Plastic Design of Eccentrically Loaded Fasteners

A. L. ABOLITZ

Bolts, rivets, and other fasteners in structural connections are often used in single or multiple shear. If the force incident upon a group of such fasteners is eccentric, the fastener group is subjected to a moment in addition to the force. The capacity of the group to transmit force is in general reduced if a moment must be transmitted at the same time. For each group of fasteners and for each direction of force a relationship exists between the values of the force F and of the moment M that may be applied simultaneously. This relationship may be termed *interaction*, and is usually expressed in formulas, curves or tables. In many published tables the eccentricity E, rather than the moment M, is used, but by means of the relationship $M = FE$ these tables can be easily converted to the type which gives M versus F.

A knowledge of the interaction is indispensable for the design or checking of a group of fasteners under an eccentric load.

A fastener group is sometimes loaded by a pure force F_0, without any moment. The capacity of a group for pure force F_0 is simply the sum of the capacities of the individual fasteners. A group may also be subjected to a pure moment M_0, unaccompanied by force; an example is furnished by the bolts in a flanged connection transmitting power in a rotating shaft. The capacity of a group for pure moment M_0 is the sum of the contributions of the individual fasteners; this individual contribution is the force exerted by the fastener times the distance to the center of rotation.

In this paper interaction curves and formulas are given not in terms of M and F, but in terms of the dimensionless fractions m and f, where $m = M/M_0$ and $f = F/F_0$. Also, for the present purpose, the faying area of a fastener group is considered to extend to a point located one-half the fastener spacing beyond the extreme fastener, and the overall dimension or diagonal D refers to this faying area. Thus D for a line of bolts* 3 in. on centers with 5 bolts in the line is 15 in. (not 12 in.). Similarly, D for a group of 2 such lines 4 in. on centers is the diagonal of an 8 x 15-in. rectangle, which is 17 in.

The importance of the problem of fastener groups under eccentric loading has long been recognized. The traditional method of calculating interaction was based on the elastic theory and assumed that the resisting force of each fastener against rotation about a center was proportional to its distance from the center, so that the fastener at the greatest distance from this center would exert its full resistance, while the nearer fasteners would exert only a fraction of their full resistance. This elastic method underestimates the capacity of fastener groups subject to sizeable moments.

More recently this fact has been recognized, and on the basis of tests (described by Mr. T. R. Higgins in *Engineering News Record*, May 21, 1964) an empirical method employing "effective" eccentricities has been evolved and incorporated in the current 6th Edition of the AISC Manual. This AISC Manual method gives results in better agreement with the actual strength of fastener groups over a considerable range, and is therefore a great improvement over the elastic method. Nevertheless it suffers from several shortcomings: It applies only to one direction of force; it is not easily adaptable to fastener groups not tabulated in the Manual; it is too conservative for large eccentricities and too liberal for small eccentricities; and its empirical nature and lack of a rational basis is intellectually unsatisfying.

A third method, set forth in this paper, involves the use of the plastic theory. This theory postulates that under heavy stress each fastener will exert its full resistance, irrespective of its location. On this basis the interaction for a given fastener group can be calculated. In Fig. 1 the results of the tests reported by Mr. Higgins are plotted together with the relevant interaction curves found by the plastic method, and it will be seen that the differences are well within a reasonable allowance for experimental scatter. The plastic method appears to agree with experimental evidence and to be largely free of the shortcomings of the other two methods.

Interaction formulas and curves based on the plastic method have been worked out in "Mathematical Deri-

A. L. Abolitz is Structural Engineer, Mueser-Rutledge-Wentworth-Johnston, New York, N. Y.

* *Wherever the word "bolts" appears in this paper it should be understood to refer to any type of fastener.*

vations," which follows. These have been plotted for some regular bolt layouts in Figs. 2 through 6, and summarized in Table I. Examples illustrate the application of these formulas and curves.

The assumptions underlying the plastic method, as well as the resulting charts, are valid for any structural elements uniformly resisting a "rigid body" displacement; for example, they may be used in analyzing a group of piles under an eccentric wind or earthquake load. The principal limitation on the plastic method hinges on the attainment of the full plastic capacity. If this is questionable because of dynamic effects and fatigue, embrittlement due to extremely cold temperatures, or other causes, the plastic method should not be used pending further research.

Groups of fasteners in tension, or in tension combined with shear, are in general suitable for plastic analysis. The formulas and charts given here must, however, be modified for use in such cases.

The formulas and charts apply to welds as well as to fasteners. However, full plastic capacity must be assured in all cases. It is hoped that future experimental and theoretical research and practical experience with the plastic method will result in important refinements and a widened scope of application.

MATHEMATICAL DERIVATIONS

Nomenclature

A Faying area of fastener group, sq in.
B Load carrying capacity of one fastener, kips
B_1 B/b, kips/in.
B_2 B/A, kips/sq in.
D Overall dimension or diagonal of faying area, in.
E Eccentricity M/F, in.
F Resultant force on fastener group, kips
F_0 Carrying capacity of fastener group for force through **G**, kips

