termed the *equilibrium position;* or, if the system is divergent, until collapse occurs.

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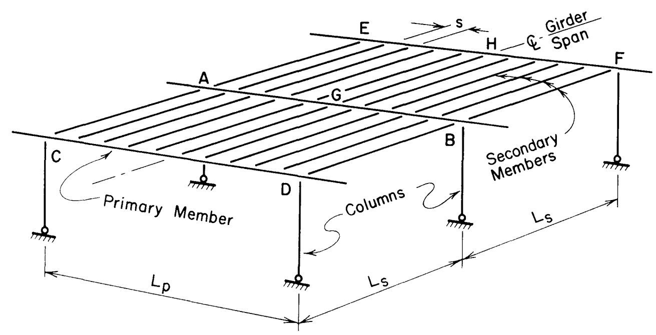

Figure 1

In the case of simply-supported members, the deflection due to dead and live loads on the structure, or accidental negative camber, can initiate ponding. In the case of continuous members having identical stiffness in adjacent spans, ponding action can be initiated by small accidental differences in the levels of these spans before loading. In such circumstances the higher level spans unload, causing accelerated deflections in the adjacent lower spans, and the ponding effect will be similar to that which occurs in a simple span. In either case, it is evident that, to prevent collapse, the equilibrium position must be reached before the maximum flexural stresses in the member reach yield point.

This phenomenon is by no means unique to steel construction. However, due to steel's high strength-to-weight ratio, as compared with other building materials, the problem may be more acute. It is further accented by the introduction of high strength steels and the popularity of plastic design. Both these factors tend to produce designs of shallower depths and therefore more flexible systems.

Figure 1 shows the system under investigation. The primary member under discussion is interior girder **AB.** The secondary member considered is the beam **GH**, which frames into the girder at its mid-span. This, of course, is the critical secondary member. Figure 2 shows the deflected position of members **AB** and **GH** at the equilibrium position previously defined. Note that the supporting primary members at both ends are assumed to be proportional in stiffness and loading so as to have the same deflection Δ. Figure 3 illustrates the loads imposed on each member by the ponded rain. In each case, the deflected elastic curve for the members is assumed as a half sine wave. The ponding loads on the primary member, due to deflection of both the primary and secondary members, are assumed to vary as the ordinates of a half sine wave. The reactions of secondary member on the primary member are assumed continuously distributed rather than as discreet concentrated loads. The ponding loads on the secondary member, however, take the shape of a sine curve due to the deflection of the secondary member plus a uniform load due to the deflection of the primary member.

Considering the primary member, the following observations can be drawn from the load diagram:

$$W_p = \frac{2}{\pi}\,\gamma L_s(\Delta_o + \Delta_w)L_p$$

and

$$W_s = \frac{(2)^2}{(\pi)^2}\,\gamma L_s(\delta_w - \delta_{1w})L_p + \frac{2}{\pi}\,\gamma L_s(\delta_o + \delta_{1w})L_p$$

The bending moment at mid-span of the primary member is:

$$M = \frac{\gamma L_s L_p^2(\Delta_o + \Delta_w)}{\pi^2} + \frac{2\gamma L_s L_p^2(\delta_w - \delta_{1w})}{\pi^3} + \frac{2\gamma L_s L_p^2(\delta_o + \delta_{1w})}{8\pi}$$

The mid-span deflection due to the ponding rain can be calculated by the conjugate beam method as:

$$\Delta_w = \frac{\gamma L_s L_p^4}{\pi^4 E I_p}\left[(\Delta_o + \Delta_w) + \frac{2}{\pi}(\delta_w - \delta_{1w}) + \frac{\pi}{4}(\delta_o + \delta_{1w})\right]$$

and letting

$$\frac{\gamma L_s L_p^4}{\pi^4 E I_p} = C_p$$

$$\Delta_w = C_p\Delta_o + C_p\Delta_w + \frac{2}{\pi}C_p\delta_w - \frac{2}{\pi}C_p\delta_{1w} + \frac{\pi}{4}C_p\delta_o + \frac{\pi}{4}C_p\delta_{1w}$$

