Buckling of Framed Domes

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THE USE OF STEEL framed domes has increased considerably in recent years.[1] Domes of the order of 1,000 ft in base diameter are now being designed for sports stadiums and other large structures where it is desirable to cover large open areas. As the size of domes increases, general buckling analysis becomes an important part of the design. The recent failure of a 307 ft diameter dome in Bucharest[2] has caused dome designers to make a careful analysis of their design for buckling.

Wright[2] and the author[3,4] have arrived at the same equation for the general buckling of a reticulated or a framed shell whose bending and membrane properties are identical in all directions. Many domes[1] are designed in such a way that the ribs running in a meridian direction are stiffer than the horizontal rings.

The purpose of this paper is to give the results of the extension of the previous buckling analysis to include the buckling load for a dome whose properties vary in different directions. This, of course, is only one of several important design considerations for large domes.

GENERAL EQUATIONS

Numerous framed domes have been analyzed by using shell analogy methods[5,6]. The method consists of finding an effective shell membrane thickness and an effective shell bending thickness for the framed dome or shell. Tests on framed domes and stiffened shells indicate good agreement between theory and experiment.

The effective membrane thickness in the meridian direction is

$$t_{m_1} = \frac{A_1}{d_1} \tag{1}$$

where A_1 is the area of the rib and d_1 is the rib spacing. The effective bending thickness in the meridian direction is

$$t_{B_1} = \frac{I_1}{d_1} \tag{2}$$

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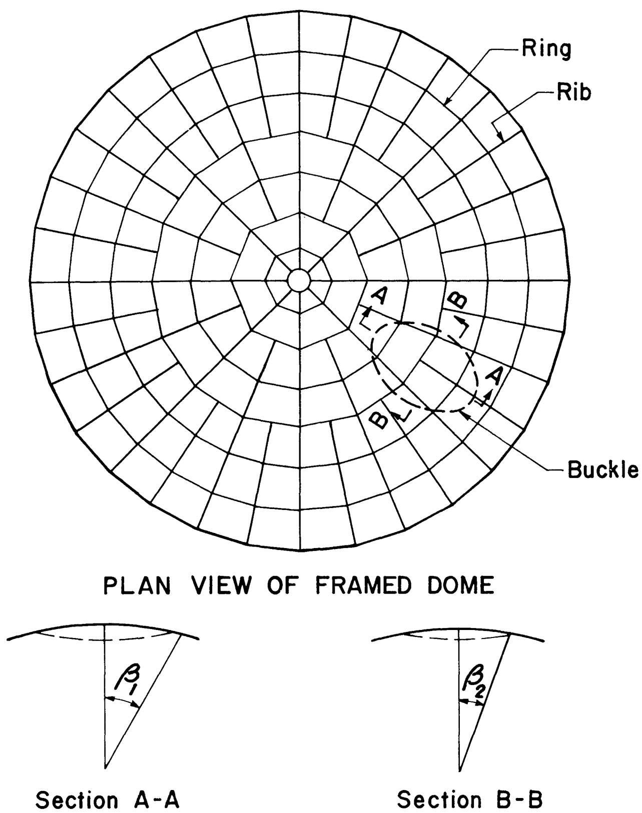

Figure 1

where I_1 is the moment of inertia of the rib. Similarly the effective membrane and bending thicknesses in the ring direction are

$$t_{m_2} = \frac{A_2}{d_2} \tag{3}$$

and

$$t_{B_2} = \frac{I_2}{d_2} \tag{4}$$

If

$$t_{m_1} = t_{m_2} = t_m \qquad t_{B_1} = t_{B_2} = t_B \tag{5}$$

the critical buckling pressure is[3,4,6]

$$p_{cr} = 0.366E\left(\frac{t_m}{R}\right)^2\left(\frac{t_B}{t_m}\right)^{3/2} \tag{6}$$

where E is the modulus of elasticity and R is the radius of curvature of the dome at the point of interest. On the other hand, if

