Application of AISC *Specification* Requirements for Second-Order Analysis and Stability Design

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ABSTRACT

Design for stability is inherent in the proper design of every steel structure. As such, every engineer using the AISC *Specification* (2022) must understand the requirements for stability design and how their own methods (including computer analyses) address the relevant considerations. This discussion provides specific, concise guidance on the application of AISC *Specification* requirements for stability design and second-order analysis for the practicing engineer.

Keywords: stability, direct analysis method, effective length, first-order analysis.

INTRODUCTION

This paper provides the practicing engineer a specific yet concise guide to the relationship between design for stability and second-order analysis as presented in the AISC *Specification for Structural Steel Buildings* (AISC, 2022), hereafter referred to as the AISC *Specification*. In their work on AISC committees and in practice, the authors have observed that the current guidance in Part 2 of the AISC *Steel Construction Manual* (AISC, 2023) does not provide specific guidance on the AISC *Specification* requirements for second-order analysis. While there is a very detailed treatment in AISC Design Guide 28, *Stability Design of Steel Buildings* (Griffis and White, 2013), this goes beyond the needs of most practicing engineers and typical projects. Accordingly, the authors have prepared a discussion of design for stability with specific guidance on the application of second-order analysis, expanding on the treatment in the AISC *Steel Construction Manual*.

The paper includes a discussion of design for stability and of second-order analysis, as well as tables for approximate second-order analysis for P-Δ effects. Additionally, there is a glossary of terms, a diagrammatic presentation of methods of second-order analysis, and a design example.

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REQUIRED STRENGTH, DESIGN FOR STABILITY, EFFECTIVE LENGTH, AND SECOND-ORDER EFFECTS

In the AISC *Specification*, reliable performance is achieved by ensuring that the available strength of members and connections equals or exceeds the required strength. As discussed in Chapter C of the AISC *Specification*, it is essential to this method that stability be provided for the structure as a whole and each of its elements. Stability considerations can affect either the required strength (demand) or the available strength (capacity), or both, depending on the method of design for stability.

While many computer analysis programs have the capability to implement relevant stability-design requirements (including second-order analysis), it is imperative that the engineer understand which requirements are implemented (and how), as well as the limits of applicability of each method, to ensure that the design is appropriate. This paper includes tables that provide specific guidance on applicability limits and requirements for different methods of stability design and second-order analysis. This information can guide effective design for stability using either computer or hand analysis. Additionally, tables provided in this paper can be used in approximate manual calculations of second-order effects to provide higher confidence that a computer program's second-order analysis has been properly implemented.

Design for Stability

The five general considerations for stability design are listed in AISC *Specification* Section C1. These are consideration of:

- Flexural, shear, and axial member deformations, and all other component and connection deformations that contribute to the displacements of the structure.

<table>
<tr><th colspan="5">Table 1. Methods of Addressing Stability-Design Considerations</th></tr>
<tr><th colspan="2">Stability-Design Consideration</th><th>Direct Analysis Method</th><th>Effective Length Method</th><th>First-Order Method</th></tr>
<tr><td colspan="2">(a) All deformations that contribute to the displacements of the structure</td><td colspan="3">Analysis of model that includes all significant sources of flexibility</td></tr>
<tr><td rowspan="2">(b) Second-order effects</td><td>System P-Δ effects (including P-δ effect on P-Δ)</td><td colspan="2">Second-order analysis</td><td>Additional lateral load</td></tr>
<tr><td>Member P-δ effects</td><td colspan="2">B_1 amplifier or inclusion of member P-δ effect in second-order analysis</td><td>B_1 amplifier</td></tr>
<tr><td>(c) Geometric imperfections (system)</td><td>Effect on structural response</td><td>Minimum notional load or modeling of imperfections</td><td colspan="2">Minimum notional load</td></tr>
<tr><td rowspan="3">(c) Geometric imperfections (member),
(d) Stiffness reduction due to inelasticity, and
(e) Uncertainty in strength and stiffness</td><td>Effect on structural response</td><td>Stiffness reduction</td><td>Effective length factor</td><td>Additional lateral load</td></tr>
<tr><td></td><td colspan="3">Member strength formulae</td></tr>
<tr><td>Effect on member strength</td><td colspan="3"></td></tr>
</table>

- Second-order effects (including both *P*-Δ and *P*-δ effects).
- Geometric imperfections.
- Stiffness reductions due to inelasticity—including the effect of partial yielding of the cross section, which may be accentuated by the presence of residual stresses.
- Uncertainty in system, member, and connection strength and stiffness.

The AISC *Specification* allows any rational method to address these stability considerations, including advanced analysis per Appendix 1. The AISC *Specification* also provides three simpler approaches: the direct analysis method, the effective length method, and the first-order analysis method. Table 1 shows how the direct analysis method, the effective length method, and the first-order method address each of the five general considerations for stability design. (Detailed requirements are presented in Table 2.)

The differences in how each of the three methods addresses stability considerations are as follows:

- The *direct analysis method*, presented in AISC *Specification* Chapter C (Section C2), is the most comprehensive and versatile of the three methods in incorporating the stability-design considerations into determination of required strength. This is achieved using notional loads (or modeling system imperfections), reduced stiffness, and a second-order analysis. The effects of member out-of-straightness and residual stresses on member strength are addressed in the determination of the available strength, through the column design strength formulas using an effective length equal to the unbraced length (i.e., K = 1.0) for all framing systems; lesser values of K can be justified by analysis.
- The *effective length method*, presented in AISC *Specification* Appendix 7 (Section 7.2), incorporates most of the stability considerations (such as system imperfections and second-order effects) into the determination of required strength. The effects of member out-of-straightness and residual stresses on member strength, and the effects of member out-of-straightness and residual stresses on structure stiffness, are dealt with in the determination of available strength through the column design strength formulas using effective lengths that may exceed member lengths (i.e., $K \geq 1.0$). Engineers should note that the use of reduced column available strength addresses these effects on the column (as opposed to the use of increased column required strengths in the direct analysis method) but does not address the corresponding load effects on beams and connections. The AISC *Specification* permits $K = 1.0$ for braced-frame and shear-wall structures. For moment-frame structures and mixed systems, $K = 1.0$ may be used when the ratio of second-order drift to first-order drift is less than or equal to 1.1. For moment-frame structures and mixed systems in which the second-order to first-order drift ratio exceeds 1.1, K is determined in accordance with AISC *Specification* Section 7.2. Effective length factors are determined by elastic buckling analysis, effective-length equations, or, more commonly, use of

alignment charts such as Figures C-A-7.1 and C-A-7.2 in the Commentary to the AISC *Specification* (provided that the associated assumptions are satisfied). Effective length factors, K, must be modified to K_2 to address the effect of gravity-only columns (often referred to as leaning columns), as described in Geschwindner (2002) and the Commentary to AISC *Specification* Section 7.2. Use of the effective length factor K_2 is distinct from consideration of the effects of leaning-column loads in a second-order analysis: the former addresses the vertical load being stabilized against sidesway buckling, and the latter addresses lateral-load amplification due to P-Δ stiffness reduction. Stability in gravity-only load combinations is addressed by means of a minimum lateral load.

- The *first-order analysis method*, presented in AISC *Specification* Appendix 7 (Section 7.3), is a conservative simplification of the direct analysis method, incorporating the stability-design considerations into determination of required strength. In lieu of a second-order analysis with reduced stiffness, however, this method utilizes a first-order analysis, with system-level stability-design requirements addressed through the application of an additional lateral load proportional to the story gravity load and corresponding either to the lateral story drift or to a minimum based on initial imperfections. (For convenience, the drift limit for each load combination may be used to conservatively determine the required additional lateral load in lieu of a calculated lateral drift.) This additional lateral load corresponds to the combined effects of the second-order story-drift (P-Δ) and member imperfections for the most severe condition possible within the limitations for this method (Griffis and White, 2013). There are significant limits on application of this method: a limit on moment-frame column axial stress to preclude any effect of column inelasticity on stability and a limit on axial forces in moment-frame beams to preclude conditions where beam P-δ effects could affect the column. (Both of these limits are evaluated with LRFD loads or with ASD loads amplified by α = 1.6.) While a second-order analysis is not required, the procedure is only permitted if the second-order story-drift amplification does not exceed 1.5; this may be demonstrated using an approximate second-order analysis such as determining the B_2 factor per AISC *Specification* Appendix 8 (and tabulated in Tables 4 and 5). As only a first-order analysis is required, the effect of gravity sway is not addressed, and therefore, nominally vertical columns are required. A limit on B_2 lower than 1.5 should be considered for systems with significant gravity sway. Additionally, member P-δ effects must be addressed by applying the amplification factor B_1 to total moments per Appendix 7, Section 7.3.2(b). The required strengths are taken as the forces and moments obtained from the analysis and the effective length factor is K = 1.0.