Solving for Δ_w:

$$\Delta_w = \alpha_p\left(\Delta_o + \frac{\pi}{4}\delta_o\right) + \alpha_p\left(\frac{2}{\pi}\delta_w + \frac{\pi}{4}\delta_{1w} - \frac{2}{\pi}\delta_{1w}\right) \qquad (1)$$

where

$$\alpha_p = \frac{C_p}{1 - C_p}$$

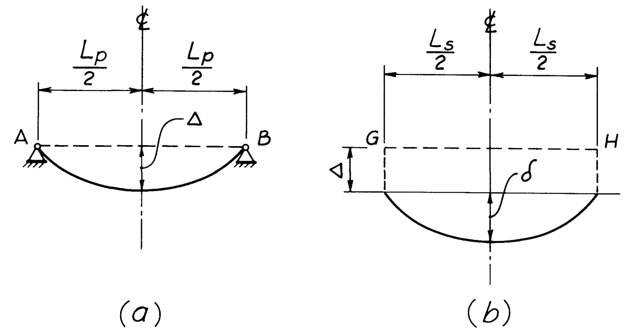

Figure 2

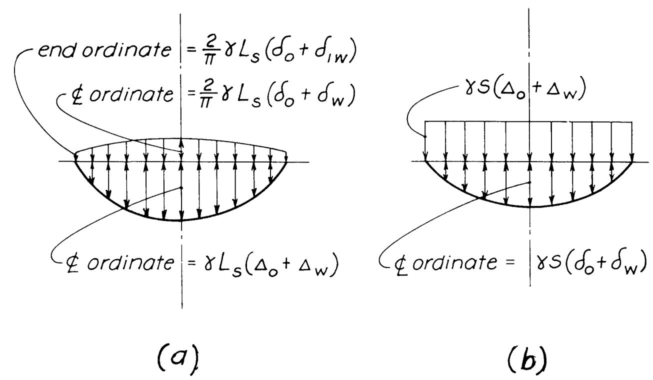

Figure 3

By similar deduction the mid-span deflection of the critical secondary member due to ponded rain can be expressed as:

$$\delta_w = \alpha_s\left(\delta_o + \frac{\pi^2}{8}\Delta_o\right) + \frac{\pi^2}{8}\alpha_s\Delta_w \qquad (2)$$

where

$$\alpha_s = \frac{C_s}{1 - C_s}$$

Also,

$$\delta_{lw} = \alpha_s\delta_o \qquad (2a)$$

By combining Equations (1), (2) and (2a),

$$\Delta_w = \frac{\alpha_p\left(\Delta_o + \frac{\pi}{4}\delta_o\right) + \frac{\pi}{4}\alpha_p\alpha_s(\delta_o + \Delta_o)}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \qquad (3)$$

and

$$\delta_w = \alpha_s\left(\delta_o + \frac{\pi^2}{8}\Delta_o\right) + \frac{\frac{\pi^2}{8}\alpha_p\alpha_s\left(\frac{\pi}{4}\delta_o + \Delta_o + \frac{\pi}{4}\alpha_s\delta_o - \frac{2}{\pi}\alpha_s\delta_o\right)}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \qquad (4)$$

It is evident that as the quantity $(\pi/4\alpha_p\alpha_s)$ approaches unity (or $\alpha_p\alpha_s$ approaches $4/\pi$) the ponding deflections Δ_w and δ_w will approach infinity. Therefore one could conclude that if the parameter $(\alpha_p\alpha_s)$ were less than $4/\pi$, Δ_w and δ_w would have a finite limit, termed the equilibrium position. However, another factor must be considered. The analysis so far has assumed elasticity on the part of the structural members. If stresses in any member of the system were to exceed the elastic limit before reaching the point of theoretical equilibrium, a runaway condition could ensue with respect to ponding. Therefore, in addition to complying with the stipulation concerning the parameter $(\alpha_p\alpha_s)$, the design should also ensure that the maximum flexural stress of any member be maintained below the yield point of the material.

In order to develop a criterion that is suitable for design office use, the following substitutions were made:

$$\rho = \frac{\delta_o}{\Delta_o} = \frac{C_s}{C_p} \tag{5}$$

This is self-evident from the observation that both the deflection and the flexibility constant for a beam are directly proportional to L^4/EI.

