Fundamentals of Orthotropic Plate Design

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THE USE OF steel bridge deck supported by closely spaced stringers and floor beams can be traced back to the "battledeck floor" introduced by the American Institute of Steel Construction in the 1930's. However, economy in design has been realized only in recent years with steel deck constructed of a welded monolithic unit with longitudinal ribs and transverse floor beams. Such a monolithic construction is now commonly known as an orthotropic plate.

Orthotropic plate construction has resulted in substantial reduction of dead weight for long span bridges built in Europe, and more recently has found applications in the design of medium and short span bridges in the United States. An outstanding example of long span orthotropic plate construction is the Save River Bridge in Belgrade, Yugoslavia, which has an 856 ft span. This new bridge, weighing 3800 tons, replaced a destroyed suspension bridge of 6800 tons.[1] The saving in weight of steel is partially offset by the increased cost of fabrication, but the net saving is still impressive. At present, there are more than 40 orthotropic plate bridge decks in Germany, and several more in other countries including Canada.

An intensive study has been made by AISC on the economy of orthotropic plate design as opposed to several conventional designs for bridges of approximately 300 ft spans, using the AASHO *Standard Specifications for Highway Bridges* and the AWS *Specifications for Highway Bridges*. The results of the study indicate that orthotropic plate design compares very favorably cost-wise with other conventional designs of similar span lengths.[2] A greater saving is expected for longer spans where the reduction of dead weight would be a more important factor. Even for short spans, the price differential between a reinforced concrete deck and a thin wearing surface on an orthotropic plate may result in a net saving, in addition to a considerable saving in the substructure cost. An example of orthotropic plate design which meets Interstate Highway Standards for bridge spans of less than 100 ft has been presented recently.[3] This latter design indicates that the total weight of an orthotropic plate bridge is about one-third of the weight of composite construction consisting of reinforced concrete and steel. In a more recent paper[4] based on studies made for the AISC on two-lane, simple-span steel deck bridges with 80, 120 and 150 ft spans, longer spacing for transverse stiffeners is suggested in order to achieve greater economy. It was observed that the estimated bids on this design by several steel fabricators varied considerably, reflecting primarily the uncertainty of the actual cost because of the lack of experience in this type of construction. It appears that the economy of orthotropic plate design will be estimated conservatively for some time until greater experience is gained by fabricators.

Although a number of orthotropic plate bridges have been built in recent years in Europe, there is not complete agreement on the method of analysis and design. The basic equations describing the stresses and deformations in a theoretical orthotropic plate can be formulated directly in accordance with the linear theory of elasticity. However, the correctness of any theory depends on the assumptions made in the formulation of the mathematical equations. Among many earlier studies in the development of the analysis and design of orthotropic plates, the contribution made by Pflügger[5], Giencke[6,7], Pelikan and Esslinger[8], and Mader[9] is especially noteworthy.

From a practical point of view, simplified methods are acceptable for design purposes so long as the errors are small, and the reliability of the solution can be checked by experimental data for a wide range of variables, such as loading conditions and bridge geometry.

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With more experience gained in design, construction and research, a generally accepted design procedure will eventually evolve from current practices.

In the United States, the development of a standard design procedure has been given great impetus through the publication of a design manual for orthotropic plate by the AISC.[2] The recommended design procedure in the manual is based on a simplified method developed by Pelikan and Esslinger.[8] While design computations based on this simplified method may still require a large amount of numerical work, design charts as well as shortcut design formulas are provided in the AISC manual to facilitate numerical computations.

New types of wearing surfaces for orthotropic plate bridge decks have been developed to provide durable bond to steel and high resistance to skidding and to water penetration. The success in the application of new wearing surfaces has removed the last obstacle in the construction of orthotropic plate decks. In the meantime, the first orthotropic plate bridge meeting American Interstate Highway Standards is now under construction over the Mississippi River at St. Louis, Mo.