With higher second-order effects, there is increased sensitivity of the response to small changes or uncertainties in vertical loading or lateral stiffness, amplifying the inaccuracies resulting from simplifications inherent in each stability-design method. For this reason, the effective length method and the first-order analysis method are limited to systems in which the magnitude of second-order story-drift amplification does not exceed 1.5. Higher second-order story-drift amplification is permitted for the direct analysis method; the commentary to Section C1 suggests a limit of 2.5 (determined with reduced stiffness, corresponding to a value of 1.9 determined with nominal properties).

Table 2 presents a comparison of the requirements and limitations for the direct analysis method, the effective length method, and the first-order analysis method, including the "simplified method" (an adaptation of the effective-length method addressed later in this discussion).

Methods of Second-Order Analysis

Second-order effects are the additional forces and displacements due to applied loads as the structure transitions from the undeformed to the deformed geometry such that there is equilibrium between internal forces and external loads acting in their displaced positions. These effects can be categorized as P-Δ effects on the structure lateral displacement and corresponding internal forces, P-δ effects on member flexural forces, and P-δ influence on structure P-Δ. See AISC *Specification* Commentary to Section C2.1 and AISC Design Guide 28 Appendix Section A.2.1 for more discussion of P-Δ and P-δ effects.

Each of the stability-design methods in Table 2 requires a determination of second-order effects: The direct analysis method and the effective length method both require a second-order analysis, and the first-order analysis method requires determination that the magnitude of second-order amplification does not exceed 1.5. The Commentary to AISC *Specification* Chapter C provides guidance on methods of second-order analysis and presents second-order analysis results for several benchmark problems to facilitate checking the adequacy of analysis methods and computer programs. Ziemian and Ziemian provide multiple benchmark frames (2021). See Griffis and White for guidance on the use of benchmark problems (2013).

Table 3 summarizes AISC *Specification* requirements and recommendations for three methods of second-order analysis: general second-order analysis that captures both P-Δ and P-δ effects, P-Δ-only second-order analysis, and an approximate method of second-order analysis by means of amplified first-order analysis, as described in the following.

<table>
<tr><th colspan="6">Table 2. Summary Comparison of Methods for Elastic Stability Design</th></tr>
<tr><th rowspan="2"></th><th colspan="2">Direct Analysis Method</th><th colspan="2">Effective Length Method</th><th rowspan="2">First-Order Analysis Method</th></tr>
<tr><th>General</th><th>Limited</th><th>General</th><th>Simplified</th></tr>
<tr><td>Limitations on Use</td><td>None[a], [b]</td><td>Nominally Vertical columns[a]</td><td>Nominally vertical columns
$\Delta_{2nd}/\Delta_{1st} \le 1.5$[c]</td><td>Nominally vertical columns
$\Delta_{2nd}/\Delta_{1st} \le 1.5$[c]
$B_1 \le B_2$[d]</td><td>Nominally vertical columns
$\Delta_{2nd}/\Delta_{1st} \le 1.5$[c]
$\alpha P_r/P_{ns} \le 0.5$[e]
Beam $\alpha P_r/P_e \le 0.08$[e]</td></tr>
<tr><td>Analysis Type</td><td>Second-order elastic</td><td colspan="2">Second-order elastic or amplified first-order elastic[f]</td><td>Amplified first-order elastic[f]</td><td>First-order elastic[g]</td></tr>
<tr><td>Geometry of Structure</td><td>Initial imperfections</td><td colspan="2">Undeformed geometry (with notional loads) or initial imperfections[h]</td><td colspan="2">Undeformed geometry</td></tr>
<tr><td>Minimum or Additional Lateral Loads</td><td>None</td><td colspan="3">Minimum[i]; a times 0.2% of the story gravity load[e], [h]</td><td>Additive; greater of 0.42% or $2.1\alpha(\Delta_{1st}/L) \times$ story gravity load[e], [j]</td></tr>
<tr><td>Member Stiffnesses</td><td colspan="2">$0.8EA$ and $0.8\tau_b EI$[k]</td><td colspan="3">Nominal EA and EI</td></tr>
<tr><td>Column Available Strength[l]</td><td colspan="2">K = 1 for all frames</td><td colspan="2">K = 1 for braced frames and shear-wall structures.
K determined from sidesway buckling analysis, from effective-length equations, or from nomograph and adjusted to K_2 for moment frames[m]</td><td>K = 1 for all frames</td></tr>
<tr><td>AISC Specification Reference</td><td colspan="2">Chapter C</td><td colspan="2">Appendix 7, Section 7.2</td><td>Appendix 7, Section 7.3</td></tr>
</table>

[a] The commentary to AISC *Specification* Section C1 recommends that $\Delta_{2nd}/\Delta_{1st} \le 2.5$ using reduced stiffness.
[b] AISC *Specification* Section C2.2b requires that system imperfections be modeled explicitly for systems with sloped columns.
[c] $\Delta_{2nd}/\Delta_{1st}$ is the ratio of maximum second-order story drift to maximum first-order story drift, which can be taken equal to B_2 per AISC *Specification* Appendix 8. (The B_2 factor may be determined using Table 4 or Table 5.) $\Delta_{2nd}/\Delta_{1st}$ is determined using LRFD load combinations or a multiple of 1.6 times ASD load combinations for the vertical load.
[d] The Simplified Method is limited to systems for which the value of the B_1 amplifier never exceeds that of the B_2 amplifier.
[e] For ASD, $\alpha = 1.6$. Amplification by α and determination of appropriate ASD-level member-design forces are discussed under "Methods of Second-Order Analysis."
[f] See Table 3 for methods of second-order analysis and associated requirements.
[g] Amplification of non-sway moments due to member curvature is required; this is achieved by applying the amplification factor B_1 to total moments per Appendix 7, Section 7.3.2(b).
[h] Notional loads are computed with appropriate load factors for the combinations being considered. Direct modeling of imperfections may be used in lieu of notional loads for the direct analysis method per AISC Specification Section C2.2a.
[i] For the direct analysis method, the notional load is additive if $\Delta_{2nd}/\Delta_{1st} > 1.7$ using reduced stiffness or $\Delta_{2nd}/\Delta_{1st} > 1.5$ using nominal properties.
[j] The maximum value of the drift ratio Δ/L for all stories shall be used. Δ is the first-order interstory drift due to the LRFD or ASD load combination, as applicable. Where Δ varies over the plan area of the structure, Δ shall be the average drift weighted in proportion to vertical load or, conservatively, the maximum drift.
[k] The stiffness-reduction factor τ_b is a function of $\alpha P_r/P_{ns}$; see AISC *Specification* Equation C2-2b.
[l] Available strength is calculated in accordance with the provisions of Chapters D through K, as applicable, with the effective length based on the value of K listed in the table.
[m] K = 1 is permitted if $\Delta_{2nd}/\Delta_{1st} \le 1.1$.

- A *general second-order analysis* is an analysis that establishes equilibrium between internal and external forces in the deformed state and meets the requirements of AISC *Specification* Section C2.1 to capture both P-Δ and P-δ effects. Such analyses are typically iterative incremental analyses that employ either stability functions or geometric stiffness matrices that update coordinate locations with each loading increment. Each load combination therefore requires its own iterative analysis and superposition is not appropriate. Most computer structural analysis programs that support iterative analysis are capable of capturing P-δ effects by subdividing members into segments such that the deformed shape is reasonably well represented (White and Hajjar, 1991). If this subdivision is not automated, the engineer must explicitly divide these elements into segments when creating the analytical model. White et al. (2021) provide specific guidance for the number of elements required to accurately capture P-δ effects for various conditions, including for both prismatic and non-prismatic members (see also Griffis and White, 2013). If such segmentation is not employed, member P-δ effects may be addressed outside of the analysis by amplifying member moments as discussed below, subject to the limitations in AISC *Specification* Section C2.1(b) for a P-Δ-only analysis.