Substituting Equation (5) into Equations (3) and (4) yields:

$$\Delta_w = \frac{\alpha_p\Delta_o\left[1 + \frac{\pi}{4}\alpha_s + \frac{\pi}{4}\rho(1 + \alpha_s)\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{6}$$

and

$$\delta_w = \frac{\alpha_s\delta_o\left[1 + \frac{\pi^3}{32}\alpha_p + \frac{\pi^2}{8\rho}(1 + \alpha_p) + 0.185\alpha_s\alpha_p\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{6a}$$

Noting that deflection is proportional to stress,

$$\frac{f_w}{f_o} = \frac{\Delta_w}{\Delta_o}$$

and placing as a limitation on the stress induced by ponding,

$$f_w \leq \frac{F_y}{\text{F.S.}} - f_o$$

then

$$\Delta_w \leq \left[\frac{\frac{F_y}{\text{F.S.}} - f_o}{f_o}\right]\Delta_o = \left[\frac{1}{\text{F.S.}}\frac{F_y}{f_o} - 1\right]\Delta_o$$

A factor of safety (F.S.) of 1.25 against yielding is suggested for design office use.

By substituting into Equation (6):

$$\left[\frac{1}{\text{F.S.}}\frac{F_y}{f_o} - 1\right]_p \geq \frac{\alpha_p\left[1 + \frac{\pi}{4}\alpha_s + \frac{\pi}{4}\rho(1 + \alpha_s)\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{7}$$

Similarly for the secondary member:

$$\left[\frac{1}{\text{F.S.}}\frac{F_y}{f_o} - 1\right]_s \geq \frac{\alpha_s\left[1 + \frac{\pi^3}{32}\alpha_p + \frac{\pi^2}{8\rho}(1 + \alpha_p) + 0.185\alpha_s\alpha_p\right]}{1 - \frac{\pi}{4}\alpha_p\alpha_s} \tag{8}$$

Equations (7) and (8) afford a relatively easy method for checking the ponding stability of a two-way roof system.

Figures 4 and 5 are design aids which plot the relationship between the parameters C_p and U_p, and C_s and U_s, for Equations (7) and (8), respectively. The term U represents the left side of these equations. To use these charts, tentatively select member sizes, as usual, on the basis of the design loading. Then from the known characteristics, compute the values of U_p, U_s, C_p, and C_s. To check the primary member, enter Fig. 4 at the left with the value of U_p. Proceed to the right to the intersection with the curve representing the flexibility constant of the secondary member (C_s). Descend to the abscissa and read the maximum flexibility constant of the primary member to satisfy Equation (7). If the actual C_p is larger than this value, it indicates that the system is potentially unstable and the design should be revised. A similar procedure can be used, employing Fig. 5, to check the secondary member.

As a further simplification, the parameters C_p and C_s can be computed from the following expressions in which γ, π and E have been replaced by their numerical equivalent:

$$C_p = \frac{l_s l_p^4}{32 \times 10^4 I_p}$$

$$C_s = \frac{s l_s^4}{32 \times 10^4 I_s}$$

where l_s, l_p and s are in feet and I_s and I_p are in in.^{4}

It is important to note that the span involved in this analysis is the distance between *support points* (i.e., column spacing) and *not* between splice points. The flexibility limitation obtained by this analysis should be applied to simple and continuous spans alike. This is because unequal deflections in adjacent continuous spans can result in a greater accumulation in one

