Bracing of Continuous Columns

TOR B. URDAL

IT IS GENERALLY accepted that bracing provides elastic support. As shown in this article, the lateral supports for the buckling condition of continuous columns depend on elasticity in three different ways:

Case 1: Reactions are proportional to the deflections with respect to the original neutral axis.

Case 2: Reactions are proportional to the lateral deflection with respect to the adjacent points of support.

Case 3: Reactions are proportional to the relative displacement of successive supports.

The three cases are shown in Fig. 1. The required rigidities of supports, as calculated below, are based on the criterion of "full bracing,"[1] i.e., the bracing should be equivalent to unyielding supports. The procedure is to make the spring constant of the actual bracing, k_{act}, at least equal to the required spring constant, k_{req}, derived from the ideal k_{id} of perfectly straight, continuous columns.

CASE 1

Required Spring Constant—Consider a straight, continuous, centrally loaded column, free to rotate, but prevented from translation at all supports. When the column buckles into half sine-waves, and the second derivative of the column deflection curve equals zero at all intermediate supports:

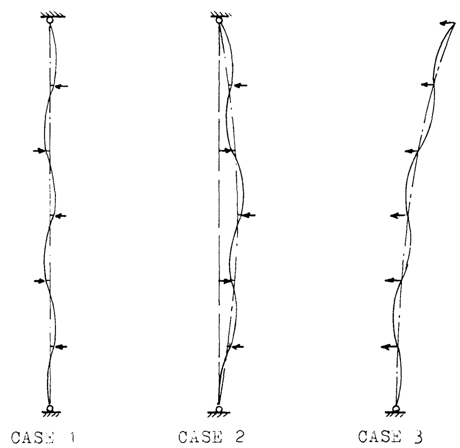

Fig. 1. Buckling of continuous columns

Tor B. Urdal is a Consulting Engineer, Oslo, Norway.

$$\frac{d^2y}{dz^2} = -\frac{M}{EI} = 0 \qquad (1)$$

The effective lengths equal the distances between the supports. Theoretically, hinges could be inserted at each support so that the column is divided into pinned-end members. This would not alter statics, because the hinged and continuous columns are statically identical.

The spring constant for the elastic supports of this statically determinate system can be found by loading the column axially with P_{cr}.* The supports may have some minor lateral deflections on account of the axial load in the bracing, but total work is zero. If the member **i-j**, Fig. 2 (the chord represents the actual curved member), deforms at an angle $(\Delta_i - \Delta_j)/h$, the corresponding work is:

$$\frac{1}{2}\frac{P_{cr}}{h}(\Delta_i - \Delta_j)^2 \qquad (2)$$

The work being done at support **i** is expressed as:

$$-\frac{1}{2}k_{id}(\Delta_i)^2 \qquad (3)$$

* $P_{cr} \approx 1.9P_a$, *where* P_a = *allowable load in AISC Specification.*

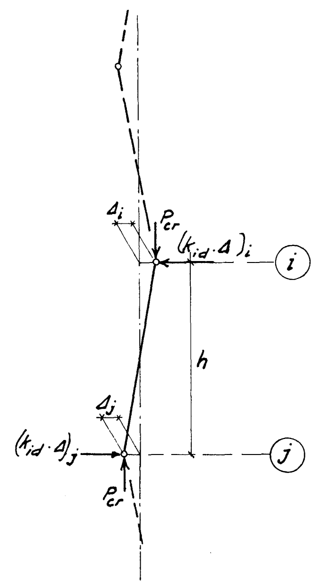

Fig. 2. Lateral deflections of supports (Case 1)

If the column has constant cross-section and equal spacing of supports (i.e., ultimate load P_{cr} is the same for all members), summation of the total work gives:

$$\tfrac{1}{2}\,\frac{P_{cr}}{h}\sum(\Delta_i - \Delta_j)^2 - \tfrac{1}{2}k_{id}\sum(\Delta_i)^2 = 0 \qquad (4)$$

and

$$k_{id} = \frac{2P_{cr}}{h}\left(1 - \frac{\sum\Delta_i\Delta_j}{\sum(\Delta_i)^2}\right) \qquad (5)$$

When $-\Delta_i = \Delta_j$ (regular zigzag displacement of supports), the results from Eq. (5) are in good agreement with the correct values for full bracing of continuous columns:

$$k_{id} = \frac{2P_{cr}}{h}, \frac{3P_{cr}}{h}, \frac{3.4P_{cr}}{h}, \frac{3.6P_{cr}}{h}$$

at intermediate points **1**, **2**, **3**, or **4**, respectively.*

When P_{cr}/h varies, k_{id} is no longer constant. If the column has a moderate variation of P_{cr}/h and a sufficient number of supports, we may assume (for reasons of continuity) that the work being done at an intermediate support **i** is approximately equal to half the work required to rotate members **h-i** and **i-j**. With notations from Fig. 3,

$$\tfrac{1}{2}(k_{id})_i(\Delta_i)^2 = {}^{11}\!/_{22}\left(\frac{P_{cr}}{h}\right)_h(\Delta_i - \Delta_h)^2 + {}^{11}\!/_{22}\left(\frac{P_{cr}}{h}\right)_i(\Delta_i - \Delta_j)^2 \qquad (6)$$