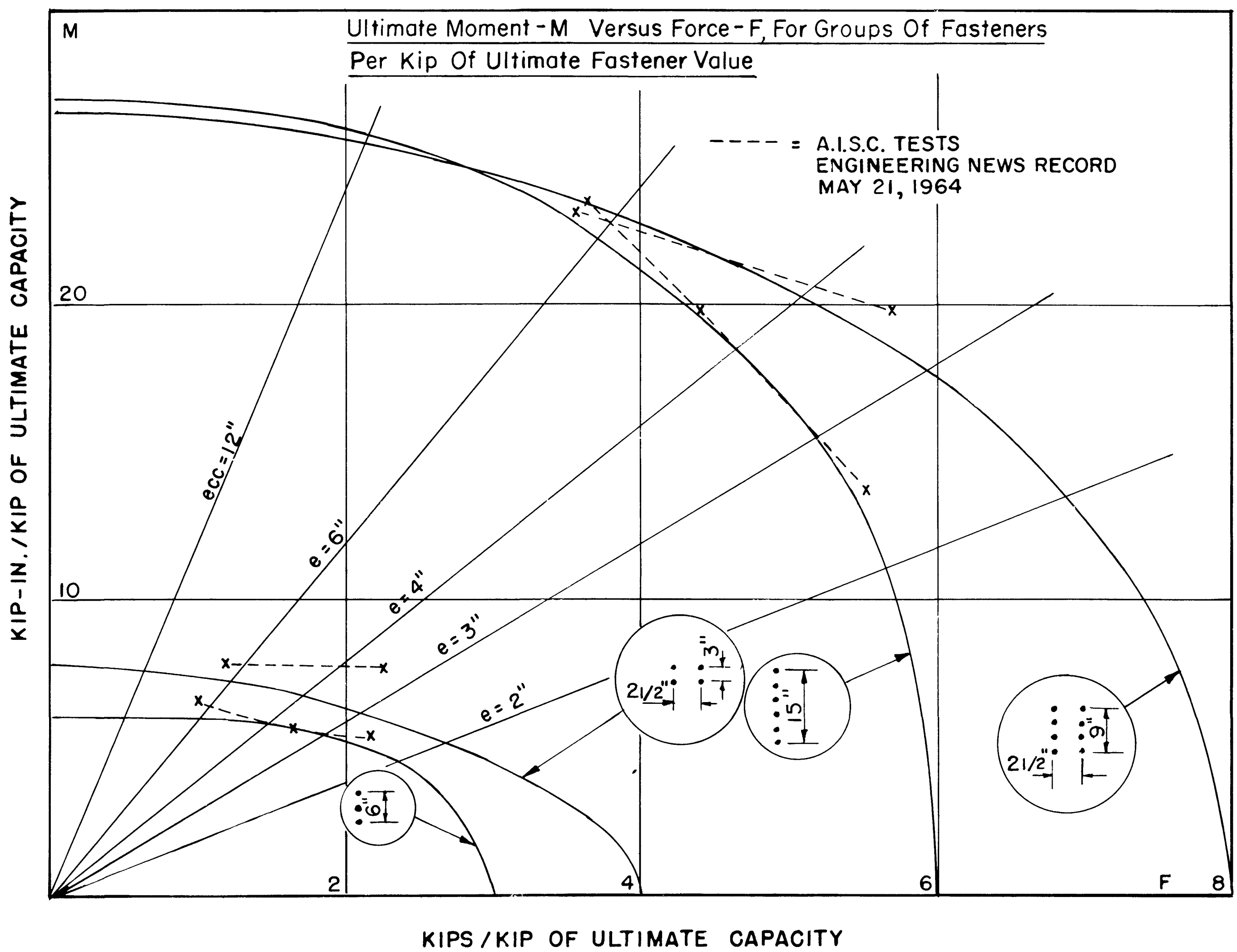

Figure 1

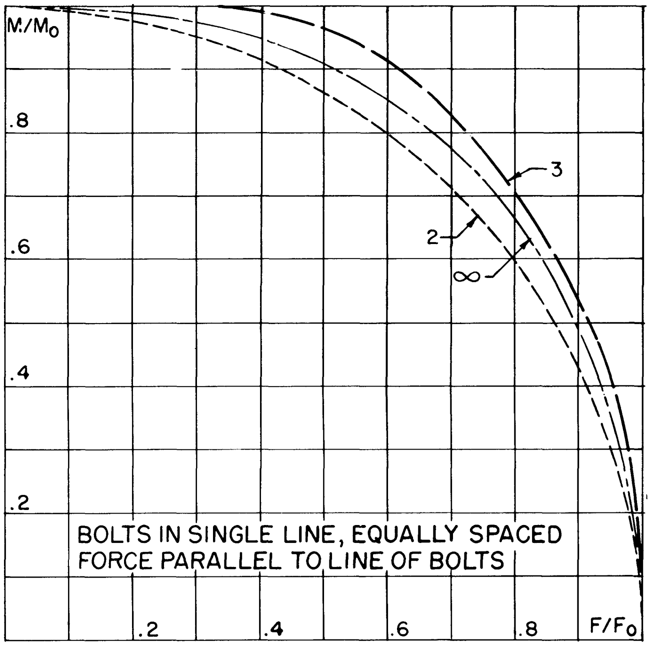

Figure 2

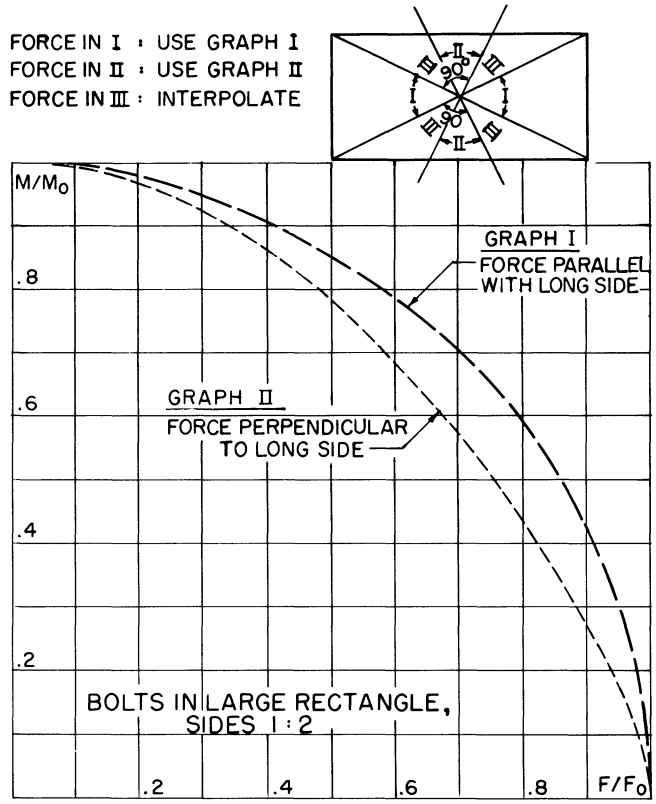

Figure 4

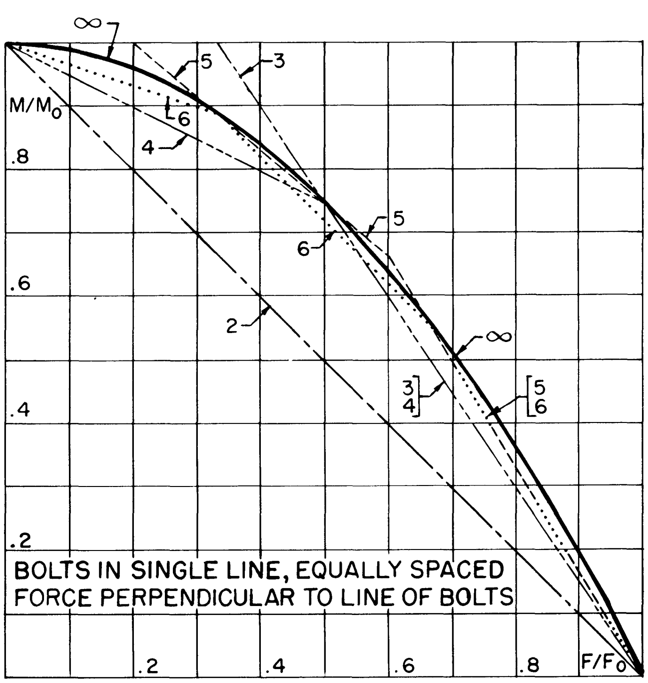

Figure 3

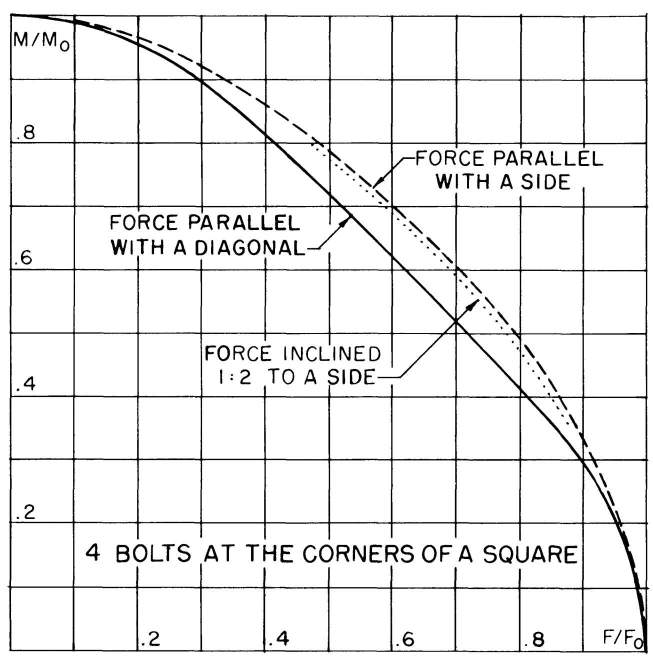

Figure 5

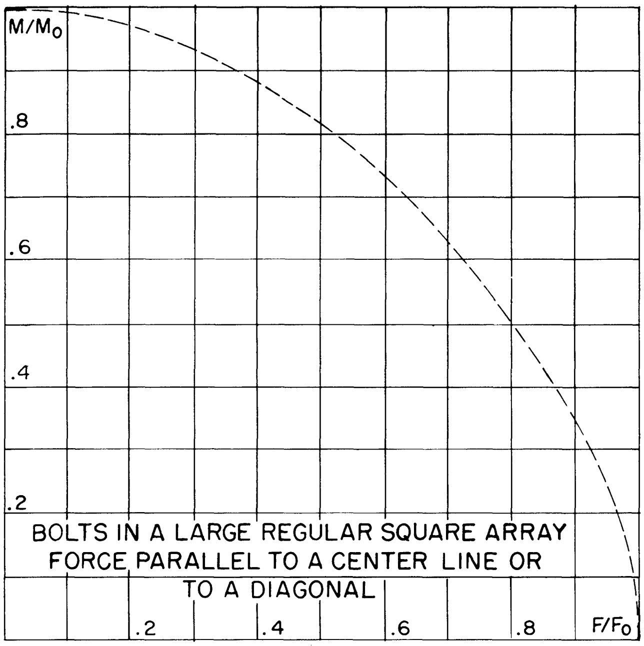

Figure 6

H Component of F in the x-direction, kips

I Polar moment of inertia of the B-values of the fastener group about an axis normal to the faying surface, kips $\times$ sq in.

M Moment of F about **O** or moment generally, kip-in.

M_0 Carrying capacity of fastener group for moment without force, kip-in.

N Number of fasteners in the group

R Distance from a center to the most remote fastener, in.

V Component of F in the y-direction, kips

C Center of rotation of fastener group under the given force

G Center of gravity of the B-values of the fastener group.

O Center of rotation of the fastener group under a moment without force.

b Fastener spacing, center to center, in.

f F/F_0

h H/F_0

k M_0/DF_0

m M/M_0

n Number of bolts in line.

r Distance from a center to any fastener, in.

v V/F_0

Additional symbols are explained where they occur. The line or lines of fasteners are placed parallel to the y-axis, unless otherwise shown. Mathematical symbols and subscripts (except the subscript 0) have their usual significance; "log" denotes a natural logarithm.

General Relation—In any bolt group, regular or irregular, whenever all bolt forces B act in the same direction, their resultant passes through the center of gravity **G**, is parallel to the forces, and is equal to their sum. This resultant is denoted by F_0, and

$$F_0 = \Sigma B \tag{1a}$$

If all the bolts are equal, i.e., have the same B, and if the number of the bolts in the group is N,

$$F_0 = NB \tag{1b}$$