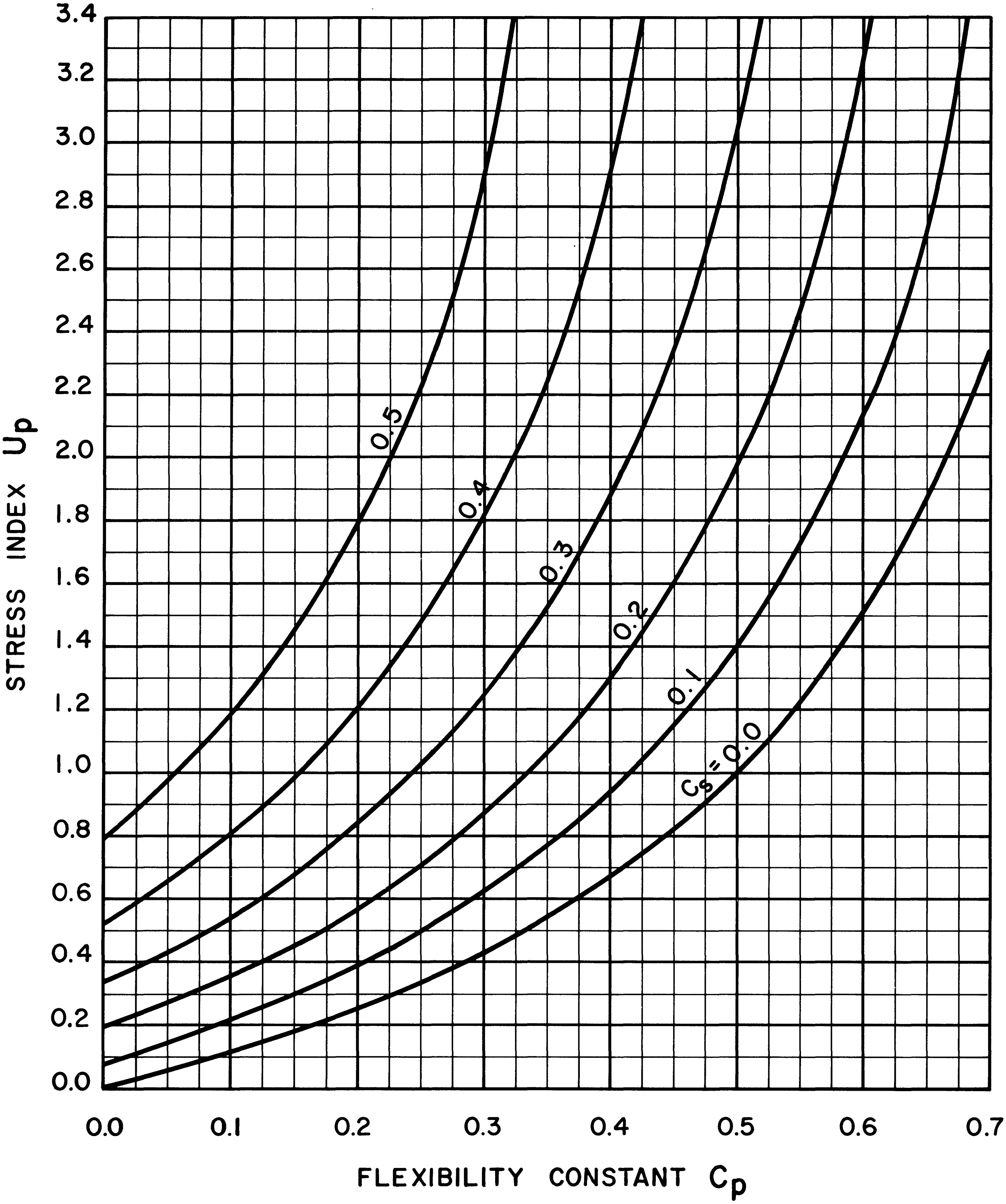
STRESS INDEX U_p
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
0.4
0.3
0.2
0.1
C_s=0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
FLEXIBILITY CONSTANT C_p