ORTHOTROPIC PLATE BRIDGE DECK

An orthotropic plate deck may consist of steel plate stiffened at the bottom by welded stringers and floor beams spaced uniformly in longitudinal and in transverse directions. Since the stiffeners have different geometrical properties in two mutually perpendicular directions and are not symmetrical with respect to the middle surface of the deck plate, the monolithic unit is sometimes called an orthotropic plate with eccentric stiffeners. There are two basic types of stiffeners for orthotropic plates, commonly known as torsionally soft ribs and torsionally stiff ribs. The former consists primarily of open slender sections such as bars which offer little resistance to torsion, and the latter includes closed box sections which provide considerable torsional resistance. Although the design of orthotropic plates with torsionally stiff ribs usually involves more numerical computations, the basic principles of analysis are applicable to decks using either type of stiffeners. In the case of steel deck bridges with long rib spans, the spacing of the transverse stiffeners or floor beams may be quite large and non-uniform. Such bridges must be treated as continuous rib spans over elastic supports.

The orthotropic plate decks may be used in connection with a variety of bridge types, including single-web girders, box girders, cable-stiffened girder bridges, truss bridges, suspension bridges and arch bridges. Examples of various types of bridges using orthotropic plate decks may be found in the orthotropic design manual published by AISC. The decks of these bridges were usually shop welded in separate units of convenient sizes for transportation, and the field splices were either riveted,

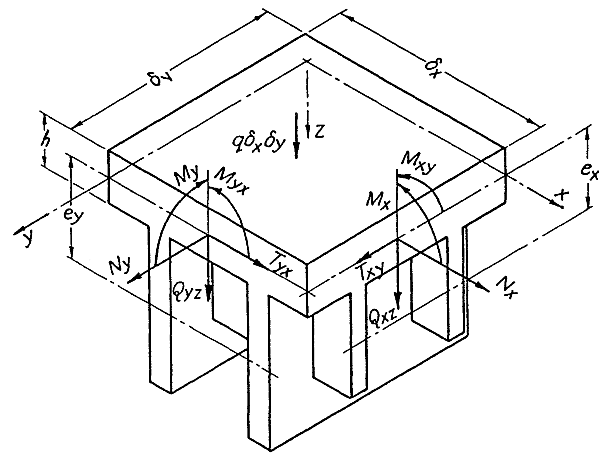

Fig. 1. Typical element of torsionally soft orthotropic plate

bolted or welded. In most cases, the steel deck plates were also used as the top flange of the supporting girders or floor beams, thus reducing further the dead weight of the supporting members.

Because of the monolithic construction, a deck plate is called upon to serve three distinct structural functions: (1) together with the stiffeners, to serve as a component of the main supporting members; (2) together with the stiffeners, to serve as the bridge floor between supporting members; and (3) to serve as the deck plate between stiffeners. The structural behavior of an orthotropic plate deck may be considered as the superposition of the actions of these three separate systems.

The bridge deck is first analyzed as an orthotropic plate with inflexible supporting edges. Next, the supporting members are permitted to deflect, thus causing additional stresses in the deck. The main problem in this step has been the determination of the width of the deck that should be included as top flange of the supporting members. Finally, the localized bending stress in the deck plate between stiffeners is considered, especially at locations near concentrated loads.

STRESS-STRAIN RELATIONSHIP

The structural behavior of an orthotropic plate with torsionally soft eccentric stiffeners may be described by the forces and moments acting on an element as shown in Fig. 1. Let x, y and z be the coordinates describing the undeformed plate. Note that only the forces and moments acting on the positive faces of the element under loading q have been indicated and are defined as follows:

N_x N_y Resultant normal forces in the x and y directions, per unit length in the y and x directions respectively.

T_{xy} T_{yx} Resultant shear forces in the y and x directions per unit length in the y and x directions respectively.

Q_{xz} Q_{yz} Resultant shear forces in the z direction per unit length in the y and x directions respectively.

M_x M_y Bending moments in the x and y directions, with respect to the middle surface of the isotropic plate, per unit length in the y and x directions respectively.

M_{xy} M_{yx} Torsional moments about the x and y axes per unit length in the y and x directions respectively.

The elastic constants of the material are denoted by the modulus of elasticity E, the modulus of rigidity G and Poisson's ratio ν. The geometrical properties of the orthotropic plate are defined as follows:

h Thickness of the deck plate.