If the bolts are uniformly spaced in a line of faying length D, as previously defined, and their resistance per unit length is B_1,

$$F_0 = DB_1 \tag{1c}$$

Finally, if the bolts are uniformly spaced over a faying

Table I

Description of Fastener Group	Pure Moment M_0	Interaction Formulas and Curves			
		Force Parallel to Longer Axis of Faying Area		Force Normal to Longer Axis	
		Formula	Fig.	Formula	Fig.
2 bolts	$= \frac{1}{4} F_0D$	$f^2 + m^2 = 1$	2	$f + m = 1$	3
3 bolts in line	$= \frac{2}{9} F_0D$	$\frac{1}{4}(3f - 1)^2 + m^2 = 1$ and $m \leqslant 1$	2	$\frac{1}{4}(3f - 1) + m = 1$ and $m \leqslant 1$	3
More than 3 bolts in one line or in a narrow rectangle	$\approx 0.25\ F_0D$	$f \approx \tan\beta \log_e \cot \frac{1}{2}\beta$ $f \sin\beta + m\cos\beta \approx 1$	2	$f^2 + m \approx 1$	3
Bolts forming rectangle with side ratio 1:2	$\approx 0.265\ F_0D$	Use chart	4	Use chart	4
4 bolts at the corners of a square diamond	$= \frac{1}{4} F_0D$	In the range $0.854 \geq f \geq 0.354$, $f + m = 1.207$	5	Same formula	5
Bolts forming a square faying area	$\approx 0.27\ F_0D$	Use chart	6	Use chart	6

area A, and their resistance per unit area is B_2,

$$F_0 = AB_2 \tag{1d}$$

Whenever the resultant force F acting on a bolt group is not a pure force (does not pass through the center of gravity **G**), the group will tend to rotate about some point **C**, called the center of rotation, and each bolt reaction will be perpendicular to the radius r_c drawn from **C** to the bolt in question. Thus the reactions will no longer be parallel to each other, and their resultant will be smaller than their numerical (scalar) sum. Therefore the greatest force that may be resisted by a bolt group is F_0, so that F/F_0, also denoted f, is always a fraction not greater than 1.