- A *P-Δ-only second-order analysis* is permitted by AISC *Specification* Section C2.1(b), which allows neglecting the influence of P-δ on P-Δ in the analysis subject to certain limitations.[1] (See Table 3.) Within these limits, the influence of P-δ on P-Δ is negligible, increasing forces no more than 1.0% and displacements by no more than 3.2% (Sabelli and Griffis, 2021). However, P-δ effects on member moments must still be considered; this is typically done by means of the B_1 amplifier from Appendix 8 on the member moments from the P-Δ-only second-order analysis. Although the iterative method may be used in a P-Δ-only second-order analysis, the noniterative geometric stiffness method is more common and more convenient. In the non-iterative method, the geometric stiffness matrix is modified based on a single defined vertical-load combination, permitting superposition of the results from analyses of separate load cases in factored load combinations. Some error may result from the difference between the vertical load combination used for stiffness modification and the vertical load combination used in superposition. Greater vertical loads results in lower stiffness, and thus greater second-order effects, so the largest vertical load combination should be used to determine the stiffness when used for a group of load combinations handled by superposition. For example, the engineer may establish one geometric-stiffness model to be used for all load combinations using the largest vertical load combination, or the engineer may establish one geometric-stiffness model to be used for gravity-only load combinations using the largest vertical load combination, and a different geometric-stiffness model to be used with lateral load combinations, for which the factors on the gravity loads are smaller and hence the magnitude of the vertical load is smaller.
- An *approximate second-order analysis* addresses P-Δ and P-δ effects by means of amplifiers on first-order analysis forces, as defined in AISC *Specification* Appendix 8. The B_1 amplifier addresses P-δ effects on member non-sway moments. The B_2 amplifier addresses P-Δ effects on members and the system, including the influence of P-δ on P-Δ amplification. B_2 is therefore applicable to all member and connection forces resulting from lateral loading or translation (including member shear and connection forces). The influence of P-δ on P-Δ amplification is typically addressed by means of a coefficient R_M in AISC *Specification* Equation A-8-7 for B_2; where AISC *Specification* Section C2.1(b) allows neglecting such effects, this coefficient may be taken as 1.0. Superposition of the results of analyses may utilize the B_2 amplifier specific to each vertical load combination, or a single B_2 factor corresponding to the largest vertical load combination of a group of load combinations may be applied for convenience. For example, the engineer may calculate one B_2 amplifier to be used with gravity-only load combinations and a (lower) B_2 amplifier to be used with lateral-load combinations. The B_2 amplifier may also be applied to the first-order displacement to approximate the second-order displacement; this has less than 2% error for $B_2 \leq 1.5$ (Sabelli and Griffis, 2021).

Additional discussion of these three methods can be found in Appendix A.

For asymmetrical geometry or loading, gravity forces may induce lateral sway. This gravity-induced lateral sway is subject to the same amplification due to second-order effects as is the sway from lateral loads. While this effect is captured directly in second-order analyses that include the gravity load, amplified first-order analyses require separation of translation and no-translation forces to correctly capture this effect, as discussed in the Commentary to AISC *Specification* Appendix 8. Such separation of forces is unnecessary when the gravity-induced sway is negligible compared to the lateral-load-induced sway, such as for the common case of a symmetrical structure with symmetrical vertical loading.

The full-story gravity load must always be included in the determination of P-Δ effects. This includes all forces coming from levels above and from floor and cladding loads at the current level, supported by both the gravity system (the so-called leaning gravity columns and walls, including laterally supported cladding) and the lateral force-resisting system. (The vertical seismic load effect

[1] ASCE/SEI 7 (2022), Section 12.8.7, provides a method of accounting for P-Δ effects that consists of amplifying forces and displacements by $1/(1-\theta)$, where θ is the stability coefficient. (Using AISC symbols, $\theta = P_{story}\Delta_H/HL$.) This method does not account for P-δ influence on P-Δ, and, as such, it only satisfies the requirements of the AISC *Specification* within the range for which AISC *Specification* Section C2.1(b) allows P-Δ-only second-order analysis. Furthermore, θ in ASCE/SEI 7 utilizes a different vertical load than the strength load combinations. The amplifier $1/(1-\theta)$ is identical to the B_2 amplifier for conditions in which there is no P-δ influence on P-Δ (i.e, $R_M = 1.0$) if the same vertical loads are used for both amplifiers.

Table 3. Summary Comparison of Methods for Second-Order Analysis for *P*-Δ and *P*-δ Effects

Type	Second-Order Analysis		Approximate Second-Order Analysis
Description	**General Second-Order Analysis**	***P*-Δ-Only Second-Order Analysis**	**Amplified First-Order Analysis**
Limitations on use in AISC *Specification*	None	Nominally vertical columns $\Delta_{2nd}/\Delta_{1st} \le 1.5$[a], [b] $P_{mf}/P_{story} \le 1/3$[c]	Nominally vertical columns
Recommended limit	None	$B_1 \le 1.2$ for members having significant effect on overall structural response[d]	
P-δ effect on members	Addressed within analysis by subdividing members	Addressed through B_1 amplifier on member moment	
P-δ influence on P-Δ	Addressed within analysis by subdividing members	Not directly addressed[d]	Addressed by the R_M factor or sidesway buckling analysis[d]
Method[e]	Incremental iterative analysis[e]	Noniterative geometric stiffness	B_1 amplifier on member moments; B_2 amplifier on lateral-load effects[e], [f]
Story gravity load[g]	Included in the analysis	Included in the determination of the geometric stiffness	Included in the calculation of the B_2 amplifier
Superposition of analysis results	Not applicable	Applicable with appropriate vertical load combination groupings	
AISC *Specification* reference	C2.1		C2.1; Appendix 8
Amplification of gravity-induced sway	Directly addressed in the analysis		Addressed by separate no-translation and translation analyses[h]
P-Δ effect on system	Directly addressed in the second-order analysis		Addressed through B_2 amplifier on lateral-translation forces
Second-order drift Δ_2	Directly determined in the second-order analysis		Approximated as the first-order drift amplified by B_2[a]
Factor for ASD (α)	1.6 for all loads	1.6 (gravity loads for determination of geometric stiffness reduction and axial forces for determination of B_1)	1.6 (gravity loads [αP_{story}] for determination of B_2 and axial forces for determination of B_1)

[a] $\Delta_{2nd}/\Delta_{1st}$ is the ratio of maximum second-order story drift to maximum first-order story drift, which can be taken equal to B_2 per AISC *Specification* Appendix 8. (The B_2 factor may be determined using Table 4 or Table 5.) $\Delta_{2nd}/\Delta_{1st}$ is determined using LRFD load combinations or a multiple of 1.6 times ASD load combinations.

[b] The limit of $\Delta_{2nd}/\Delta_{1st} \le 1.5$ for an analysis using full stiffness properties corresponds to $\Delta_{2nd}/\Delta_{1st} \le 1.7$ for a reduced-stiffness analysis as required for the direct analysis method.

[c] P_{mf}/P_{story} is the ratio of gravity load supported by columns that are part of moment-resisting frames in the direction of translation being considered to the total gravity load on the story.

[d] See AISC *Specification* Commentary Section C2.1, "Effect of Neglecting P-δ" and commentary to Appendix 8.

[e] Note that incremental iterative analysis and amplified first-order analysis can be utilized in a P-Δ-only second-order analysis. A P-Δ-only second-order analysis is often performed using the geometric-stiffness matrix method

[f] The elastic critical buckling strength may be determined from AISC *Specification* Equation A-8-7 or from a sidesway buckling analysis. B_2 factors may be obtained directly from Table 4 or Table 5.

[g] Story gravity load includes loading from levels above and on nonframe columns and walls, and the weight of wall panels laterally supported by the lateral force-resisting system. It need not include the vertical component of the seismic load.

[h] Separate no-translation and translation analyses are not required for the simplified method discussed later in this section.

need not be included.) Computer models of the lateral system should account for leaning columns to capture the P-Δ effects of gravity load not on the frame columns. For iterative second-order analysis, these leaning-columns deform laterally with the lateral force-resisting system, and their thrust adds to the lateral load at the equilibrium condition. For noniterative geometric-stiffness second-order analysis, the full-story gravity load must be included in the determination of the geometric stiffness matrix; P-Δ effects from the gravity loads on leaning columns are adequately represented in that method. For the approximate second-order analysis, the full story gravity load must be included in the calculation of the B_2 amplifier.

Typically, large areas of the building are considered in calculating P_{story}, and live-load reduction is permitted based on the tributary area. These reductions may be implemented relatively easily in a geometric-stiffness-matrix analysis or in computing a B_2 amplifier. For incremental second-order analysis, Ziemian and McGuire (1992) provide a method of using compensating forces at columns to achieve the appropriate reductions at the system level without affecting member-level forces.

