Design for Local Web Shear at Brace Connections: An Adaptation of the Uniform Force Method

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ABSTRACT

Recent literature has examined local shear forces in beams in chevron braced frames (Fortney and Thornton, 2015). Subsequently, design methods based on optimal stress distributions to address these shears were developed (Sabelli and Arber, 2017; Sabelli and Saxey, 2021). This paper extends those design methods to gusset connections at columns, utilizing the adaptability of the Uniform Force Method to facilitate design to reduce required member shear strength. The design model presented reduces the required member shear strength required, as compared to the conventional application of the Uniform Force Method. The model allows for redistribution of force from the beam interface to the column interface using a "bypass method," as well as utilizing the gusset plate as part of a moment connection using the "haunch method." Finite element analyses are used to confirm the adequacy of a design employing these methods.

Keywords: gusset plates, braced frames, truss connections.

INTRODUCTION

Design of brace connections requires consideration of local forces induced in the surrounding framing members. Although the discussion of these forces initially focused on beam midspan connections (Fortney and Thornton, 2015), such forces also occur at beam-column-brace connections. These local forces are typically missed by analysis methods that neglect connection dimensions. Richards et al. (2018) studied midspan gussets; their finite element analyses confirm the presence of these local forces, as well as redistribution of stresses similar to those posited by Sabelli and Arber (2017). [A design procedure for beam-to-column connections with full-height gussets based on such a stress redistribution is presented in Sabelli and Saxey (2021).]

The Uniform Force Method (UFM) (Thornton, 1991; Muir and Thornton, 2014; AISC, 2017) is commonly used to analyze forces at gusset-plate connections. It is a powerful tool that permits designers to proportion and optimize connections. This study presents an adaptation of the UFM specifically developed to allow the designer to proportion and analyze connections to reduce required member shear strength and thus to reduce the instances of connections requiring web doublers. Similar to the UFM as originally developed, the methods presented in this paper rely on the lower bound theorem as presented by Thornton (1984) for similar connections, demonstrating adequate strength through investigation of an advantageous load path in a ductile connection and examining forces at gusset edges.

This paper addresses the UFM and adds two options. To further optimize designs, a "bypass" method is developed that permits assigning more of the brace force to the column than is possible using the UFM. This approach recognizes that shear yielding of the beam does not constitute formation of a complete mechanism and that inelastic shear deformation of the beam web requires inelastic deformation of the column web or gusset. (Even inelastic deformation of both the beam web and column web does not constitute a complete mechanism, and additional strength can be mobilized as is discussed in the bypass section.)

Additionally, a "haunch" method is developed, permitting the use of the gusset as part of a moment-resisting connection. Equations are presented for UFM forces; for UFM forces in combination with bypass forces; and for UFM, bypass, and haunch forces combined.

This paper begins with the derivation of design equations for the three methods, along with the combined force equations. For clarity, a summary table presents the equation numbers that apply for the design quantities for each method. Design examples are presented for each method. Finally, there is a brief presentation of validating finite element analyses of one of the design examples.

Each of the methods presented (the adapted Uniform Force Method and the bypass method and haunch method

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enhancements) depends on the lower-bound theorem to justify the use of a relatively simple design model. While the design model is not intended to produce forces matching those of more sophisticated analytical models, the methods are expected to result in designs with adequate strength. The finite element analyses performed confirm this strength. Additionally, the analyses produce force distributions similar to those determined using the design model.

This paper focuses exclusively on the design of braced-frame-connections to achieve the required strength of each component for a defined set of forces. The design forces considered are brace axial forces, beam axial and shear forces, and (for the haunch method) beam moment. Inelastic deformation demands related to brace buckling are not considered; as such, the methods are appropriate for wind design and for the design of buckling-restrained braced frames but are not sufficient for providing the required ductility of special concentrically braced frames.