Equation (6) is correct for the case of a column of uniform strength with braces spaced equally; however, no great inaccuracy will result if the bracing is spaced only approximately equally. $\Delta_j = \Delta_h = -\Delta_i$ gives:

$$(k_{id})_i = 2\left(\frac{P_{cr}}{h}\right)_h + 2\left(\frac{P_{cr}}{h}\right)_i \qquad (7)$$

where the two terms refer to the members **h-i** and **i-j**.

Practical columns have out-of-straightness and eccentricity of loading. This can be taken account of by substituting Δ_0 for these imperfections. Most demanding is an initial S-shape corresponding with the supports. This is not likely to occur for more than three (perhaps in rare cases, four) successive supports of a continuous column. The numerical factors of Eq. (7) are conservative when the influence of Δ_0 is included, but this is hardly of economical importance. Restricting the deflections of the supports by taking, for instance, $\Delta_i = \Delta_0$, the required spring constants and reactions are given by[1]:

$$k_{req} = 2k_{id} \text{ and } S_{req} = 2k_{id}\Delta_0 \qquad (8)$$

* *Translation prevented at end supports.*

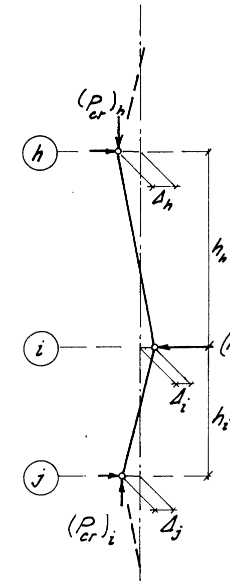

Fig. 3. Lateral deflections of supports (Case 1)

Example of Actual Spring Constant—In multistory buildings, the lateral supports of a continuous column may be dependent on the bending stiffness of the beams. If the displacements of the stories have no important effect, the column comes under Case 1.

Sufficient support is provided when:

$$k_{act} \geqq k_{req} \qquad (9)$$

Figure 4a shows a column **A** supported by the bending stiffness of the beam **A-B**. Figure 4b shows the moment diagram due to unity load at **A**. The lateral deflection is:

$$1\cdot\Delta = \frac{M_1a(a+b)}{3EI} = \frac{a^2(a+b)}{3EI} \qquad (10)$$

$$k_{act} = \frac{1}{\Delta} = \frac{3EI}{La^2} \qquad (11)$$

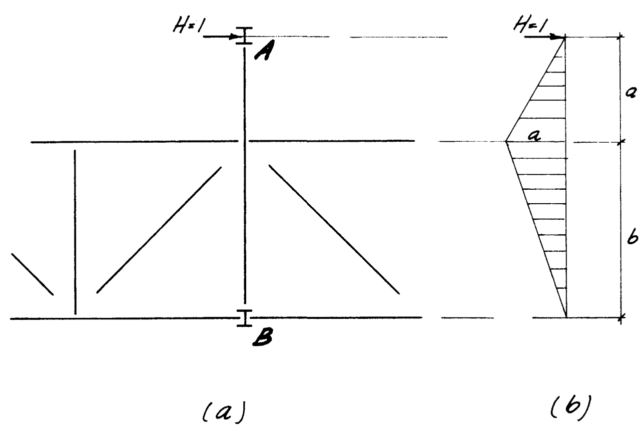

Fig. 4. Beam as lateral support

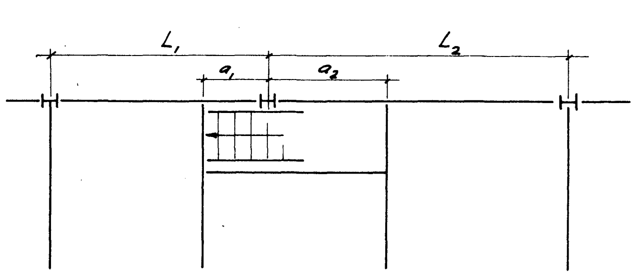

Fig. 5. Two beams as lateral support

Figure 5 shows a column in an exterior wall, supported by a beam on either side. With beam lengths L_1 and L_2 and overhangs a_1 and a_2, the spring constant for this case is:

$$k_{act} = \frac{3EI_1}{L_1 a_1^2} + \frac{3EI_2}{L_2 a_2^2} \qquad (12)$$

CASE 2

Required Spring Constant—Equations (5), (7), and (8) may be used for single columns as described. But Case 1 is not relevant for a number of columns (or compression chords of trusses, etc.) with parallel bracing. The bracing excludes the possibility that the reactions are proportional to the same neutral axis. Case 2 is shown in Fig. 6. The two intermediate members of a continuous column are supposed originally to be located on the same axis (Fig. 6a). On account of the axial load, the support **i** may have a deflection Δ_i with respect to a straight line connecting supports **h** and **j**:

$$\tfrac{1}{2}(k_{id})_i(\Delta_i)^2 = \tfrac{1}{2}\left(\frac{P_{cr}}{h}\right)_h(\Delta_i)^2 + \tfrac{1}{2}\left(\frac{P_{cr}}{h}\right)_i(\Delta_i)^2 \qquad (13)$$

and

$$(k_{id})_i = \left(\frac{P_{cr}}{h}\right)_h + \left(\frac{P_{cr}}{h}\right)_i \qquad (14)$$

Eccentricity Δ_0 of the original column axis (Fig. 6b) would increase the moments $P_{cr}\Delta$ to $P_{cr}(\Delta + \Delta_0)$, and Eq. (14) would be increased by $(1 + \Delta_0/\Delta)$. However, it is reasonable to assume that the bracing will adjust initial random imperfections of individual columns. As generally accepted, the resulting Δ_0 is small, and its influence may be neglected. This gives $k_{req} = k_{id}$.

For a number of columns k_{req} would be the sum, but taking P_{cr} for each one of them would be too conservative. Instead of basing calculations on the properties of the columns, $\sum P_{cr}$ is replaced by $F\sum P$:

$$(k_{req})_i = F\sum\left(\frac{P}{h}\right)_h + F\sum\left(\frac{P}{h}\right)_i \qquad (15)$$

where F is the load factor, and $\sum(P/h)_h$ and $\sum(P/h)_i$ are the sums for all columns **h-i** and **i-j**.

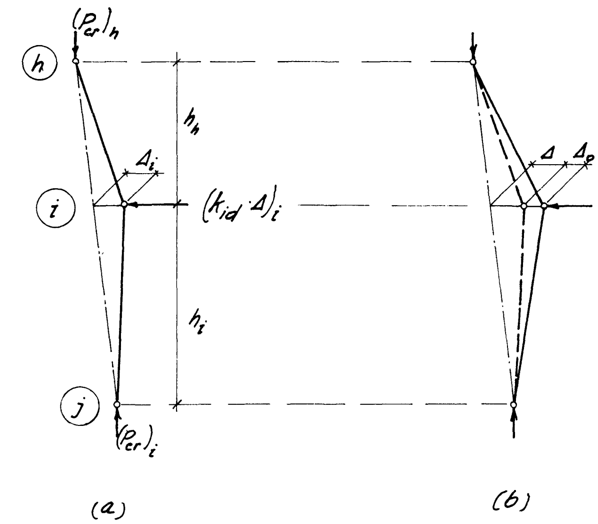

Fig. 6. Relative lateral deflection of support (Case 2)

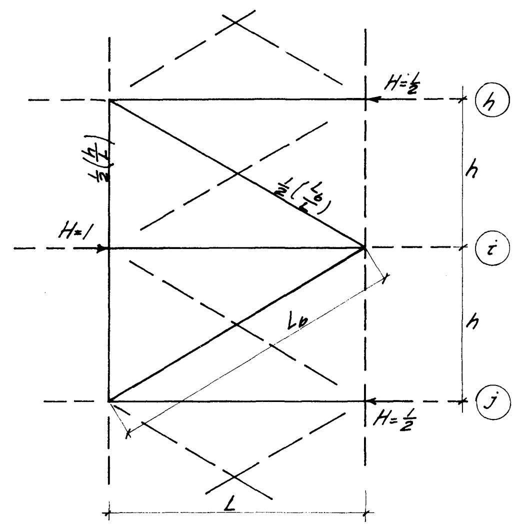

Fig. 7. Diagonal bracing (Case 2)

In general, bracing is designed for lateral load or made to comply with some minimum requirement. A criterion based upon the stability condition, Eq. (15), is presented below.