In general, representing the gravity load using a single mass with a single displacement is adequate for two-dimensional analyses but will not capture amplification of plan torsion or the deformation of a nonrigid diaphragm. For iterative second-order analysis, distributed leaning columns corresponding to the actual column locations is preferred for buildings in which torsional movement (plan rotation) or diaphragm deformation may be significant (White and Hajjar, 1991). For the geometric stiffness method, the effects of plan torsion and diaphragm flexibility are captured if the stiffness modifications are based on the entire story mass and mass moment of inertia, not merely on the mass supported by the lateral frames; it is not necessary to model a column or set of columns to represent the leaning columns as long as the total gravity load and spatial gravity load distribution are accurately modeled. For approximate second-order analysis, amplification of plan torsion or the deformation of a nonrigid diaphragm can be bounded by using the maximum value of drift or by adjustments to the drift [Flores et al., 2018; ASCE, 2022 (Commentary to section 12.8.7)].

Both P-Δ effects and P-δ effects are nonlinear with respect to loading and thus cannot be scaled directly between LRFD and ASD. In the AISC *Specification*, adequate reliability is based on LRFD-level loading, with ASD providing similar reliability by adjustment of the action with nonlinear effect. As P-Δ effects are nonlinear with respect to vertical loads on the system, ASD vertical loads must be amplified with the load-adjustment factor α, to which the AISC *Specification* assigns a value of 1.6 for ASD [Section C2.1(d) and Appendix 8, Section 8.1.2]. Similarly, P-δ effects in members subject to flexure are nonlinear with respect to axial compressive force, which are amplified by this same factor for ASD design. Engineers using ASD should be attuned to having these effects properly captured with the application of α where required. Commentary to Section B3 discusses the differences between ASD and LRFD required strength under lateral load combinations.

When using an iterative second-order analysis for ASD design, AISC *Specification* Section C2.1(d) requires amplification of all loads (gravity, lateral, etc.) by α, prior to performing the second-order analysis. (Notional loads, which are already amplified by α in Equation C2-1, should not be amplified a second time.) Subsequent to the iterative second-order analysis, the member forces determined using this method are divided by α to be at ASD level.

Amplification of all ASD loads by α is unnecessary (and generally cumbersome) for the geometric-stiffness method or the approximate second-order analysis method. Instead, for these methods, α is typically only applied in the determination of the second-order effect; the loads used in the analysis are not amplified by α, nor are the member forces from that analysis subsequently divided by α. (The factor is applied for certain checks; see Table 2.) For ASD design using the geometric-stiffness method, the vertical loads used in the calculation of the stiffness reduction must be amplified by α; the resulting reduced stiffness is used with ASD-level forces. Similarly, P_{story} must be amplified by α for calculation of the B_2 amplification factor for approximate second-order analysis; this factor is applied to ASD-level load effects. For both the methods, in determining the B_1 amplification factor for P-δ magnification of member moment when using ASD, Equation A-8-3 amplifies axial compressive forces by α, resulting in the appropriate magnification of the ASD moment, which is combined with ASD-level axial forces using Chapter H of the AISC *Specification*.

The drift is determined directly in second-order analysis. For the amplified first-order method, the second-order drift may be approximated as the drift from a first-order analysis amplified by B_2 from Equation A-8-7 for the load combination being considered per AISC *Specification* Appendix 7, Section 7.2.1. See also LeMessurier (1977), Griffis and White (2013), and Sabelli and Griffis (2021) for drift amplification.

B_2 Amplifier for P-Δ Effects for Approximate Second-Order Analysis

As discussed earlier, approximate second-order analysis may be performed by amplifying forces from a first-order analysis for P-Δ and P-δ effects using amplifiers B_1 and B_2. Such an approximate second-order analysis may be used with the direct analysis method or the effective length method (including the simplified method discussed later in this section); it may also be used to confirm the applicability of the first-order-analysis method.

In such an analysis, the B_2 amplifier addresses P-Δ effects on the system, including the P-δ influence on structure P-Δ. The system stiffness is integral to the B_2 amplifier, and thus, the calculated drift or the drift limit may be used in its determination.

Determination of the $\mathbf{B_2}$ *Amplifier Using First-Order Drift*

The B_2 amplifier can be defined in terms of first-order system lateral stiffness (H/Δ_H), P-Δ stiffness reduction ($\alpha P_{story}/L$), and P-δ influence on structure P-Δ (R_M) by combining AISC *Specification* Equations A-8-6 and A-8-7:

$$B_2 = \frac{1}{1-\left(\dfrac{\alpha P_{story}}{R_M H}\right)\left(\dfrac{\Delta_H}{L}\right)} \tag{1}$$

where

H = total story shear, kips (N)

L = height of story, in. (mm)

P_{story} = total vertical load supported or braced by the story, kips (N)

R_M = stiffness-reduction coefficient to account for member P-δ influence on structure P-Δ

α = ASD/LRFD force level adjustment factor, equal to 1.0 (LRFD) or 1.6 (ASD)

Δ_H = first-order interstory drift, in. (mm)

The first-order interstory drift, Δ_H, is equivalent to Δ_{1st} in Tables 2 and 3. The stiffness-reduction coefficient R_M is defined by AISC *Specification* Equation A-8-8 in terms of P_{mf}/P_{story}, where P_{mf} is the total vertical load in moment frames columns.

Note that the ratio Δ_H/L is the so-called drift ratio for the lateral load H. The drift Δ_H should be determined from an analysis consistent with the stability design method used: reduced stiffness for the direct analysis method and nominal properties for the effective length method (and for verifying applicability of the first-order method). The value of B_2 so determined is appropriate for that stability-design method.

AISC *Specification* Section C2.1(b) states that P-δ influence on P-Δ amplification may be neglected for systems with second-order story-drift amplification less than 1.5 (1.7 for reduced stiffness) and in which no more than one-third of the gravity load is supported on moment frame columns. In such cases, taking R_M equal to 1.0 gives the same result as performing a P-Δ-only second-order analysis. Note that AISC *Specification* Equation A-8-8 for R_M gives a conservative, lower-bound value; see Sabelli and Griffis (2021) for a more precise formulation.

Determination of the $\mathbf{B_2}$ *Amplifier Using Drift Limit*

Sabelli et al. (2021) present a method for using the drift limit to bound the second-order effect, permitting determination of second-order amplification prior to design and analysis. This method can be used to obtain values of the second-order amplifier, B_2, based on the second-order drift ratio Δ_2/L adapting Equation 3 in Sabelli and Griffis (2021):

$$B_2 = 1+\frac{\alpha P_{story}\Delta_2}{HL} \tag{2}$$

The second-order drift, Δ_2, must not exceed the drift limit, Δ_{all}, and thus may conservatively be taken as equal to it. For seismic design, the drift is amplified by the deflection-amplification factor, C_d. (This factor may be taken as 1.0 for wind design.) Thus:

$$C_d\Delta_2 = \Delta_{all} \tag{3}$$

and

$$B_2 = 1+\left(\frac{\alpha P_{story}}{C_d H}\right)\left(\frac{\Delta_{all}}{L}\right) \tag{4}$$

Values so determined are the upper bound of second-order amplification for structures meeting the drift limit. These values are generally reasonable for drift-governed systems but may be excessively conservative for systems with drift significantly below the limit.

The vertical force, αP_{story}, used in Equation 4 should be consistent with analysis used for determining drift-limit compliance. For seismic design, the strength evaluation and drift evaluation typically utilize the same vertical load. For wind design, the serviceability drift evaluation typically utilizes a lower vertical load than the strength evaluation, and thus the amplifiers for the two evaluations may be significantly different. (Note that serviceability loads are not identical to Allowable Strength Design load combinations, and the factor α does not apply. For guidance on serviceability loads and drift limits, see Griffis, 1993.)

Inherent in this method is the assumption that second-order analysis is used to determine drift. Use of unamplified first-order analysis to determine drift is unconservative generally (LeMessurier, 1977) and incompatible with this method. That is, Equation 2 utilizes the second-order drift, and thus, values of the amplifier B_2 obtained using Equation 4 (which is derived from Equation 2) correspond to systems stiff enough such that the second-order drift meets the drift limit. A system in which the first-order drift is at the drift limit is more flexible than assumed in Equation 4 and thus the amplifier values will not be correct. To ensure the validity of this method (and of the design), the first-order drifts must be amplified to capture second-order effects when determining conformance to the drift limits. Amplification by the B_2 factor is typically sufficient to capture these effects.

Tables for Determination of the B_2 *Amplifier for Approximate Second-Order Analysis*

Tables 4 and 5 provide values for the B_2 amplifier based on first-order drift (Table 4) and the drift limit (Table 5). Table 5 may be used as an upper bound; once a drift value is determined by analysis, Table 4 may provide a lower value for systems stiffer than required to meet the drift limit.