Member Shear Checks

The presence of local shears in beam and column webs is necessary for static equilibrium. All beam-column-brace (and truss) connections must resist these forces, regardless of the connection design and analysis techniques used. The shear is due to both the force normal to the member axis and to the moment at the gusset-flange interface. While different analysis methods assign different force distributions, there is no method that eliminates both normal force and moment in connections of members with non-zero depth.

For gusset-plate connections, local member shear may be at a maximum in either of two locations for each member (considering the column above and below the beam as separate members). The maximum member shear may be due to the total normal force, which is fully delivered by the gusset at the section nearest the workpoint. For the column, one such section is just above the beam top flange in Figure 1 (section C1); for the beam, it is at the beam-to-column connection (section B1). Additionally, moments at the gusset-to-column or gusset-to-beam interface may be large enough such that the associated shear in the member at or near the mid-length of the gusset (sections C2 and B2) may exceed the value at C1 and B1.

The member shear checks at C1 and B1 are straightforward. These are simply the normal forces transverse to the member axis delivered by the gussets (horizontal forces for the column; vertical forces for the beam). In the case of a beam such as shown in Figure 1, the transverse forces from the gusset above and below are additive. Similarly, column-shear forces from gussets on opposite column flanges are additive.

At sections C2 and B2, the member shear is a function of both the portion of the normal force delivered between the gusset end and the section in question, and the moment on the gusset-flange interface. The combined effects of flexure and normal force can be analyzed using a number of models, three of which are shown in Figure 2. The elastic and (conventional) plastic distributions are adapted from the AISC *Steel Construction Manual* (2017), hereafter referred to as the AISC *Manual*. The third method, the "concentrated stress method," is described by Sabelli and Saxey (2021).

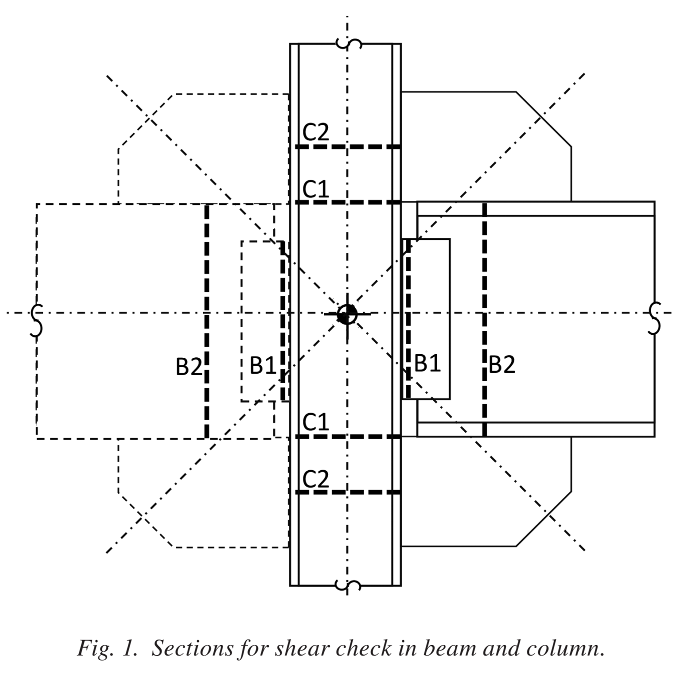


Fig. 1. Sections for shear check in beam and column.

[The results from the concentrated stress method match the "optimized plastic method" from the AISC *Manual* (2017) for cases in which the section is fully plasticized.] In each diagram the flexural stresses are sufficiently large so that the shear at an interior section (B2) is greater than the shear at B1. Note that the precise location of the section B2 varies among the stress-distribution models in Figure 2. For a given combination of moment and normal force, the length required for a limit on the maximum stress differs for each model.

In this paper the conventional plastic distribution is used. In that distribution, the sections B2 and C2 are always at the gusset mid-length, which simplifies the equations derived based on the moments at the gusset interfaces. However, the concentrated stress method requires a lower force to transmit the same moment over the same length and thus can provide for a more efficient design (Sabelli and Saxey, 2021).