Figure 4

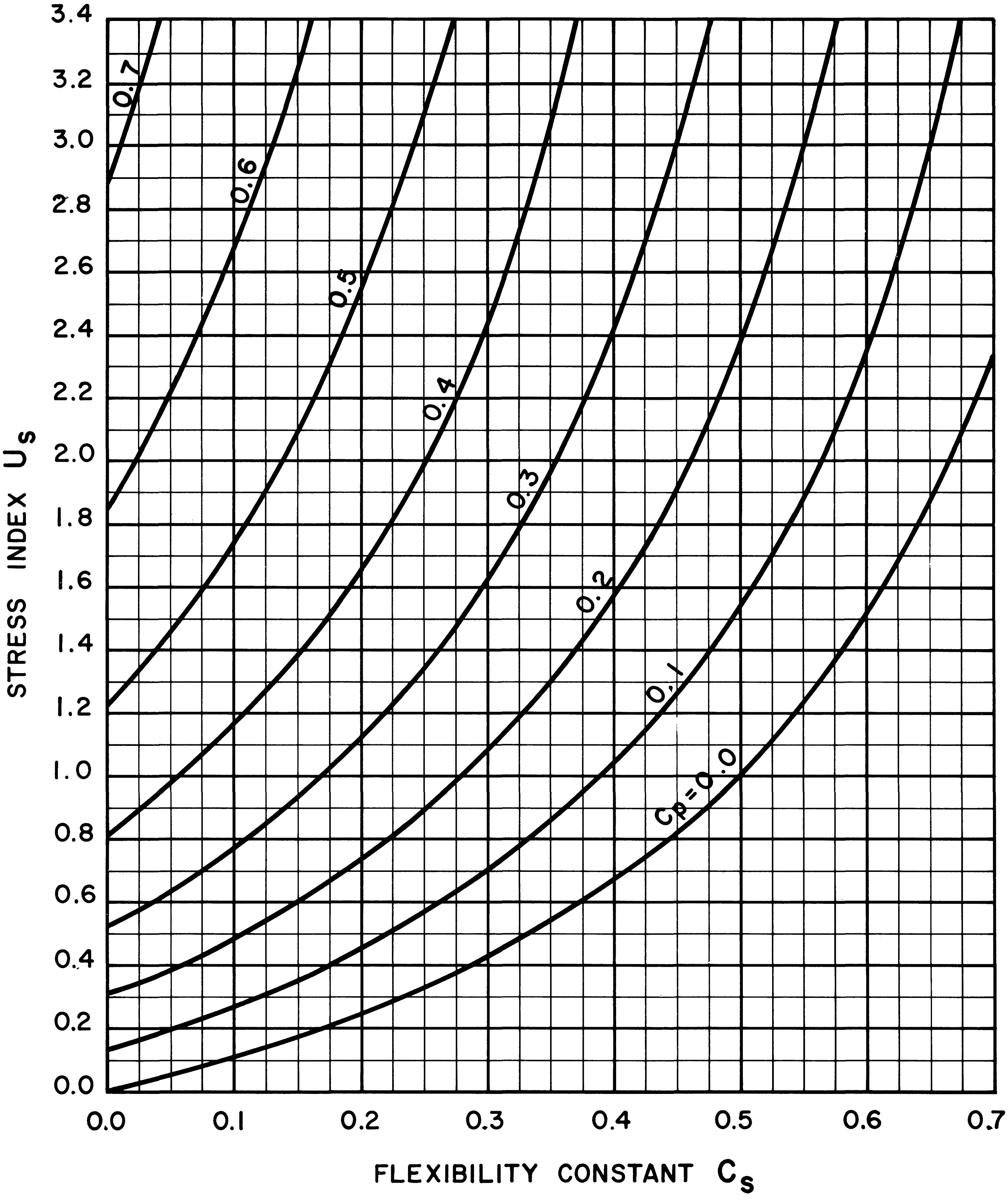
STRESS INDEX U_S
FLEXIBILITY CONSTANT C_S
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
C_P=0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Figure 5

span which, in turn, tends to unload the adjacent spans. This reduces the restraint at the ends of the ponded span, increases the deflection due to ponding in that span, and ultimately causes the continuous beam to act as though it were simply supported.

Another consideration to bear in mind relates to the calculation of U_p and U_s. In these terms, the value of f_o is the stress associated with all dead and live loads which are likely to be on the roof at the time that ponding commences. This would include any anticipated water load due to reservoir action of curbs and similar architectural features.

DISCUSSION AND CONCLUSIONS

The most desirable method to preclude the effect of ponding is to provide sufficient slope to the roof surface along with adequate drainage facilities to prevent the accumulation of rain water in the first instance. For the slope to be sufficient, the upward pitch provided to a roof surface must exceed the downward slope of the beam's elastic curve at or near the support point, caused by all gravity loads. Experience as well as theoretical considerations indicate that a pitch of ⅛-in. per ft will suffice for this purpose under normal conditions of *free* drainage. However, the hydraulics of roof drainage is actually a very complex problem which requires careful study and is not included in the scope of this paper. In many cases, it is not feasible to drain a roof area without incurring the risk of some accumulation.

This analysis presented has been tested against several cases of roof collapse attributed to ponding. In each case the instability would have been predicated by a significant margin.

REFERENCES

1. *Chinn, James* Failure of Simply-Supported Flat Roofs by Ponding of Rain, *AISC Engineering Journal, April, 1965.*
2. *Haussler, R. W.* Roof Deflection Caused by Rainwater Pools, *Civil Engineering, October, 1962.*
3. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, *American Institute of Steel Construction, New York, N. Y., April 17, 1963.*

NOMENCLATURE

C_p Flexibility constant of primary member $= \dfrac{\gamma L_s^4 L_p}{\pi^4 EI_p}$

C_s Flexibility constant of secondary member $= \dfrac{\gamma S L_s^4}{\pi^4 EI_s}$

E Modulus of elasticity (psi)

F.S. Factor of safety

F_y Yield point of member considered (psi)

I_p Moment of inertia, primary member (in.^{4})

I_s Moment of inertia, secondary member (in.^{4})

L_p Span of primary member (in.)