The values in Table 4 are calculated for pairs of load ratio modified by the P-δ stiffness-reduction coefficient ($\alpha P_{story}/R_M H$) versus drift index (Δ_H/L), based on Equation 1. The values in Table 5 are calculated for pairs of load ratio modified by the deflection-amplification factor ($\alpha P_{story}/C_d H$) versus drift-limit ratio (Δ_{all}/L), based on Equation 4. The drift values and limits provided in Tables 4 and 5 include those for seismic design per ASCE/SEI 7 (2022), Table 12.12-1, as well as drift limits commonly used for wind serviceability design. The vertical loads, αP_{story}, used in the modified load ratios are effectively at LRFD level. Both tables require that the lateral load H be the load used for determination of drift or drift-limit compliance. The resulting value of B_2 is based on the system stiffness (Table 4) or the minimum required effective (second-order) system stiffness (Table 5) and is, therefore, applicable to the strength design for any level of lateral load (ASD or LRFD). In Table 4, the load ratio is modified by the P-δ stiffness-reduction R_M. In Table 5, it is modified by the seismic deflection-amplification factor C_d.

In Tables 4 and 5, values of B_2 that are less than or equal to 1.1 are indicated in the white region of the table; in this region, an effective length factor of 1.0 may be used, regardless of the method of stability design. The light gray portions of the table represent the regions where B_2 is greater than 1.1 and no greater than 1.5; in this region, the effective length method may require an effective length factor that is greater than 1.0. The dark gray represents the regions where B_2 is greater than 1.5 and no greater than 2.5; in this region, the direct analysis method is permitted but the first-order and effective-length methods are not. The black-shaded region is where B_2 is greater than 2.5; it is recommended that the structure be stiffened when B_2 is greater than this limit.

The reduced-stiffness model used in the direct analysis method is not intended to amplify the calculated drift; the engineer may account for this by increasing the drift limit when using a reduced-stiffness model. However, the recommended limit of 2.5 for the direct analysis method applies to determination of B_2 using the reduced-stiffness model.

Interpolation may be used with both Tables 4 and 5. In Table 4, interpolation is conservative between values of $\alpha P_{story}/R_M H$ and between values of Δ_H/L (or between values of Δ_H), resulting in a value of B_2 larger than would be determined by direct calculation. In Table 5, linear interpolation between, and extrapolation beyond, values of Δ_{all}/L and $\alpha P_{story}/C_d H$ are valid due to the simplicity of Equation 4.

Simplified Method for Determination of Required Strength

When a quick, conservative stability-design solution is desired, the following "simplified method" presented by Carter and Geschwindner (2008) can be used. This method is based on the effective length method for stability design, utilizing the approximate second-order analysis method in AISC *Specification* Appendix 8 and is, therefore, subject to the limitations on those two methods as presented in Tables 1 and 2.

Additionally, to permit the use of a single amplifier, the method is limited to systems in which the value of the amplifier B_1 does not exceed that of B_2. This is typically true for members that are not subject to significant transverse loading between their ends (a condition that makes it unlikely that B_1 is greater than 1.0); this is generally the case for columns. Additionally, even though the beams in a frame may be subject to transverse loads, they tend to have low values of B_1 due to the low ratio of αP_r to P_{e1} (used in AISC *Specification* Equation A-8-3). While these two general conditions apply to many (or even most) buildings, the engineer is nevertheless responsible for ensuring that their design is within the limitations for this method.

If B_1 does not exceed B_2, it is conservative to amplify the total moment from a first-order analysis ($M_{nt} + M_{lt}$) by B_2 to determine the required strength, M_r, effectively setting B_1 equal to B_2. Thus, AISC *Specification* Equation A-8-1 becomes:

$$M_r = B_1 M_{nt} + B_2 M_{lt} \le B_2 \left(M_{nt} + M_{lt} \right) \tag{5}$$

where

B_1 = multiplier to account for P-δ effects

M_{lt} = first-order moment due to lateral translation of the structure only, kip-in. (N-mm)

M_{nt} = first-order moment with the structure restrained against lateral translation, kip-in. (N-mm)

Similarly, for convenience, the other total forces from a first-order analysis, such as total axial force ($P_{nt} + P_{lt}$), can be amplified conservatively by B_2 to determine the required strength, P_r, and AISC *Specification* Equation A-8-2 becomes:

$$P_r = P_{nt} + B_2 P_{lt} \le B_2 \left(P_{nt} + P_{lt} \right) \tag{6}$$

where

P_{lt} = first-order axial force due to lateral translation of the structure only, kips (N)

P_{nt} = first-order axial force with the structure restrained against lateral translation, kips (N)

Table 4. B_2 Amplifier for Approximate Second-Order Analysis for P-Δ Effects Using First-Order Drift									
		$\alpha P_{story}/R_M H$							
Δ_H/L	Δ_H	10	20	40	60	80	100	160	200
0.0005	L/2000	1.01	1.01	1.02	1.03	1.04	1.05	1.09	1.11
0.0010	L/1000	1.01	1.02	1.04	1.06	1.09	1.11	1.19	1.25
0.0020	L/500	1.02	1.04	1.09	1.14	1.19	1.25	1.47	1.67
0.0025	L/400	1.03	1.05	1.11	1.18	1.25	1.33	1.67	2.00
0.0040	L/250	1.04	1.09	1.19	1.32	1.47	1.67		
0.0050	L/200	1.05	1.11	1.25	1.43	1.67	2.00		
0.0067	L/150	1.07	1.15	1.36	1.67	2.14			
0.0100	L/100	1.11	1.25	1.67	2.50				
0.0150	L/67	1.18	1.43	2.50					
0.0200	L/50	1.25	1.67						
0.0250	L/40	1.33	2.00						$B_2 > 2.50$; stiffen structure

Notes:
1. White region: $1.0 < B_2 \le 1.1$
2. Light gray region: $1.1 < B_2 \le 1.5$
3. Dark gray region: $1.5 < B_2 \le 2.5$
4. Black region: $B_2 > 2.5$

Table 5. B_2 Amplifier for Approximate Second-Order Analysis for P-Δ Effects Using Drift Limit (Second-Order Drift)									
		$\alpha P_{story}/C_d H$							
Δ_{all}/L	Δ_{all}	10	20	40	60	80	100	160	200
0.0005	L/2000	1.01	1.01	1.02	1.03	1.04	1.05	1.08	1.10
0.0010	L/1000	1.01	1.02	1.04	1.06	1.08	1.10	1.16	1.20
0.0020	L/500	1.02	1.04	1.08	1.12	1.16	1.20	1.32	1.40
0.0025	L/400	1.03	1.05	1.10	1.15	1.20	1.25	1.40	1.50
0.0040	L/250	1.04	1.08	1.16	1.24	1.32	1.40	1.64	1.80
0.0050	L/200	1.05	1.10	1.20	1.30	1.40	1.50	1.80	2.00
0.0067	L/150	1.07	1.13	1.27	1.40	1.53	1.67	2.07	2.33
0.0100	L/100	1.10	1.20	1.40	1.60	1.80	2.00		
0.0150	L/67	1.15	1.30	1.60	1.90	2.20	2.50		
0.0200	L/50	1.20	1.40	1.80	2.20				
0.0250	L/40	1.25	1.50	2.00	2.50				$B_2 > 2.50$; stiffen structure

Notes:
1. White region: $1.0 < B_2 \le 1.1$
2. Light gray region: $1.1 < B_2 \le 1.5$
3. Dark gray region: $1.5 < B_2 \le 2.5$
4. Black region: $B_2 > 2.5$

By applying the factor B_2 to all forces, the simplified method amplifies both lateral-load and gravity-sway effects, obviating the need for separate translation and no-translation analyses otherwise required for systems with significant gravity sway using the amplified first-order analysis method. To permit such an approach, the gravity load causing lateral sway must be amplified by the factor B_2 in determining the lateral drift Δ_H such that the gravity-load effect on the lateral load-resisting system is captured in the B_2 amplifier selected. Application of the factor B_2 is not required for axial forces on members that do not have forces resulting from or inducing lateral translation, such as vertical leaning columns designed as pin ended. As discussed in the Methods of Second-Order Analysis section, the B_2 amplification applies to all lateral-load effects, including member shear and connection forces. B_2 amplification factors may be obtained from Table 4 or Table 5, as discussed in the following.

The simplified method consists of six steps:

Step 1: Establish story drift limit and the corresponding lateral load. Together these represent the minimum required lateral stiffness of the structure.