The foregoing holds for both the traditional elastic (strictly speaking, quasi-elastic) method and for the plastic method. The basic difference between the two methods is manifested when the value of each bolt reaction is considered. If the distance from the center **C** to the bolt most remote from it is R_c, then, in the elastic method, the bolt reaction is Br_c/R_c, its moment about **C** is Br_c^2/R_c, and the moment about **C** for the whole group is

$$\text{Elastic } M_c = \Sigma Br_c^2/R_c = I_c/R_c \tag{2}$$

(In the following, the discussion refers to the plastic method only, unless specifically stated otherwise.)

In the plastic method, as already mentioned, the bolt reaction is simply B (regardless of how far from C the bolt is located), its moment about **C** is Br_c, and for the whole group

$$M_c = \Sigma Br_c \tag{3a}$$

or, for equal bolts,

$$M_c = B\Sigma r_c \tag{3b}$$

If all the bolts are equidistant from **C**,

$$M_c = (\Sigma B)r_c = F_0 r_c \tag{3c}$$

If all the bolts are uniformly spaced in a faying line of length D,

$$M_c = B_1 \int_D r_c \, dD \tag{3d}$$

and if they are uniformly spaced over an area A,

$$M_c = B_2 \int_A r_c \, dA \tag{3e}$$

If the area is that of a circle or a circular sector with center **C**, radius R, and angle θ, then $dA = rd\theta dr$, and

$$M_c = B_2 \int_0^R \int_0^\theta r(rd\theta dr) = \tfrac{1}{3}B_2R^3\theta = \tfrac{2}{3}(B_2\tfrac{1}{2}R^2\theta)R = \tfrac{2}{3}F_0R \tag{3f}$$

For an area in the shape of a right triangle, refer to Fig. 8. In the triangle **CPB** draw two rays a small angle $d\theta$ apart from **C** to **PB**. The area between the rays may be treated as a circular sector of radius $l_1 \sec \theta$ and angle $d\theta$.

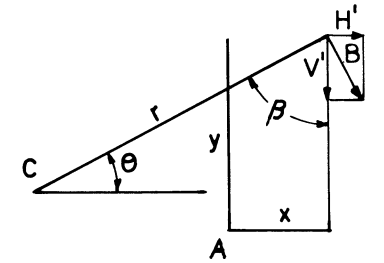

Figure 7

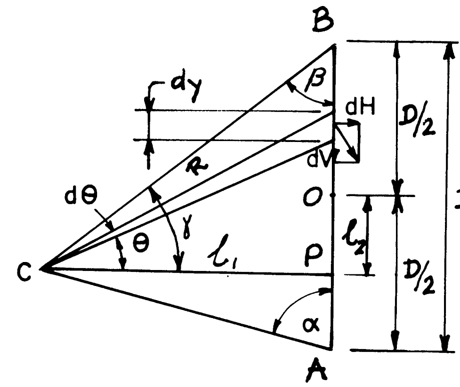

Figure 8

For such a sector $dM_c = \tfrac{1}{3}B_2(l_1 \sec \theta)^3 d\theta$; integrate for the whole triangle, obtaining

$$\begin{aligned} M_c &= \tfrac{1}{6}B_2 l_1[\sec\gamma \tan\gamma + \log(\sec\gamma + \tan\gamma)] \\ &= \tfrac{1}{3}F_0R(1 + \sin\beta \tan\beta \log \cot \tfrac{1}{2}B) \end{aligned} \tag{3g}$$

For comparison, the elastic

$$M_c = \tfrac{1}{3}F_0R(\tfrac{1}{2} + \sin^2\beta) \tag{3h}$$

The right triangle just discussed is mainly useful because oblique triangles, rectangles and polygons generally can be subdivided into right triangles. For example, a regular hexagon may be broken up into 12 right triangles with $\beta = 60°$. For **C** at the center of the hexagon, the moment values are: plastic, 0.303 F_0D; elastic, 0.208 F_0D. This example demonstrates that the difference between the two methods may be very considerable.

It will be noted that the plastic M_c's of several bolt groups are additive, provided, of course, they all refer to the same center. Similarly, if a subgroup is removed from a larger bolt group, the M_c of the remaining bolts equals the difference of the M_c's of the group and the subgroup. This observation is often useful in computation.

If the center of rotation **C** is known, the resistance of a bolt group may be found in the manner indicated in

Fig. 7. The resistance B of a bolt with coordinates x and y, referred to any origin **A**, is given by the following:

$$V' = B \sin \beta;\ H' = B \cos \beta;\ M_A' = H'y + V'x \quad \text{(4a, b, c)}$$

The vertical force V (parallel to the y-axis), the horizontal force H (parallel to the x-axis), and the moment M_A may be found by summing V', H', and M_A' for the whole bolt group.

It is of interest to find, for a given bolt group, the location of **C** at which M_c is a minimum. Call this point **0** and the corresponding moment M_0. Since $M_0 = \Sigma Br_0$, we may instead locate **0** for the condition $\Sigma Br_0 =$ minimum. Differentiating with respect to any direction x, $\Sigma B dr_0/dx = \Sigma Bx/r_0 = 0$. Denoting the angle between each r_0 and the x-direction by θ, the expression locating **0** may be written:

$$\Sigma B \cos \theta = 0;\ \text{or, for equal bolts, } \Sigma \cos \theta = 0 \quad \text{(5a, b)}$$

It will be noted that the component of the bolt reaction in the x-direction is $H' = B \cos \theta$ (Equation (4b)). $B \cos \theta = 0$ therefore means that the total component of the reaction of the group in any arbitrary x-direction is zero. In other words, the minimum moment M_0 is a pure moment; this is why, elsewhere in this paper, **0** is termed the center of pure rotation.

To avoid any misunderstanding, it is emphasized that a pure force passes not through **0** but through the center of gravity **G**, as previously noted. Now the expression locating **G** is not Equation (5) but a different one, viz, $\Sigma Br_G \cos \theta = 0$. Thus the center of pure rotation **0** and the center of gravity **G** are not necessarily the same. For irregular bolt layouts—those lacking symmetry about either axis—they are in fact generally distinct points. For examples of this see Figs. 10 and 14.