A_x A_y Cross-sectional areas of the ribs in the x and y directions per unit length along the y and x axes respectively.

e_x e_y Distance from the middle surface of the isotropic plate to the centroid of the stiffeners in the x and y directions respectively.

S_x S_y Static moments of A_x and A_y with respect to the middle surface of the isotropic plate respectively.

I_x I_y Moments of inertia of A_x and A_y with respect to the middle surface of the isotropic plate respectively.

I Moment of inertia of the isotropic plate with respect to its middle surface, per unit length of the plate, equals $h^3/12(1 - \nu^2)$.

The partial derivatives with respect to x and y are represented by superscripts prime ($'$) and dot ($\dot{}$) respectively in the subsequent equations.

Let u, v and w be the displacement components of a point of the middle surface of the deck plate in the x, y and z directions respectively. Then, the horizontal displacements of a point at a depth z may be expressed by

$$u(z) = u - zw'$$

$$v(z) = v - zw^{\cdot}$$

Thus, the corresponding normal strains in x and y directions are found to be

$$\epsilon_x = u'(z) = u' - zw''$$

$$\epsilon_y = v^{\cdot}(z) = v^{\cdot} - zw^{\cdot\cdot}$$

The distribution of these normal strains over the depth of the orthotropic plate is shown in Fig. 2. The shearing strain $\gamma_{xy}(z)$ for a point at a depth z is given by

$$\gamma_{xy}(z) = u^{\cdot}(z) + v'(z)$$

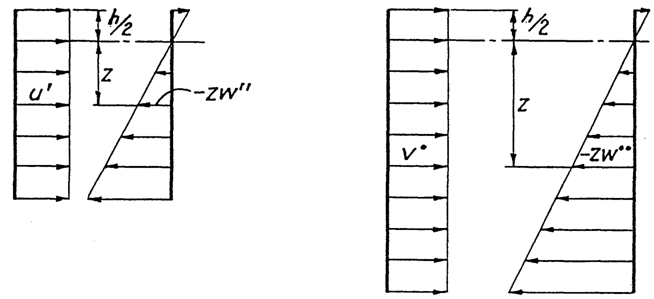

Fig. 2. Distribution of normal strains

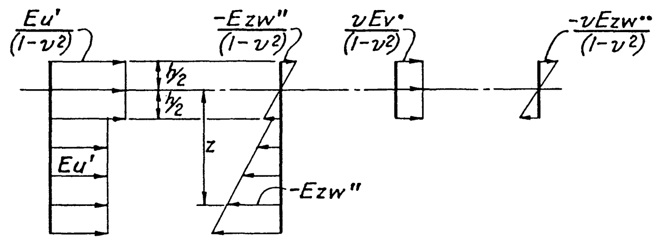

Fig. 3. Distribution of normal stress σ_x

The stresses in the deck plate are related to the strains as follows:

$$\sigma_x = \frac{E}{(1 - \nu^2)} u'(z) + \frac{\nu E}{(1 - \nu^2)} v^{\cdot}(z)$$

$$\sigma_y = \frac{E}{(1 - \nu^2)} v^{\cdot}(z) + \frac{\nu E}{(1 - \nu^2)} u'(z)$$

$$\tau_{xy} = G\,\gamma_{xy}(z)$$

The stresses in the stiffeners are given by

$$\sigma_x = Eu'(z)$$

$$\sigma_y = Ev^{\cdot}(z)$$

$$\tau_{xy} = 0$$

As an illustration of the distribution of the normal stresses, the variation of σ_x over the depth of the orthotropic plate is shown in Fig. 3.

In the case of orthotropic plates with torsionally stiff ribs, it is not necessary to consider stiffeners in both directions since such an arrangement will be rather unwieldy from the standpoint of construction. Hence, the more realistic case consists of stiffeners with closed box sections in the direction parallel to the x-axis and stiffeners with open slender sections in the direction parallel to the y-axis as shown in Fig. 4. The forces and moments acting on the elements can also be related to displacements as in the case of plates with torsionally soft ribs.