Step 2: Perform a first-order elastic analysis. Gravity load cases must include a minimum lateral load at each story equal to 0.002 times the story gravity load, where the story gravity load is the load introduced at that story, independent of any loads from above.

Step 3: Determine the ratio of the total story gravity load to the lateral load determined in Step 2. For an ASD design, this ratio must be multiplied by 1.6. If Table 4 is used, this ratio is divided by the coefficient R_M for moment-frame structures. If Table 5 is used, this ratio is divided by the factor C_d for seismic design.

Step 4: Multiply all of the forces and moments from the first-order analysis by the value of B_2 obtained from Table 4 or Table 5. (Axial forces in leaning columns need not be amplified.) Use the resulting forces and moments as the required strengths for the designs of all members and connections. Note that B_2 must be computed for each story and in each principal direction.

Step 5: For all cases where the B_2 amplifier is 1.1 or less, the effective length may be taken as the unbraced length (i.e., K = 1.0). For cases where the B_2 amplifier is greater than 1.1 but does not exceed 1.5, determine the effective length factor as described in the Design for Stability section for the effective-length method. For cases where the value exceeds 1.5, the structure is too flexible to permit use of this method; either the direct analysis method should be used, or the structure must be stiffened.

Step 6: Compute the approximate second-order drifts by amplifying first-order drifts from Step 2 using the amplifier B_2 (and the factor C_d for seismic design). Compare to the drift limit set in Step 1. Revise the design as needed.

Note that using the drift limit (rather than the calculated first-order drift) in Table 4 is a conservative simplification. Using Table 5 removes this simplification but conservatively assumes the (second-order) drift is equal to the drift limit. Regardless of which table is used in the first iteration prior to analysis, iteration using the calculated first-order drift (with Table 4) can reduce the B_2 amplifier (Sabelli et al., 2021). For more information on this simplified method, see Carter and Geschwindner (2008).

CONCLUSION

Proper analysis and design include consideration of stability and the conditions that affect stability. The AISC *Specification* provides several practical approaches, each one valid within the limitations specified. The individual stability effects can be handled either by determining a more accurate (usually greater) value of required strength (demand) through more detailed modeling and analysis, or by imposing reductions to the available strength (capacity). Practical simplifications can often acceptably be made without unduly affecting the economy of the design; some such methodologies and approaches are described in this paper.

APPENDIX A

Diagram of Three Methods of Second-Order Analysis

Second-order effects increase deflection, and P-Δ effects (including P-δ influence on P-Δ) make the structure more flexible. Each of the three methods of second-order analysis discussed in this paper can be considered to address structure-level second-order stiffness reduction in a different way.

Figure A-1 shows first-order and second-order forces and displacements for the three methods of second-order analysis discussed earlier. For all three methods, the vertical load, P_{story}, and the external lateral load, H, are the same, but the second-order displacements and internal forces vary between methods. [To highlight differences, high values of P_{mf}/P_{story} (1.0) and B_2 (1.8) are used.]

For general second-order analysis (point 1), the stiffness is reduced by both the P-Δ and the P-δ effects. The external force H causes a displacement, Δ_2, determined by second-order analysis. The internal forces correspond to this lateral deformation and the internal second-order stiffness, which includes P-δ softening. In the figure, the second-order effects are represented by the displacement amplification factor, D_{AF}, and the force amplification factor, F_{AF}, based on LeMessurier's work (1977) and discussed in Griffis and White (2013).

For P-Δ-only second-order analysis (point 2), the P-δ effects on P-Δ are not included, and thus both the second-order displacement and the second-order forces are underestimated. (The condition in Figure A.1 with P_{mf}/P_{story} = 1.0 is well outside the range permitted for P-Δ-only second-order analysis, and thus, the degree of underestimation exceeds that permitted by the AISC *Specification*.) The internal forces correspond to the second-order lateral deformation and the lateral stiffness without P-δ softening. Both displacements and forces are amplified by $1/(1-\theta)$, with θ determined using the appropriate vertical forces for strength design. (This is identical to B_2 computed with $R_M = 1.0$.)

For approximate second-order analysis, a first-order analysis is performed (point 3), and first-order forces are amplified by the factor B_2, which may also be used to approximate second-order displacements (point 4). Note that the force amplification is overestimated due to the simplified equation for R_M in AISC *Specification* Appendix 8. Displacement is slightly underestimated.

If there are no P-δ effects on P-Δ, the differences between the three methods disappear, and points 1 and 4 move to point 2. Similarly, the differences between methods are much less significant for cases with smaller second-order effects.

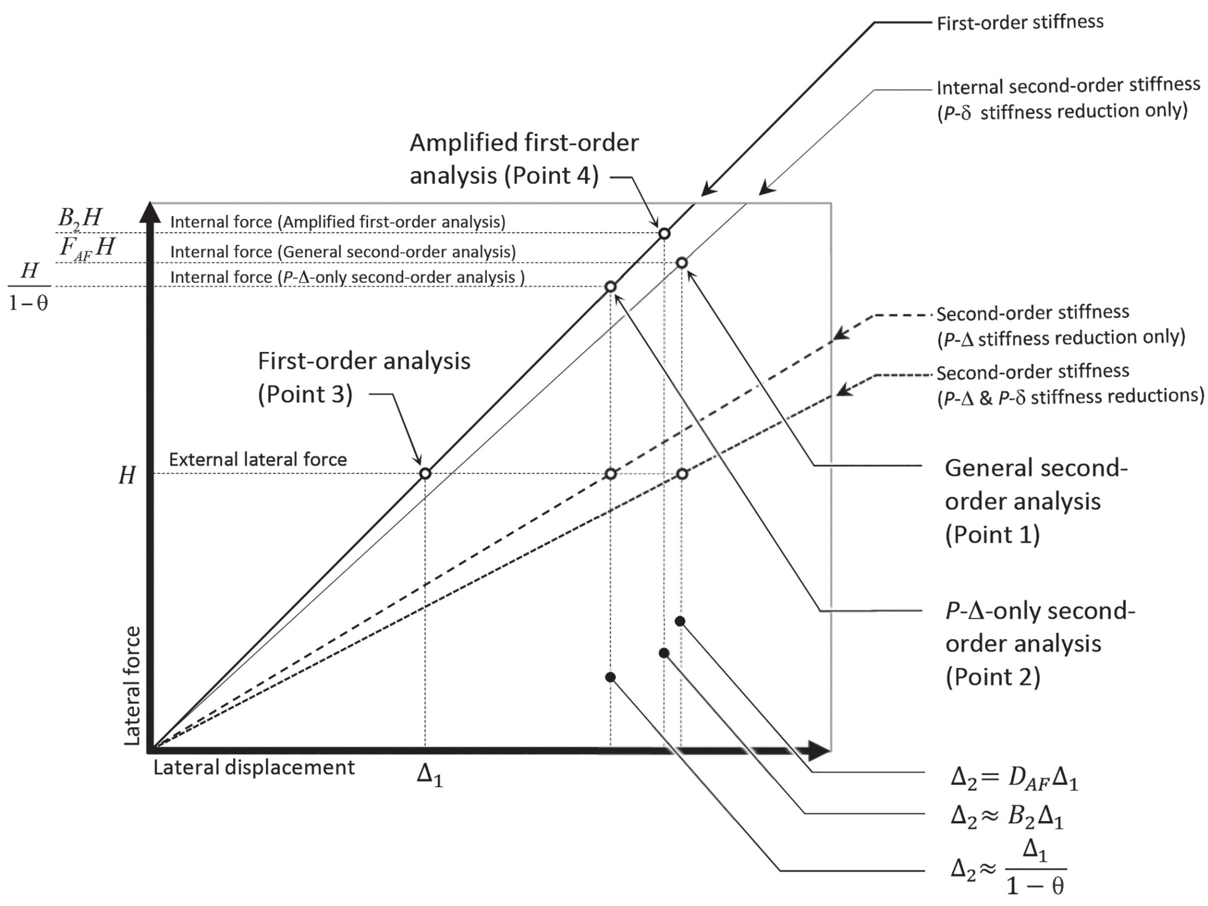

Fig. A-1. Diagram of methods of second-order analysis.

APPENDIX B

Design Example

Application of stability-design and second-order-analysis requirements are illustrated in the following example. The example utilizes amplified first-order analysis in combination with tables and equations in the paper to determine second-order forces and displacements (i.e., approximate second-order analysis) using the direct analysis method. For comparison purposes, results from a true second-order analysis are presented as well.