Equation (5) has an interesting consequence. Suppose the center **0** has been correctly located for a bolt

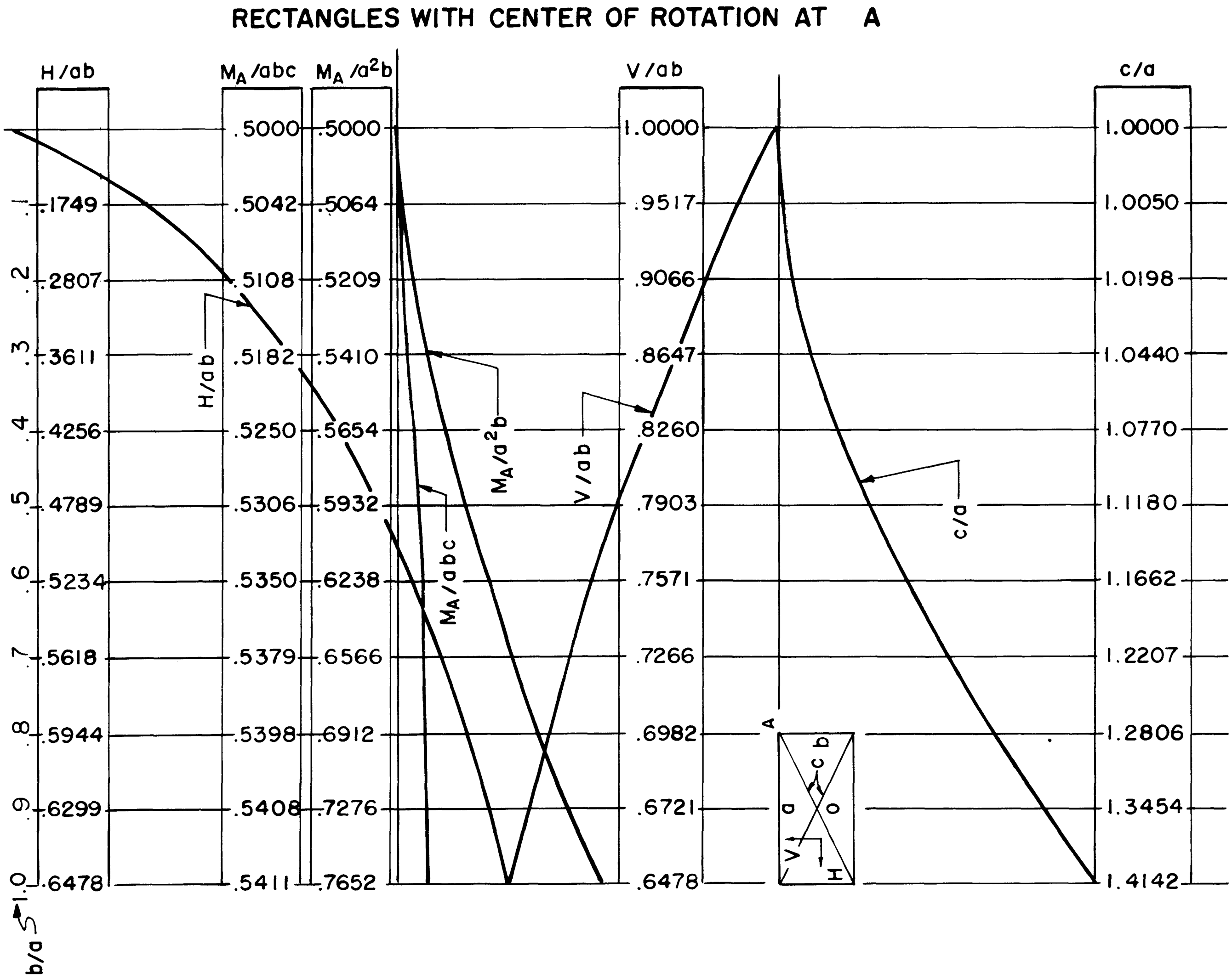

Figure 9

group. Draw rays from **0** to the bolts. The bolts may be shifted any distance along the rays, toward **0** or away from it, without affecting the location of the center of pure rotation. As an example, this center for a group of 4 bolts located at the corners of a quadrilateral, even an irregular one, is at the intersection of the diagonals.

Pure Moment—The calculation of M_0 is essential for the plastic method. For regular bolt layouts—those symmetrical about two axes—this is facilitated by the fact that both **0** and **G** are obviously located at the origin of coordinates.

In the case of n bolts, each of capacity B, spaced b inches apart in a single line, the overall faying dimension D is nb, $F_0 = nB$, and for odd numbered n,

$$M_0 = \tfrac{1}{4}(n^2 - 1)Bb = \tfrac{1}{4}\left(1 - \frac{1}{n^2}\right)F_0 D \qquad (6a)$$

For even numbered n, also for large n,

$$M_0 = \tfrac{1}{4}n^2 Bb = \tfrac{1}{4}F_0 D \qquad (6b)$$

For comparison, the elastic

$$M_G = \tfrac{1}{6}F_0 D \qquad (6c)$$

A rectangular faying area may be cut up into 8 equal right triangles, each with a vertex at the center of the rectangle. Equation 3(g) may now be used on the triangles, and the sum of their M_G's about the center of the rectangle is the required M_0 of the rectangle. Writing $M_0 = kF_0D$, k is taken as $\frac{1}{2}$ the values of M_A/abc in Fig. 9. The values of k vary within a narrow range, from 0.27 for a square to 0.25 for a very long rectangle.

Force in Any Direction Combined with Moment—The pure force F_0 and the pure moment M_0 having been discussed, attention will now be directed towards the usual case of force combined with moment. The force F in general has components V and H parallel to the y- and x-axes. The fractions v and h stand for V/F_0 and H/F_0, and m for M/M_0, as before. Referring again to Fig. 8, note that

$$l_1 = CP = D \sin\alpha \sin\beta / \sin(\alpha + \beta)$$

and

$$l_2 = OP = D \sin(\alpha - \beta)/2\sin(\alpha + \beta)$$

To deal with the case of numerous bolts uniformly spaced in a line of faying length D, draw two rays a small angle $d\theta$ apart, as before.