$$t_{m_1} \neq t_{m_2} \qquad t_{B_1} \neq t_{B_2} \tag{7}$$

then Equation (6) no longer holds. The writer has resolved the basic stability equations for stiffened shells[4] using variable membrane and bending thicknesses and the result is

$$p_{cr} = 0.183E\left(\frac{t_{m_1}}{R}\right)^2 \frac{C^{1/2}F^{1/2}}{D}\left(\frac{t_{B_1}}{t_{m_1}}\right)^{3/2} \tag{8}$$

where

$$A = \frac{t_{m_2}}{t_{m_1}}$$

$$B = \frac{t_{B_2}}{t_{B_1}}$$

$$\begin{aligned} C = {} & \frac{1}{2} G^4 + G^3(1 - G) + G^2(1 - G)^2 + \\ & \frac{1}{2} G(1 - G)^3 + \frac{1}{10}(1 - G)^4 + \frac{1}{4} G^4(A - 1) + \\ & \frac{2}{3}(A - 1)\, G^3(1 - G) + \frac{3}{4}(A - 1)G^2(1 - G)^2 + \\ & \frac{2}{5}(A - 1)\, G(1 - G)^3 + \frac{1}{12}(A - 1)(1 - G)^4 \end{aligned}$$

$$D = \frac{1}{2} G^2 + \frac{1}{2} G(1 - G) + \frac{1}{6}(1 - G)^2$$

$$\begin{aligned} F = {} & \frac{3}{2} + \frac{9}{4}(B - 1) + \frac{3}{2}(B - 1)^2 + \frac{3}{8}(B - 1)^3 + \\ & \frac{1}{2} B^3 + \frac{3}{4} B^2(1 - B) + \frac{1}{2} B(1 - B)^2 + \\ & \frac{1}{8}(1 - B)^3 \end{aligned}$$

$$G = H\frac{\beta_1}{\beta_2}$$

$$\beta_1 = \left(\frac{t_{m_1}}{R}\right)^{1/2}\left(\frac{t_{B_1}}{t_{m_1}}\right)^{3/4}$$

$$\beta_2 = \left(\frac{t_{m_2}}{R}\right)^{1/2}\left(\frac{t_{B_2}}{t_{m_2}}\right)^{3/4}$$

For a given shell geometry, all of the above listed constants will be known except H, which is the proportionality constant between the wave angles of the buckled segment (β_1 and β_2, see Fig. 1). The value of H must be determined for each case such that the buckling load given by Equation (8) will be a minimum. For

$$t_{m_1} = t_{m_2} \qquad t_{B_1} = t_{B_2}$$

the buckle will be circular (see Fig. 1), H will be equal to one, and Equation (8) will reduce to Equation (6). For unequal stiffening, a value of H is assumed (greater than 0) and p_{cr} is calculated using Equation (8). This procedure is repeated until the minimum value of p_{cr} is determined. This minimum value is then the buckling load.

CONCLUSIONS

The shell analogy has been used to predict the buckling behavior of framed domes. Equation (6) agrees well with laboratory tests on stiffened shells,[4] and predicts the failure of the Bucharest dome.[2] Equation (8) provides the designer with a tool to assist him in designing large framed domes of variable geometric properties. In addition to using the equations presented herein, the designer needs to consider the dome edge conditions. It has been demonstrated both theoretically and experimentally that poor edge conditions will result in buckling loads lower than the values given by Equations (6) and (8). It has been shown that the yield strain of the material is an important parameter in shell buckling. It is to be expected that yield strain would be important in framed dome buckling also. Obviously, the results presented herein assume elastic action during the buckling process, which in general does not conform to physical reality. However, the limited experience to date on stiffened shells (used in roofs, heads for space simulator vessels, outer shells of cryogenic vessels, and bulkheads in space boosters) indicates that Equations (6) and (8) are useful in designing shell-like domes.

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