While it is not typical to perform direct analysis using tables and hand methods, this design example also demonstrates how these methods can give the engineer higher confidence in the results of a computer second-order analysis by confirmation with simpler methods. Results from computer analysis programs that have been validated using benchmark problems may nevertheless be incorrect due to missing gravity loads or other implementation errors.

The example is adapted from AISC Design Guide 28, Example 3.2 (Griffis and White, 2013). For brevity, the reader is referred to the original example for certain portions of the design not relevant to the illustration of the methods presented in this paper. The example consists of the following steps:

1. Determination of loads (see Griffis and White, 2013).
2. Determination of second-order amplification for service-level loads.
3. Selection of members to meet serviceability drift limit.
4. Determination of second-order amplification for strength-level loads.
5. Determination of member design forces.
6. Member strength checks (see Griffis and White, 2013).

Figure B-1 shows the building plan.

For brevity, the design example presented here is only for the north-south moment frames and is limited to two load combinations: a serviceability load combination used for a drift check and a strength load combination used to determine a member force. Similarly, for simplicity only, uniform loading is considered; this obviates the need to amplify gravity-sway moments.

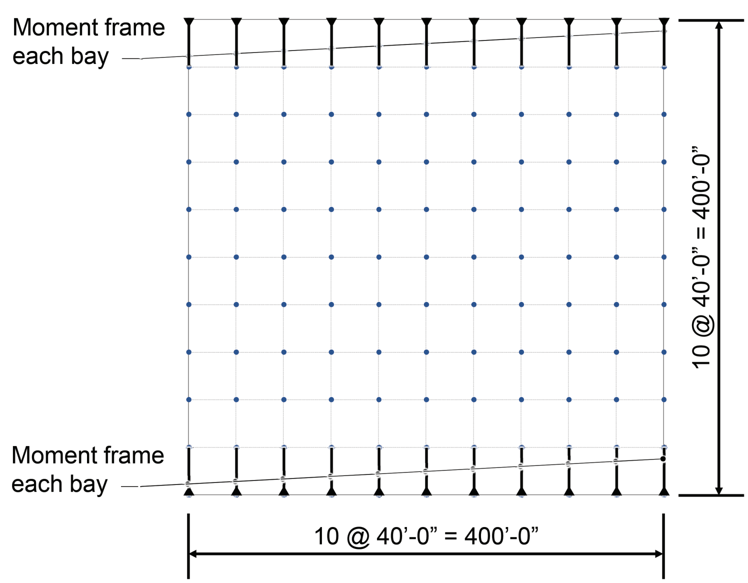

Fig. B-1. Plan of example building (from Griffis and White, 2013).

Table B.1. Load Combinations		
Designation	**Description**	**Combination**
Combination S	Service	$1.0D + 0.5S + 1.0W_y$
Combination 12	Strength	$1.2D + 0.5S - 1.6W_y - N_y$

Table B.2. Loads on Example Building			
Designation	**Symbol**	**Value (kips)**	**Comment**
Dead	D	5120	Uniformly distributed
Snow	S	4800	Nonuniform loading is not considered
Wind	W_y	120	50-year wind in building y-axis (north–south) direction
Notional	N_y	17.1	Notional load for Combination 12 in building y-axis direction $N_y = 0.002[1.2(5{,}120 \text{ kips}) + 0.5(4{,}800 \text{ kips})]$

Note that the original example utilizes ASCE/SEI 7-10 (ASCE, 2010) and thus the wind-load factors differ from those used in ASCE/SEI 7-22. Load combinations are presented in Table B.1 and loads in Table B.2.

Figure B-2 shows an elevation of the typical two-member frame.

Additionally, for purposes of illustration of drift-governed conditions, the example assumes a serviceability drift limit that differs from the original example:

$$\Delta_{service} \le \Delta_{all}$$
$$= 2.50 \text{ in.}$$
$$\frac{\Delta_{all}}{L} = \frac{2.50 \text{ in.}}{360 \text{ in.}}$$
$$= 0.00694$$

Determination of Second-Order Amplification for Service-Level Loads

To begin, Table 5 is used to determine the second-order amplification for the serviceability condition.

For wind serviceability, $C_d = 1.0$ and $\alpha = 1.0$.

$$\frac{\alpha P_{story}}{C_d H} = \frac{(1.0)(7{,}520 \text{ kips})}{(1.0)(120 \text{ kips})}$$
$$= 62.7$$

From Table 5 (or Equation 4), the estimated value of the amplifier is determined as B_2(service) = 1.43. Using this amplifier, members are selected such that the second-order drift meets the drift limit.

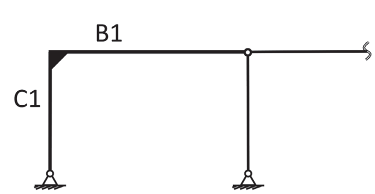

Fig. B-2. Elevation of typical frame (from Griffis and White, 2013).

Table B.3. Beam and Column Sizes		
Member	**Edge Frames**	**Interior Frames**
C1	W24×117	W24×117
B1	W24×131	W24×146

For service-level loading the maximum first-order drift can be estimated by:

$$
\begin{aligned}
\Delta_1 &\le \frac{\Delta_{all}}{B_{2(service)}} \\
&= \frac{2.50 \text{ in.}}{1.43} \\
&= 1.75
\end{aligned}
$$

Use of B_2 as a displacement amplifier is reasonable for low values of P_{mf}/P_{story}. For larger values of P_{mf}/P_{story} a second-order analysis may be more appropriate, especially for larger values of B_2. See Sabelli and Griffis (2021) for numerical comparisons.

Selection of Members to Meet Serviceability Drift Limit

Column and beam sizes are selected such that the first-order drift does not exceed this value. Sizes are given in Table B.3. (These sizes differ from the service evaluation in the Design Guide but are used in the subsequent strength evaluation.)

With the moment frame column and girder sizes listed in Table B-3, the first-order story drift from the first-order analysis is:

$$\Delta_1 = 1.72 \text{ in.}$$

For comparison and validation purposes, this first-order drift from analysis can be used to calculate B_2 more accurately:

$$
\begin{aligned}
R_M &= 1 - 0.15\left(P_{mf} / P_{story}\right) &&\text{(}\textit{Spec}\text{. Eq. A-8-8)} \\
&= 1 - 0.15(848 \text{ kips}/7{,}520 \text{ kips}) \\
&= 0.983
\end{aligned}
$$

Using Table 4:

$$
\begin{aligned}
\frac{\alpha P_{story}}{R_M H} &= \frac{1.0(7{,}520 \text{ kips})}{0.983(120 \text{ kips})} \\
&= 63.8 \\
\frac{\Delta_H}{L} &= \frac{1.72 \text{ in.}}{360 \text{ in.}} \\
&= 0.00477 \\
B_2 &= 1.46
\end{aligned}
$$

This value is determined conservatively using $\Delta/L = 0.005$ and between $\alpha P_{story}/R_M H = 60$ and 80.

Alternatively, using Equation B-1:

$$B_2 = \frac{1}{1 - \left(\frac{\alpha P_{story}}{R_M H}\right)\left(\frac{\Delta_H}{L}\right)} \tag{B-1}$$

$$
\begin{aligned}
&= \frac{1}{1 - \left[\frac{1.0(7{,}520 \text{ kips})}{0.983(120 \text{ kips})}\right]\left(\frac{1.72 \text{ in.}}{360 \text{ in.}}\right)} \\
&= 1.44
\end{aligned}
$$

Table B.4. Column and Beam End Moments from First-Order Analysis		
		Column C1 and Beam B1 End Moments (kip-ft)
Dead	M_D	130
Snow	M_S	122
Wind	M_W	166
Notional	M_N	23.6

Both the estimated first-order drift and the estimated second-order amplification for service-level loading are close to the values based on analysis. Because AISC *Specification* Equation A-8-8 for determining R_M is somewhat conservative, Equation 4, which conservatively uses the drift limit but does not use R_M, gives a slightly lower result than Equation B-1, which uses the actual first-order drift but requires R_M, for this drift-controlled example.