L_s Span of secondary member (in.)

M_p Bending moment in primary member, at mid-span (lb-in.)

S Spacing of secondary members (in.)

U_p Stress index of primary member $= \left[\dfrac{1}{\text{F.S.}}\dfrac{F_y}{f_o} - 1\right]$

U_s Stress index of secondary member $= \left[\dfrac{1}{\text{F.S.}}\dfrac{F_y}{f_o} - 1\right]$

W_p Total water load, on primary member, due to volumetric configuration of primary members (lb)

W_s Total water load, on primary member, due to volumetric configuration of secondary members (lb)

d_p Depth of primary member (in.)

d_s Depth of secondary member (in.)

f_o Extreme fiber flexural stress in a member at onset of ponding (psi)

f_p Extreme fiber flexural stress in primary member (psi)

f_s Extreme fiber flexural stress in secondary member (psi)

f_w Extreme fiber flexural stress in a member due to ponding (psi)

l_p Span of primary member (ft)

l_s Span of secondary member (ft)

s Spacing of secondary members (ft)

α_p Flexibility parameter of primary member $= \dfrac{C_p}{1 - C_p}$

α_s Flexibility parameter of secondary member $= \dfrac{C_s}{1 - C_s}$

γ Unit weight of water (lb/in.^{3})

Δ_o Deflection in primary member at onset of ponding (in.)

Δ_w Deflection in primary member due to ponding effect (in.)

δ_o Deflection in secondary member at onset of ponding (in.)

δ_w Deflection in secondary member at center line of primary member due to ponding effect (in.)

δ_{1w} Deflection in secondary member at end of primary member due to ponding effect (in.)

ρ Initial deflection ratio $= \delta_o/\Delta_o$

APPENDIX

Example 1—An industrial building has been designed with 50 ft-0 in. x 38 ft-0 in. bays. The structural members of the flat roof have been proportioned by conventional analysis. Check the design for ponding.

Given: Girders (50 ft-0 in. span): 21WF55

$I_p = 1140.7$ in.^{4}

$f_b = 23.0$ ksi

Secondary members

(38 ft-0 in. span): 20H7 open web joist

$I_s \cong 160$ in.^{4}

$f_b = 28.5$ ksi

Joist spacing: 6 ft-3 in. o.c.

Live load: 20 psf

Dead load: 15 psf

Solution:

Assume that one-quarter of L.L. is on roof at outset of ponding.

$$f_o \text{ (girder)} = 23 \times \left(\frac{15+5}{35}\right) = 13.2 \text{ ksi}$$

$$f_o \text{ (joist)} = 28.5 \times \left(\frac{15+5}{35}\right) = 16.3 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{36}{13.2}\right) - 1 = 1.18$$

$$U_s = \left(\frac{1}{1.25} \times \frac{50}{16.3}\right) - 1 = 1.45$$

$$C_p = \frac{38 \times 50^4}{32 \times 10^4 \times 1140.7} = 0.65$$

$$C_s = \frac{6.25 \times 38^4}{32 \times 10^4 \times 160} = 0.26$$

(a) Check girder:

From Fig. 4, with $U_p = 1.18$ and $C_s = 0.26$:

Allowable $C_p = 0.32 < 0.65$ **N.G.**

(b) Check joist:

From Fig. 5, with $U_s = 1.45$ and $C_p = 0.65$:

Allowable $C_s < 0 < 0.31$ **N.G.**

Therefore, neither the girders nor joists are suitable on the basis of ponding analysis, even though both are adequately proportioned on the basis of static load strength. Note that neither the girder nor the joist would have met the requirements of AISC Specification Sect. 1.13, which requires a minimum depth of 24 in. for both primary and secondary members in this case.

Example 2—Redesign the roof system of Example 1 to be adequate for ponding.