The intercept, dy, between the same rays is equal to $l_1 \sec^2\theta \, d\theta$ and $dV/dy = B_1 \cos\theta$; $dH/dy = B_1 \sin\theta$; $dM/dH = l_1 \tan\theta - l_2$. Integrating these expressions,

$$v = \frac{\sin\alpha \sin\beta}{\sin(\alpha + \beta)} \log(\cot\tfrac{1}{2}\alpha \cot\tfrac{1}{2}\beta) \qquad (7a)$$

$$h = \frac{\sin\alpha - \sin\beta}{\sin(\alpha + \beta)} \qquad (8a)$$

$$m = \frac{2\sin\alpha \sin\beta}{\sin(\alpha + \beta)}\left[\frac{\cos\alpha + \cos\beta}{\sin(\alpha + \beta)} - v\right] \qquad (9a)$$

For $\alpha = 90°$ these equations reduce to

$$v = \tan\beta \log\cot\tfrac{1}{2}\beta \qquad (7b)$$

$$h = \tan\tfrac{1}{2}(90 - \beta) \qquad (8b)$$

$$m = 2\tan\beta(1 - v) \qquad (9b)$$

For $\alpha = \beta$ they reduce to

$$v = \tan\beta \log\cot\tfrac{1}{2}\beta \qquad (7b \text{ (repeated)})$$

$$h = 0 \qquad (8c)$$

$$m = \sec\beta - v\tan\beta \qquad (9c)$$

The procedure for calculating V, H and M_c for a polygonal faying area is to join **C** to the corners of the polygon and to treat the polygon as a sum and/or difference of triangles, each with a vertex at **C**. After V and H have been found, F is given by the Pythagorean Theorem, and its direction is given by the ratio H/V. By locating **C** at a sufficient number of points and carrying out the requisite calculations the relationships between m_c, h/v and f may be determined with sufficient accuracy for tabulation or plotting. Some results of this procedure for rectangles are shown in Figs. 4 and 9.

If the *force F is in the y-direction*, **C** is located on the x-axis of symmetry (provided the bolt pattern is symmetrical about this axis). This simplifies the calculations. Results for some common bolt layouts follow:

Two bolts: $V = 2B\sin\beta$; $M = Bb\cos\beta$. Therefore, $f^2 + m^2 = 1$ (see Fig. 2).

Three bolts in line: When $\beta > 0$, $V = B + 2B\sin\beta$; when C is at the center bolt ($\beta = 0$), $+1 > V > -1$; in all cases, $M = 2Bb\cos\beta$; therefore: for $f \geq \frac{1}{3}$, $\frac{1}{4}(3f - 1)^2 + m^2 = 1$; and, for $f \leq \frac{1}{3}$, $m = 1$ (see Fig. 2).

Numerous equidistant and equal bolts in line: This case, previously discussed, is a practical approximation for any number of bolts greater than 3. The formulas, with some slight rearrangement, are given here again (see Fig. 2):

$$f = \tan\beta \log\cot\tfrac{1}{2}\beta \qquad (7b) \text{ (repeated)}$$

$$f\sin\beta + m\cos\beta = 1 \qquad (9d)$$

These formulas are also usable for the case of 2 lines of bolts and narrow rectangular faying areas. An interaction curve for rectangles with a side ratio of 1:2 is given in Fig. 4, for 4 bolts at the corners of a square in Fig. 5, and for a square faying area in Fig. 6. The data for Fig. 5 has been calculated using Equations (4), and for Figs. 4 and 6 by means of the tables of Fig. 9.

The case of *force in the x-direction* may be treated similarly. Results for some common bolt layouts follow.

Two bolts: Here, 3 different positions of **C** have to be considered: (1) Outside the bolts; (2) coincident with a bolt; (3) between the bolts. For (1), $f = 1$ and $m = 0$; for (2), $f + m = 1$; for (3), $f = 0$ and $m = 1$ (See Fig. 3).

Several bolts in line: Here, too, the interaction curves are made up of straight segments, which, when n is large, approximate the parabola $f^2 + m = 1$. (See Fig. 3.)

This parabolic curve is also usable for the case of 2 lines of bolts and narrow rectangular faying areas. The case of a rectangle with side ratio 1:2 is dealt with in Fig. 4, computed on the same basis as before. For square bolt patterns the curves are of course the same as for the case of force in the y-direction.

Irregular layouts: For an example, see Fig. 10.

Force oblique to both axes: In the case of an *oblique force* inclined at a given angle θ to the y-axis, $h/v = \tan\theta$. As a rule, points for interaction curves have to be obtained by means of Equations (4), (7), (8) and (9), using the long forms containing α of the latter three; this is somewhat tedious for manual calculation, but may be easily programmed for machine computation. Results for a few layouts follow:

Four bolts at the corners of a square, force parallel to a diagonal: Draw this diagonal along the x-axis and the other diagonal along the y-axis. **C** will lie on the y-axis.

If **C** is outside the square, $1 \geq f \geq 0.854$, $0 \leq m \leq 0.354$, and $(2f - 1)^2 + (2m)^2 = 1$

If **C** is at a corner of the square, $0.854 \geq f \geq 0.354$, $0.354 \leq m \leq 0.854$, and $f + m = 1.207$

If **C** is inside the square, $0.354 \geq f \geq 0$, $0.854 \leq m \leq 1$, and $(2f)^2 + (2m - 1)^2 = 1$. (See Fig. 5.)

Square faying area, force parallel to a diagonal: The interaction curve for this case works out to be the same as for a force parallel to a side of the square. (See Fig. 6.)