Example of Actual Spring Constant—Figure 7 shows two identical panels of diagonal bracing adjacent to support **i**. When the compression diagonals are not considered, a unity load at **i** activates the members as shown. The deflection of **i** with respect to **h** and **j** is:

$$\Delta_i = 2\left(\frac{h^2}{4L^2}\cdot\frac{h}{EA_c} + \frac{L}{4EA_g} + \frac{\tfrac{1}{2}L}{EA_g} + \frac{L_b^2}{4L^2}\cdot\frac{L_b}{EA_b}\right) \qquad (16)$$

where A_c, A_g, and A_b are the areas of the identical members of columns, girders, and bracing diagonals, respectively. Equation (16) applies for columns in the braced bay. At another column axis all intermediate girders at levels **h**, **i**, and **j** are contributing to Δ_i, and the representative value may be taken as the mean for all columns. Mostly the contributions of columns and girders may be neglected. The spring constant of the diagonals is:

$$(k_{act})_i = \frac{1}{\Delta_i} = \frac{2L^2EA_b}{L_b^3} \quad (17)$$

Equating Eqs. (15) and (17):

$$A_b = \frac{FL_b^3}{2EL^2}\left[\sum\left(\frac{P}{h}\right)_h + \sum\left(\frac{P}{h}\right)_i\right] \quad (18)$$

When both stability and strength are considered, A_b from Eq. (18) may be added to the bracing area required for lateral load.

CASE 3

Required Spring Constant—The bracing system acts as a cantilevered truss to provide the required lateral supports. The reaction at a support is then proportional to the deflection with respect to the adjacent lower support. For a small displacement Δ_i of **i**, as shown in Fig. 8, the work equation gives:

$$\tfrac{1}{2}(k_{id})_i(\Delta_i)^2 = \tfrac{1}{2}\left(\frac{P_{cr}}{h}\right)_i(\Delta_i)^2 \quad (19)$$

and

$$(k_{id})_i = \left(\frac{P_{cr}}{h}\right)_i \quad (20)$$

Replacing P_{cr} by FP and summing for all columns:

$$(k_{req})_i = F\sum\left(\frac{P}{h}\right)_i \quad (21)$$

Example of Actual Spring Constant—The pair of unit shears applied in opposite directions at floors **i** and **j** (Fig. 9) not only activate the diagonal between the two floors, but axial forces are introduced in the columns and continue to the foundations. The relative lateral deflection of **i:**

$$\Delta_i = \frac{L_{bi}^3}{L^2EA_{bi}} + \frac{1}{E}\left(\frac{h_i}{L}\right)^2\left[\sum_i^n \frac{h}{A_c} + \sum_j^n \frac{h}{A_c}\right] \quad (22)$$

When the contributions of the columns may be neglected, the spring constant of the diagonal is:

$$(k_{act})_i = \frac{1}{\Delta_i} = \frac{L^2EA_{bi}}{L_{bi}^3} \quad (23)$$

(If the diagonals are equally stressed in compression and tension, k is doubled.)

Equating Eqs. (21) and (23) gives the necessary elastic area of the diagonal below **i**:

$$A_{bi} = \frac{FL_{bi}^3}{EL^2}\sum\left(\frac{P}{h}\right)_i \quad (24)$$

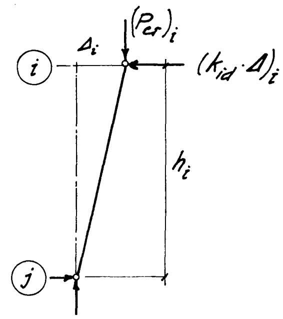

Fig. 8. Relative lateral deflection of support (Case 3)

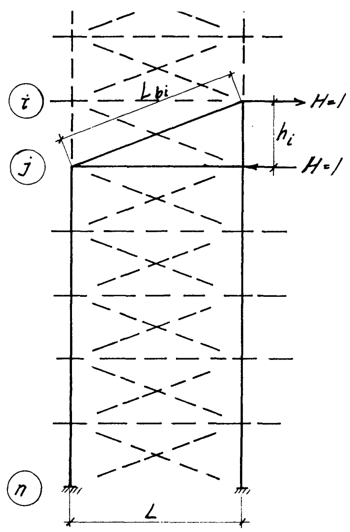

Fig. 9. Diagonal bracing (Case 3)

Combined Gravity and Horizontal Loading

$$A_{bi} = \frac{FL_{bi}^3}{EL^2}\sum\left(\frac{P}{h}\right)_i + \frac{FL_{bi}}{LF_y}P_{gi} \quad (25)$$

$P_{gi} = \sum_1^i H_i + \left(\frac{\Delta}{h}\right)_i \sum P_i$ is the force in the girder at the i'th story level. The load factor is taken as 1.7 and 1.3 in Eqs. (24) and (25) respectively.

Braced Frames—When the bracing satisfies Eqs. (24) and (25), the effective length of columns may be determined on the condition that sidesway is prevented. For plastically designed frames, the effective length factor $K = 1$, and for frames proportioned by the allowable stress design, $K < 1$.

REFERENCES

1. *Winter, George* Lateral Bracing of Columns and Beams, *Proc. Am. Soc. Civil Engrs., March, 1958.*
2. *Higgins, T. R.* Column Stability under Elastic Support, *AISC Engineering Journal, April, 1965.*
3. *Galambos T. V.* Lateral Support of Tier Building Frames, *AISC Engineering Journal, January, 1965.*