For steel deck bridges with long rib spans, the stress distribution in the deck plate is relatively simple except in the vicinity of transverse floor beams.

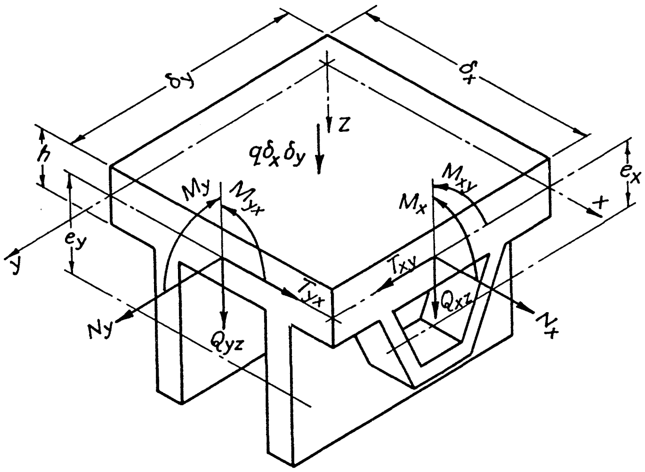

Fig. 4. Typical element of torsionally stiff orthotropic plate

GRID-TYPE ORTHOTROPIC PLATES

If the stiffeners consist of a grid system of closely spaced ribs in both directions, the deck may be treated in accordance with the orthotropic plate theory. It should be noted again that the localized bending stress in the deck plate between stiffeners must be superimposed on the stresses obtained by the orthotropic plate theory. Although the spacing in the longitudinal direction is not necessarily the same as that in the transverse direction, the spacing in each direction must be uniform and not too large. With these restrictions, the grid type orthotropic plate construction is not as economical as the long rib spans under the normal conditions of relative cost of material and labor in this country.

Assuming that the deflections are small relative to the thickness of orthotropic plates, a set of differential equations describing the structural behavior of an orthotropic plate with either torsionally soft or stiff ribs is found to be as follows:[10]

$$\bar{A}_x u'' + h_1 v^{\cdot\prime} + h_2 u^{\cdot\cdot} - S_x w''' = 0$$

$$\bar{A}_y v^{\cdot\cdot} + h_1 u^{\cdot\prime} + h_2 v'' - S_y w^{\cdot\cdot\cdot} = 0$$

$$\bar{I}_x w'''' + (2I + \eta_1) w''^{\cdot\cdot} + \bar{I}_y w^{\cdot\cdot\cdot\cdot} - S_x u''' - S_y v^{\cdot\cdot\cdot} - \eta_2 u^{\cdot\cdot\prime} - \eta_2 v''^{\cdot} = q/E$$

in which

$$h_1 = \frac{h}{2(1-\nu)} \qquad h_2 = \frac{h}{2(1+\nu)}$$

$$\bar{A}_x = \frac{h}{1-\nu^2} + A_x \qquad \bar{A}_y = \frac{h}{1-\nu^2} + A_y$$

$$\bar{I}_x = I + I_x \qquad \bar{I}_y = I + I_y$$

and η_1 and η_2 involve the geometrical properties of closed box sections for plates with torsionally stiff ribs, but $\eta_1 = \eta_2 = 0$ for plates with torsionally soft ribs.

A complete solution of this set of differential equations has been obtained for various loading and boundary conditions. The solution has also been programmed for computer in the case of single span orthotropic plates. This approach may also be extended to the analysis of continuous spans on elastic supports[11]. Because of the mathematical complexity of the solution, this method of analysis is not recommended for general use at the present time.