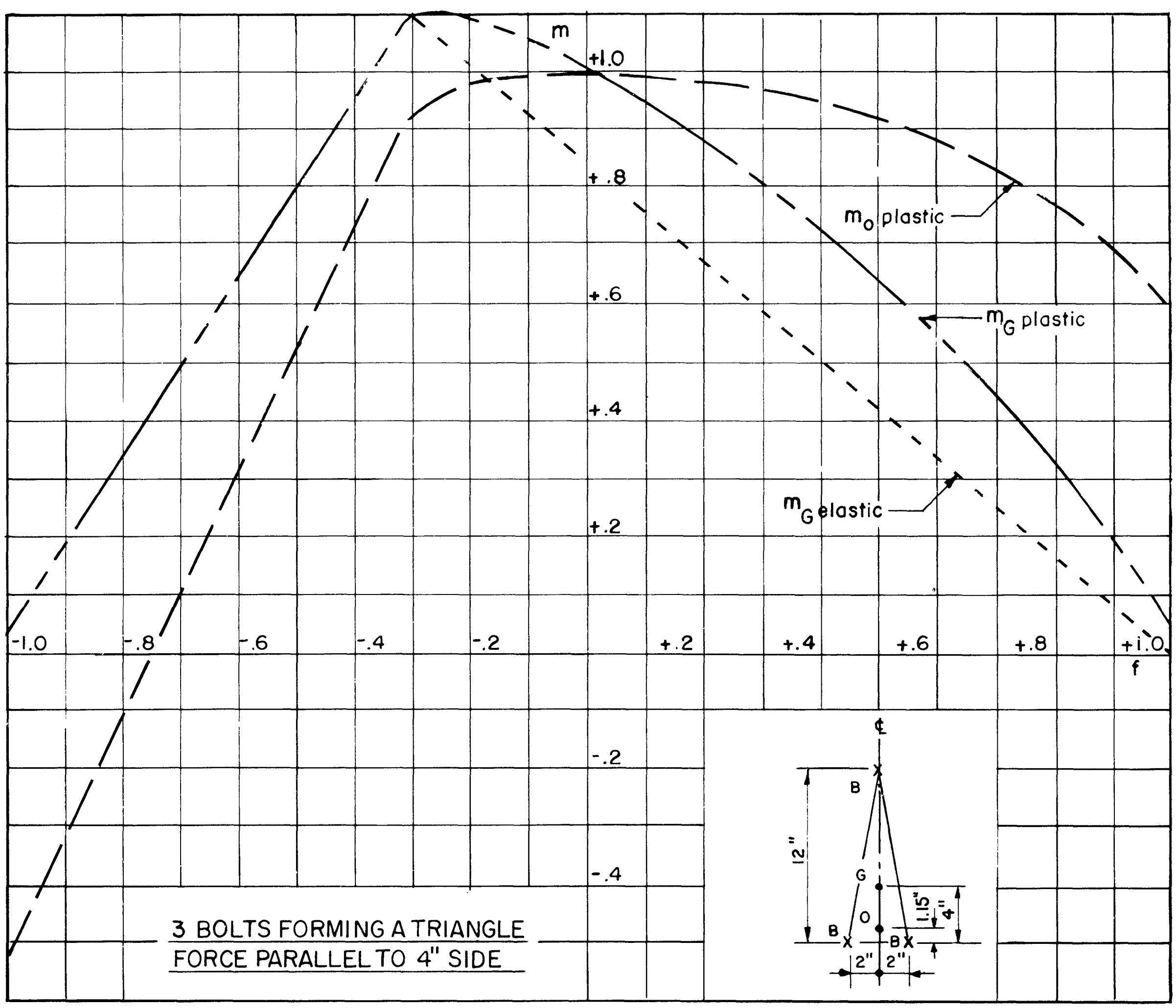

Figure 10

Rectangles, including single line (narrow rectangle): In a rectangular group the bolts contributing most to its pure moment resistance are those located near the ends of the diagonals, the reactions of these bolts being normal to the diagonals. It may therefore be assumed that the most favorable direction of force, i.e., the direction yielding the highest interaction curve for the given bolt group, is parallel to either diagonal, while the least favorable direction is at right angles to either diagonal. This assumption is borne out in the case of bolts in a single line, for which "best" and "worst" forces run parallel to the y- and x-axis respectively. Based on these considerations, an approximate rule for rectangles under oblique forces is given in Fig. 4.

EXAMPLES

Charts are generally more convenient to use than formulas. The formulas have been given mainly for the benefit of engineers who might wish to prepare tables from them.

When, as often happens, the eccentricity E is given, the following observation is helpful. Write $M = EF$, and $M_0 = kDF_0$, and divide these expressions by each other, obtaining

$$(M/M_0):(F/F_0) = E{:}kD \text{ or } m{:}f = E{:}kD$$

This equation represents a straight line through the origin, whose intersection with the appropriate interaction formula gives the required value of f and/or m. (See Fig. 11.) Note that the coefficient k is given in Table I and in most cases treated here is about $\frac{1}{4}$.

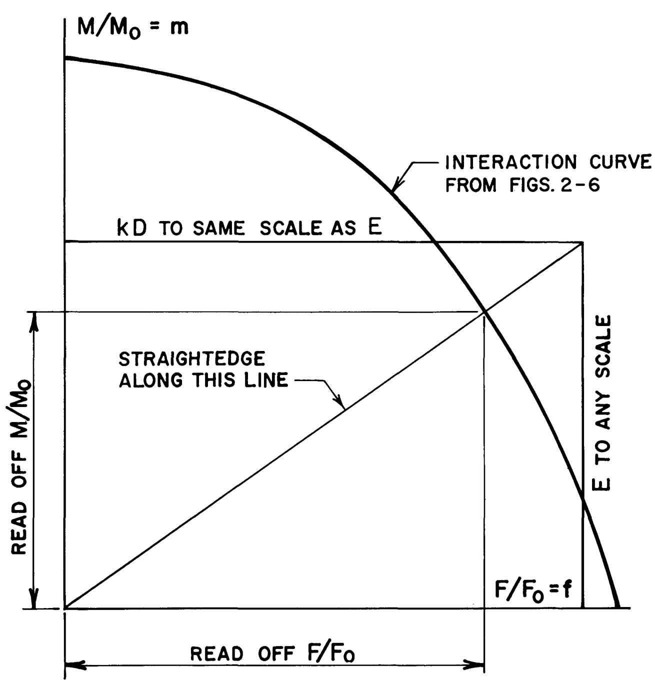

Figure 11

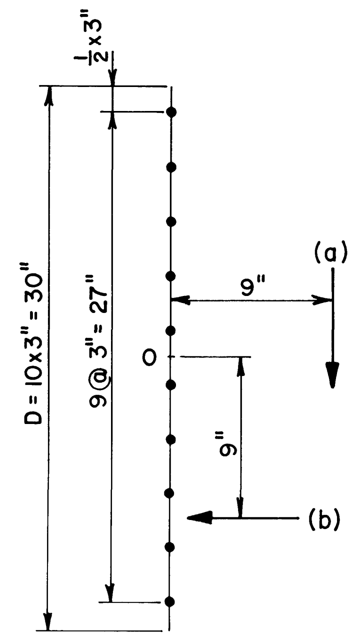

Figure 12

Example 1 (Fig. 12)

Given: What force may be imposed at an eccentricity of 9 in. on a single line of 10 bolts, 3 in. on centers of 9.0 kips capacity (a) parallel to the line and (b) perpendicular to the line?

Solution: $F_0 = 10 \times 9.0 = 90$ kips

$D = 10 \times 3$ in. $= 30$ in.

$k = \frac{1}{4}$

$kD = 7\frac{1}{2}$ in.

$E = 9$ in.

$E{:}kD = 9{:}7\frac{1}{2}$

(a) Force parallel to the line. In Fig. 2 place a straight edge on the line $m{:}f = 9{:}7\frac{1}{2}$. Read off $f = 0.67$.

$F = 0.67 \times 90 =$ **62 kips**.

(b) Force perpendicular to the line. In Fig. 3 place a straight edge along the same line as in (a). This intersects the parabolic interaction curve at $f = 0.565$.

$F = 0.565 \times 90 =$ **51 kips**.

For comparison, results by other methods are:

(a) Elastic, $47\frac{1}{2}$ kips. (AISC Manual, $75\frac{1}{2}$ kips.)

(b) Elastic, 34 kips. (No tables for this case are given in the AISC Manual.)