Determination of Second-Order Amplification for Strength-Level Loads

Next, the B_2 amplifier is calculated for member strength design for Combination 12. (Note that Combination 12 does not have the maximum vertical load. The B_2 amplifier for this combination is not appropriate for strength design using load combinations with higher vertical load.) For strength-level loading using the direct analysis method, there are two differences that affect the B_2 amplifier. First, the vertical load corresponds to the strength-level load combinations. Second, the lateral stiffness of the frame is reduced by a factor 0.8, and thus for the lateral load H, Δ_H = 1.72 in./0.8. For conditions in which the additional flexural stiffness reduction factor τ_b applies, use of Equation B-1 to capture the total direct-analysis stiffness reduction is not appropriate. The value of τ_b = 1.0 is typically confirmed after analysis, but in this case, it is obvious by inspection.

$$B_2 = \frac{1}{1-\left(\frac{\alpha P_{story}}{R_M H}\right)\left(\frac{\Delta_H}{L}\right)} \tag{B-2}$$

$$= \frac{1}{1-\left[\frac{1.0(8{,}544 \text{ kips})}{0.983(120 \text{ kips})}\right]\left(\frac{1.72 \text{ in.}/0.8}{360 \text{ in.}}\right)}$$

$$= 1.76$$

Determination of Member Design Forces

A first-order analysis is performed. The frame is modelled with reduced stiffness for the direct analysis method, with τ_b = 1.0. End moments for column C1 and beam B1 for an interior frame are presented in Table B.4.

The amplified first-order analysis results for Combination 12 are:

$$M_u = 1.2M_D + 0.5M_S + B_2(1.6M_W + M_N)$$

$$= 1.2(130 \text{ kip-ft}) + 0.5(122 \text{ kip-ft}) + (1.76)\left[1.6(166 \text{ kip-ft}) + (23.6 \text{ kip-ft})\right]$$

$$= 726 \text{ kip-ft}$$

For comparison, an iterative, incremental second-order analysis using Combination 12 is also performed, with appropriate column meshing to capture P-δ influence on structure P-Δ. The second-order analysis for Combination 12 gives:

$$M_u = 719 \text{ kip-ft}$$

From this second-order analysis, the second-order amplification using the direct analysis reduced-stiffness strength model for Combination 12 is:

$$\begin{aligned}\frac{\Delta_2}{\Delta_1} &= \frac{6.53 \text{ in.}}{3.74 \text{ in.}}\\ &= 1.75\end{aligned}$$

This value compares well with the value of 1.76 determined for B_2. The slight overestimation of forces using the B_2 amplifier (approximately 1%) can be attributed to the conservatism of AISC *Specification* Equation A-8-8 for determining R_M.

GLOSSARY

Amplifier. Factor applied to load effect from first-order analysis to approximate load effect from second-order analysis.

Approximate second-order analysis. Amplified first-order analysis approximating second-order effects by amplifiers B_1 (for member P-δ effects) and B_2 (for P-Δ effects).

Braced frame. Essentially, a vertical truss system that provides resistance to lateral forces and provides stability for the structural system.

Buckling. Limit state of sudden change in the geometry of a structure or any of its elements under a critical loading condition.

Buckling strength. Strength for instability limit states.

Drift. Lateral deflection of structure.

Drift ratio. Interstory drift divided by story height, taken at a representative location.

Effective length factor, K. Ratio between the effective length and the unbraced length of the member.

Effective length. Length of an otherwise identical compression member with the same strength when analyzed with simple end conditions.

Elastic analysis. Structural analysis based on the assumption that the structure returns to its original geometry on removal of the load.

First-order analysis. Structural analysis in which equilibrium conditions are formulated on the undeformed structure; second-order effects are neglected.

First-order stiffness. Lateral stiffness of the structure neglecting second-order effects.

Geometric imperfections:

Member imperfection. Initial displacement of points along the length of individual members (between points of intersection of members) from their nominal locations, such as the out-of-straightness of members due to manufacturing and fabrication.

System imperfection. Initial displacement of member intersections from their nominal locations, such as the out-of-plumbness of columns due to erection tolerances.

Gravity sway. Lateral drift caused by vertical gravity loads on the undeformed structure (i.e., considered without imperfections or notional loads).

Inelastic analysis. Structural analysis that takes into account inelastic material behavior, including plastic analysis.

Internal second-order stiffness. Lateral stiffness of the structure relating displacements to internal (member) forces, modified considering the reduced flexural stiffness of members with compressive axial force (P-δ stiffness-reduction).

Interstory drift. Drift at a given story relative to the drift at the story below taken at vertically aligned points.

Instability. Limit state reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry produces large displacements.

Lateral force-resisting system. Structural system designed to resist lateral loads and provide stability for the structure as a whole.

Lateral load. Load acting in a lateral direction, such as wind or earthquake effects.

Leaning column. Column designed to carry gravity loads only, with connections that are not intended to provide resistance to lateral loads.

Moment frame. Framing system that provides resistance to lateral loads and provides stability to the structural system, primarily by shear and flexure of the framing members and their connections.

Notional load. Virtual load applied in a structural analysis to account for destabilizing effects that are not otherwise accounted for in the design provisions.

P-Δ *effect.* Effect of loads acting on the displaced location of joints or nodes in a structure. In tiered building structures, this is the effect of loads acting on the laterally displaced location of floors and roofs

P-δ *effect.* Effect of loads acting on the deflected shape of a member between joints or nodes.

P-δ *stiffness-reduction*. Reduction of flexural stiffness of members due to the presence of axial compression, affecting system lateral stiffness and increasing the *P*-Δ effect.

Second-order analysis. A structural analysis that solves for equilibrium between internal and external forces in the deformed state.

General second-order analysis. A second-order analysis in which *P*-δ effects *P*-Δ effects are directly analyzed.

P-Δ only second-order analysis. A second-order analysis in which *P*-Δ effects are directly analyzed and *P*-δ effects are addressed by means of application of B_1 amplifiers.

Rigorous second-order analysis. A general second-order analysis that includes consideration of additional second-order effects related to member twist. (See AISC *Specification* Appendix 1 Section 1.2a.)

Second-order effect. Effect of loads acting on the deformed configuration of a structure; includes *P*-Δ effect, *P*-δ effect, and *P*-δ stiffness reduction.

Second-order stiffness. Lateral stiffness of the structure relating displacements to external loads, modified considering the *P*-Δ effect.

Stability. Condition in the loading of a structural component, frame, or structure in which a slight disturbance in the loads or geometry does not produce large displacements.

Stability design. Structural design that addresses the five general considerations in AISC *Specification* Section C1.

Stiffness reductions. Modifications in axial and flexural stiffness in the direct analysis method to capture destabilizing effects of member imperfections and inelasticity as well as uncertainties in strength and stiffness.

Story stiffness. Story shear divided by interstory drift. Story stiffness is sensitive to the loading profile; use of the design load profile to determine story stiffness is recommended.

SYMBOLS

A	Cross-sectional area of member, in.^{2} (mm^2)
B_1	Multiplier to account for *P*-δ effects
B_2	Multiplier to account for *P*-Δ effects
C_d	Deflection amplification coefficient for seismic analysis
D_{AF}	Displacement amplification factor
E	Modulus of elasticity of steel, ksi (MPa)
F_{AF}	Force amplification factor, similar to B_2
H	Total story shear, in the direction of translation being considered, produced by the lateral forces used to compute Δ_H, kips (N)
I	Moment of inertia in the plane of bending, in.^{4} (mm^4)
K	Effective length factor
K_2	Effective length factor modified for effect of leaning columns
L	Height of story, in. (mm)
M_{lt}	First-order moment due to lateral translation of the structure only, kip-in. (N-mm)
M_{nt}	First-order moment with the structure restrained against lateral translation, kip-in. (N-mm)
P_e	Elastic critical buckling strength of member kips (N)
P_{ns}	Member compressive strength, kips (N)
P_r	Member required strength, kips (N)
P_{lt}	First-order axial force due to lateral translation of the structure only, kips (N)
P_{mf}	Total vertical load in columns in the story that are part of moment frames, if any, in the direction of translation being considered (= 0 for braced-frame systems), kips (N)
P_{nt}	First-order axial force with the structure restrained against lateral translation, kips (N)
P_{story}	Total vertical load supported by the story using LRFD or ASD load combinations, as applicable, including loads in columns that are not part of the lateral-force-resisting system, kips (N)
R_M	Stiffness-reduction coefficient to account for member *P*-δ influence on structure *P*-Δ
α	ASD/LRFD force level adjustment factor, equal to 1.0 (LRFD) or 1.6 (ASD)
Δ_{all}	Allowable interstory drift, in. (mm)
Δ_H	First-order interstory drift, in the direction of translation being considered, due to lateral forces, in. (mm)
Δ_{1st}, Δ_1	First-order interstory drift, equal to Δ_H, in. (mm)
Δ_{2nd}, Δ_2	Second-order interstory drift, in. (mm)
τ_b	Flexural stiffness reduction factor for direct analysis method
θ	Stability coefficient from ASCE 7, Section 12.8.7

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