A simplified method of treating this type of deck system may be obtained by modifying the well known Huber's equation for anisotropic plates of constant thickness but of different properties in two orthogonal directions. Huber has derived the following equation expressing the relation between vertical load q and vertical displacement w as follows:

$$D_x w'''' + 2H w''^{\cdot\cdot} + D_y w^{\cdot\cdot\cdot\cdot} = q$$

in which the constants D_x and D_y represent the bending stiffnesses in the x and y directions; and the coefficient H is computed from the torsional rigidity and Poisson's ratio of the plate. In extending Huber's equation to an orthotropic plate with eccentric stiffeners, H becomes a function of the eccentricity and the torsional rigidity of the stiffeners also. Various approximations have been suggested by assuming different values for H. One of the simpler methods for estimating H for orthotropic plates with torsionally soft stiffeners makes use of the concept of effective stiffness introduced by Giencke.[6] This concept is based on the assumption that the horizontal strain is zero at the adjusted centroid of the cross section in each direction. The adjusted centroids in the x and y directions are located at distances $e_x{}^*$ and $e_y{}^*$ respectively below the middle surface of the deck plate. The distances $e_x{}^*$ and $e_y{}^*$ are defined as follows:

$$e_x{}^* = \frac{e_x A_x}{A_x + \dfrac{h}{(1-\nu^2)}}$$

$$e_y{}^* = \frac{e_y A_y}{A_y + \dfrac{h}{(1-\nu^2)}}$$

The moments used in this simplified method are applied at the respective adjusted centroids of a torsionally soft orthotropic plate element, and are different from those defined earlier in this paper. The normal forces, on the other hand, are applied at the middle surface of the deck plate because they are caused only by the effect of the lateral contraction of the deck plate as a result of the assumption of zero strain at the adjusted centroid. The horizontal shear forces are also acting at the middle surface of the deck plate since the stiffeners cannot resist such forces. Then, the bending stiffnesses are given by

$$D_x = E \int_{A_x} (z - e_x^*)^2 \, dA_x + \frac{E}{(1 - \nu^2)} \int_{-h/2}^{h/2} (z - e_x^*)^2 \, dz$$

$$D_y = E \int_{A_y} (z - e_y^*)^2 \, dA_y + \frac{E}{(1 - \nu^2)} \int_{-h/2}^{h/2} (z - e_y^*)^2 \, dz$$

The term H, called the effective stiffness, is defined as

$$H = D + \nu e_x^* e_y^* B + (e_x^* + e_y^*)^2 \frac{(1 - \nu)}{4} B$$

in which

$$D = \frac{Eh^3}{12(1 - \nu^2)} \qquad B = \frac{Eh}{(1 - \nu^2)}$$

If the torsional stiffness of individual ribs is not neglected, the effective stiffness H should also include the terms D_{xy} and D_{yx}, representing the torsional stiffnesses of the ribs in the x and y directions, per unit length along the y and x directions respectively.

Again, a complete solution of this differential equation for the case of a single panel has been programmed for computer. It should also be emphasized that the effect of localized bending stress in the deck plate between stiffeners should be considered. A method of treating such local bending stresses has been suggested.[12]

STEEL DECK BRIDGES WITH RIBS IN ONE DIRECTION

Generally, steel bridge decks with stiffeners arranged in one direction only prevail, as they are more economical. In Europe the spans of ribs between the floor beams are usually limited to 7 or 8 ft. In the studies made for AISC,[2,4] span lengths varying from 15 to 28 ft have been proposed for closed (torsionally stiff) ribs.

A simplified design procedure has been developed for the analysis of this type of steel deck bridges. The deck plate with longitudinal ribs is treated as a continuous orthotropic plate supported on rigid main girders and elastic floor beams as shown in Fig. 5. The design computation is broken up into two steps: (1) the moments in the deck are computed first by assuming that the continuous plate is supported on rigid floor beams; and (2) effects of the elastic flexibility of the floor beams are determined. This design procedure is described in full detail in the AISC *Design Manual for Orthotropic Steel Plate Deck Bridges*. Simplified design charts are also available in the manual. The highlights of this design procedure will be the topic of the next paper in this Journal.