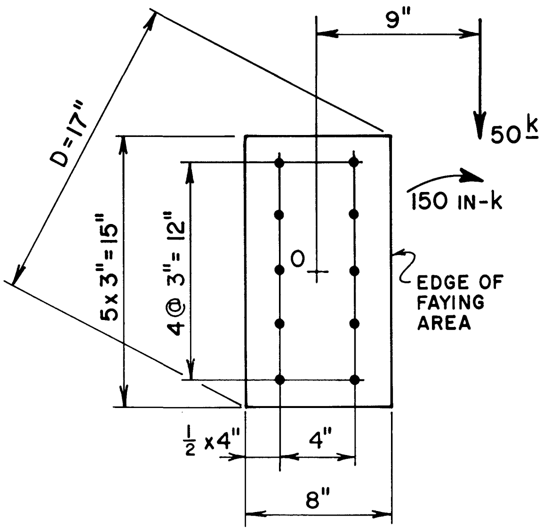

Figure 13

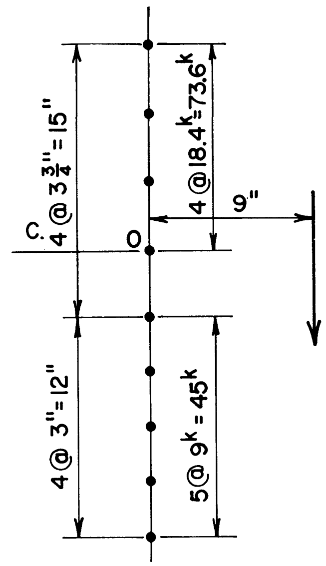

Figure 14

Example 2 (Fig. 13)

Given: The 2 × 5 bolt group shown is loaded by a vertical force of 50 kips and, in addition, by a small horizontal force at a large eccentricity which produces a moment of 150 kip-in. Find the required bolt capacity.

Solution: Let the bolt capacity be B kips. Then

$$F_0 = 10B$$
$$D = 17 \text{ in.}$$
$$k = 1/4$$
$$kD = 4\tfrac{1}{4} \text{ in.}$$
$$M_0 = kDF_0 = 42\tfrac{1}{2}B$$
$$F = 50$$
$$M = F \times 9 \text{ in.} + 150 = 600$$
$$E = 600/50 = 12 \text{ in.}$$

Use Fig. 4. Place straight edge through the origin and the point $f = 4\tfrac{1}{4}$ in., $m = 12$ in. This intersects the interaction curve at $m = 0.935$. Therefore $M_0 = M/0.935 = 600/0.935 = 42\tfrac{1}{2}B$.

$$B = \mathbf{15.1\ kips}.$$

Example 3 (Fig. 14)

Given: Five of the bolts of Fig. 12 have been replaced with four larger bolts, of capacity 18.4 kips each, as shown. What is the greatest vertical force that may be imposed at a 9 in. eccentricity?

Solution: $F_0 = 5 \times 9.0 + 4 \times 18.4 = 118.6$ kips.

This is an irregular group. To locate the center of pure rotation **0** use $\Sigma B \cos\theta = 0$. For bolts above **0**, $\cos\theta = +1$, and for those below, $\cos\theta = -1$. Therefore **0** must be located so as to have equal bolt capacities above and below it. Locate **0** as shown (adding part of the 18.4 kips of the bolt at **0** to the upper bolts and the rest to the lower bolts).

$$M_0 = 5 \times 9.0 \times (3\tfrac{3}{4} + 6) + 3 \times 18.4 \times 7\tfrac{1}{2} = 853 \text{ kip-in.}$$

$$kD = 853/118.6 = 7.2 \text{ in.};\ E = 9 \text{ in.}$$

Though the curves in Fig. 2 have been prepared for a regular pattern, their use will give sufficient practical accuracy for cases like this example. Joining the origin to the point $f = 7.2$, $m = 9$, the intersection with the curve for an infinite number of bolts gives $f = 0.66$. $F = 0.66 \times 118.6 =$ **77 kips**.

This answer may be checked by trial and error, using Equations (4). Putting the center of rotation **C** approximately $5\tfrac{3}{4}$ in. to the left of **0** and $\tfrac{3}{4}$ in. above it, it is found that H is small (it should be zero), the ratio M/V is approximately the required 9 in., and $V = F = 78$ kips, close enough to the 77 kips obtained much faster by using the interaction curve of Fig. 2.

Example 4 (Fig. 15)

Given: A bracket is connected along the perimeter of a rectangle with sides $a = 3\tfrac{1}{2}$ in. and $l = 12$ in. by means of a continuous weld of value 3.5 kips per inch. The load acting on the bracket is so eccentric that it may be approximated by a pure moment. Assuming that

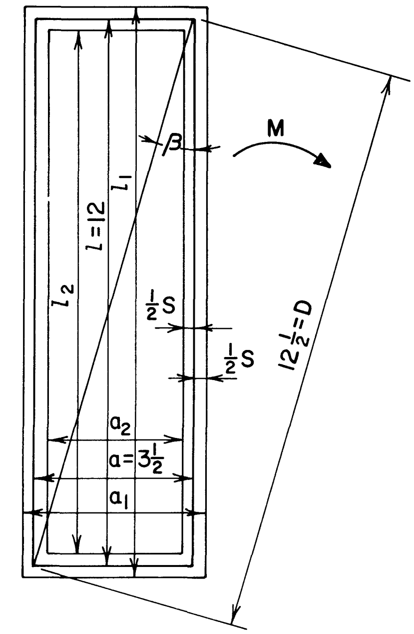

Figure 15

plastic concepts are applicable, find the moment capacity of the weld.

Solution: Let the width of the weld be s. Then the weld may be treated as the difference between two concentric rectangles, $\mathbf{a_1 l_1}$ and $\mathbf{a_2 l_2}$. Referring to Fig. 9, let $M_0 = kDF_0$ for each of these two rectangles, where k is one-half of the coefficients M_A/abc tabulated in the same figure. By performing the required algebraic manipulation and going to the limit $s \rightarrow 0$, it is easily derived that, for the weld, $M_0 = k(1 + \sin\beta \cos\beta)DF_0$.

$$M_0 = 0.259(1 + 0.96 \times 0.28) \times 12\tfrac{1}{2} \times (31 \times 3.5)$$
$$= 0.329 \times 12\tfrac{1}{2} \times 108.5 \text{ kips}$$
$$= \mathbf{446 \text{ kip-in.}}$$

For comparison, the elastic moment capacity = $(1 + 2 \sin\beta \cos\beta)DF_0$ = 347 kip-in. (The elastic capacity may be checked by extrapolating from the table in the AISC Manual, Page 4-62).

Another way of dealing with continuous welds deposited in straight line segments such as those tabulated in the AISC Manual is by means of Equations (7), (8) and (9), or by means of charts based thereon.