Solution:

Try: 24WF68 girder: $I_p = 1814.5$ in.^{4}

$f_b = 16.4$ ksi

24J8 joist: $I_s \cong 270$ in.^{4}

$f_b = 19.2$ ksi

$$f_o \text{ (girder)} = 16.4 \times \frac{20}{35} = 9.4 \text{ ksi}$$

$$f_o \text{ (joist)} = 19.2 \times \frac{20}{35} = 11.0 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{36}{9.4}\right) - 1 = 2.10$$

$$U_s = \left(\frac{1}{1.25} \times \frac{36}{11.0}\right) - 1 = 1.60$$

$$C_p = 0.65 \times \frac{1140.7}{1814.5} = 0.41$$

$$C_s = 0.26 \times \frac{160}{270} = 0.15$$

(a) Check girder:

From Fig. 4, with $U_p = 2.10$ and $C_s = 0.15$:

Allowable $C_p = 0.55 > 0.43$ **O.K.**

(b) Check joist:

From Fig. 5, with $U_s = 1.60$ and $C_p = 0.41$:

Allowable $C_s = 0.18 > 0.15$ **O.K.**

Use 24WF68 girders and 24J8 joists.

Example 3—A flat roof with bay spacings of 34 ft-0 in. x 24 ft-0 in. has been preliminarily proportioned with 14B26 girders (34 ft-0 in. span) and 12H4 joists (24 ft-0 in. span) on 6 ft-0 in. centers. Check the design for ponding.

Given: 14B26: $I_p = 242.6$ in.^{4}

$f_b = 21.0$ ksi

12H4: $I_s \cong 35$ in.^{4}

$f_b = 30.0$ ksi

Solution:

Assume that one-fifth L.L. is on roof at onset of ponding.

$$f_o \text{ (girder)} = 21.0 \times \frac{19}{35} = 11.4 \text{ ksi}$$

$$f_o \text{ (joist)} = 30.0 \times \frac{19}{35} = 16.0 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{36}{11.4}\right) - 1 = 1.50$$

$$U_s = \left(\frac{1}{1.25} \times \frac{50}{16.3}\right) - 1 = 1.50$$

$$C_p = \frac{24 \times 34^4}{32 \times 10^4 \times 242.6} = 0.41$$

$$C_s = \frac{6 \times 24^4}{32 \times 10^4 \times 35} = 0.18$$

(a) Check girder:

From Fig. 4, allowable $C_p = 0.45 > 0.41$ **O.K.**

(b) Check joist:
From Fig. 5, allowable $C_s = 0.18 \geq 0.18$ **O.K.**

Preliminary members are O.K. for ponding.

Note that although these members are adequate for ponding according to the analytic method in this paper, they could not be used for a design under the AISC Specification. Section 1.13 requires a minimum depth of 14.5 for these members, so that 16-in. deep girders and joists would actually have to be provided.

Example 4—An industrial building consists of a flat roof system supported directly on masonry walls. Open web steel joists, 24**LH**06, span a 40 ft-0 in. clear opening between walls, and 3-in. deep, 20 ga. steel deck spans 14 ft-0 in. c. to c. of joists. Check the design for ponding.

Given: 24**LH**06: $I \cong 400$ in.^{4}
$f_b = 28.8$ ksi
3-in. deck: $I = 0.83$ in.^{4}/ft
$f_b = 18.2$ ksi
Live load: 20 psf
Dead load: 10 psf

Solution: In this example it is obvious that the masonry walls act as unyielding supports, the joists act as primary flexural members, and the steel deck acts as the secondary flexural members.

Assume that one-half the live load (10 psf) is on the roof at the onset of ponding.

Check joist:

$$f_0 = 28.8 \times \frac{10+10}{30} = 19.2 \text{ ksi}$$

$$U_p = \left(\frac{1}{1.25} \times \frac{50}{19.2} - 1\right) = 1.08$$

$$C_p = \frac{14 \times 40^4}{32 \times 10^4 \times 400} = 0.28$$

From Fig. 4, allowable $C_p = 0.38 > 0.28$ **O.K.**

Check deck:

$$f_0 = 18.2 \times \frac{10+10}{30} = 12.1 \text{ ksi}$$

$$U_s = \left(\frac{1}{1.25} \times \frac{33}{12.1} - 1\right) = 1.18$$

$$C_s = \frac{1 \times 14^4}{32 \times 10^4 \times 0.83} = 0.14$$

From Fig. 5, allowable $C_s = 0.19 > 0.14$ **O.K.**

Therefore, the design is satisfactory for ponding.