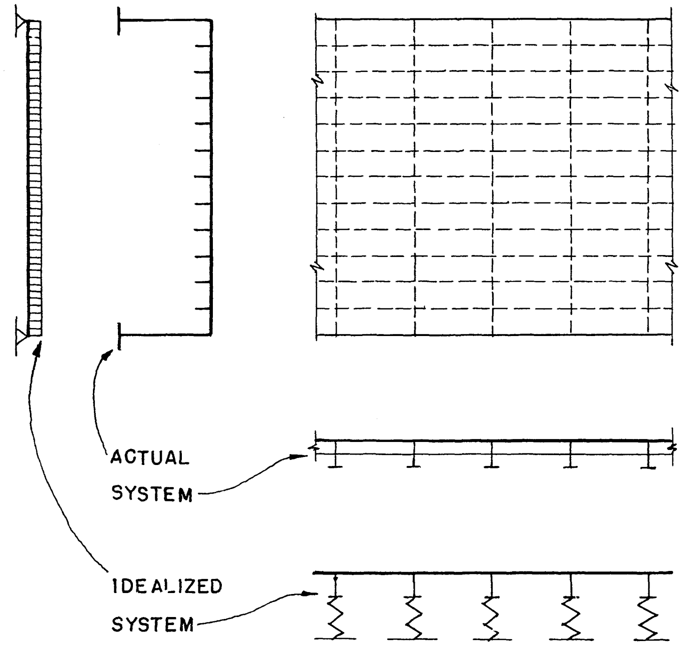

Fig. 5. Bridge deck treated as an orthotropic plate on elastic supports

DESIGN CONSIDERATIONS

Various factors influencing design must be considered in order to intelligently apply the results of analysis to design, and to develop satisfactory design criteria and specifications.

First of all, the overall economy in the design of a bridge may be achieved by the use of an orthotropic plate deck. In the case of continuous girder bridges, for example, longer spans may be made feasible because of the efficiency and light weight of the steel decks. In the design of tied arch and suspension bridges, the flexural and torsional rigidity provided by the orthotropic plate decks may be utilized advantageously.

Since the structural behavior of an orthotropic steel deck in the elastic range can be considered as a combination of three separate systems, it would seem logical that the design should be based on the combined stress in the deck plate from the direct superposition of the three systems. However, this is seldom done because different factors of safety are usually assigned to these systems. It is a well known fact that the ratio of static ultimate strength to yield strength of an orthotropic plate is much higher than that of the supporting girders. Hence, for designs based on allowable stresses, it is customary to superimpose the full amount of theoretical stresses in the girders with only a fraction of the theoretical stresses computed for the orthotropic plate. Since a plate has relatively high reserve strength after initial yielding, local stresses in the deck plate are often disregarded in design. Thus, the concept of plastic design is introduced in the combination of stresses although the design procedure appears to be based on elastic analysis only.

Fatigue strength—as opposed to static strength—of the orthotropic plate design should also be investigated, since deck plates, stiffeners and girders are subject to variation of stresses. Under such circumstances, the local stresses in the deck plate, stiffeners and supporting girders should be treated in accordance with the usual fatigue provisions in bridge design. Since the dead load is proportionally smaller for an orthotropic plate bridge than for a conventional bridge, the larger proportion of live load stress in the total design stress causes larger variation between maximum and minimum stresses. Hence, closer attention should be given to the fatigue strength in the design of such a structure.

Again, because of the larger proportion of live load stress in the total design stress, the live load deflections tend to be larger in the bridge deck and also in the supporting girders if the girders are made shallower for a given span. Generally, elastic deflections are acceptable unless perceptible deformations should occur under loads. Nevertheless, excessive relative deflections in the bridge deck should be avoided.

Finally, the dynamic effects of live loads on orthotropic plate bridges should also be considered since the vibration characteristics of such structures may differ from similar structures with conventional decks. Results of limited investigations currently available indicate that, insofar as structural safety is concerned, the dynamic effects on the bridge decks and supporting girders may be adequately treated by using the usual impact factors specified for conventional bridge types. However, perceptible vibrations in the deck may exist unless the natural frequency of vibration of the bridge lies beyond that which might be caused by the traffic on the bridge deck.

CONCLUSION

The development of the orthotropic plate bridge deck is one of the most outstanding advances in bridge design and construction in recent years. Like many other new advances, it shows great promise but it also requires careful treatment. Since design procedures have not been completely codified, a great deal of judgment must be exercised in interpreting the results of analysis and in establishing design criteria. It is hoped that the AISC design manual will serve as a guidepost for the future development in orthotropic plate design.

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