Using conventional plastic distribution, the shear at section B2 or C2 is:

$$V_{mid} = \frac{M}{e} + \frac{N}{2} \tag{1}$$

where

L = gusset length, in.

M = moment at gusset interface to beam or column flange, kip-in.

N = normal force on beam or column flange, kips

e = eccentricity in force couple resisting moment M equal to half the gusset length L, in.

This shear loading from Equation 1 must be combined with additional shear that may be present in the member.

Sign Conventions

Figure 3 shows a typical brace-connection diagram. Subscripts are employed in some equations to distinguish actions and dimensions related to one gusset or one brace from another. Gussets are designated 1 and 2. Dimensions and forces associated with each gusset are given the subscript 1 or 2.

Braces have two subscripts. The first pertains to which side of the column the gusset connects to (1 or 2). The second pertains to which of the two braces connecting to a

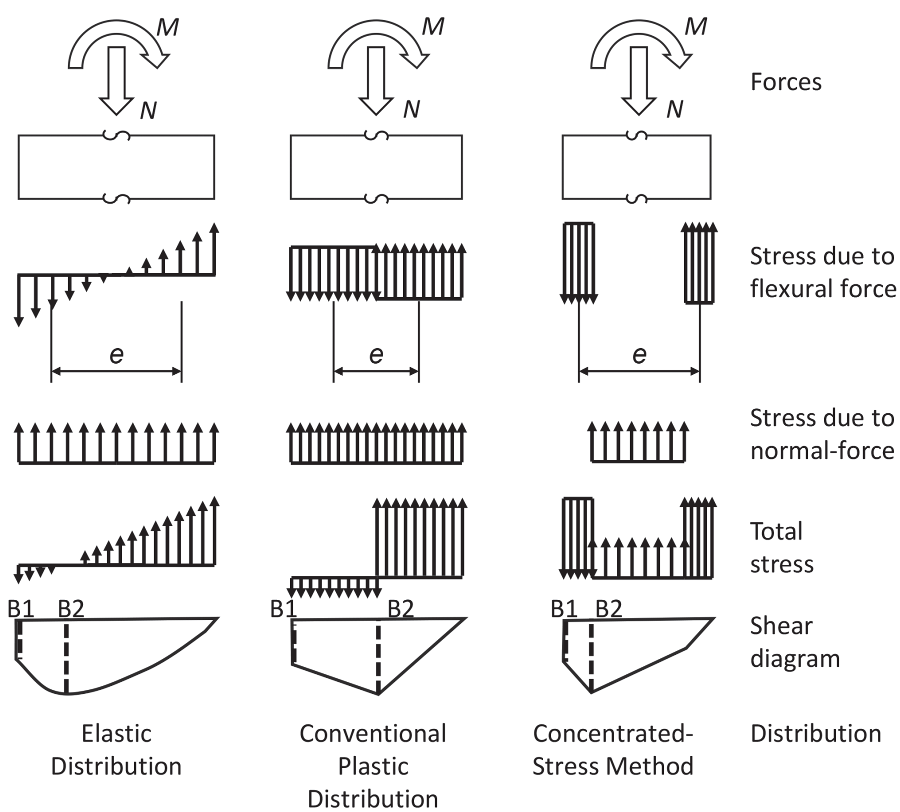


Fig. 2. Stress-distribution models.

beam is indicated (above or below). Sign conventions match the figures such that positive brace axial forces $F_{1,2}$ and $F_{2,1}$ correspond to compression and positive brace axial force $F_{1,1}$ and $F_{2,2}$ correspond to tension. Forces and angles pertaining to each brace carry the same designation subscript.

The term *workpoint* refers to the intersection of brace centerlines with the beam and column centerline. (Eccentric workpoints are not addressed in this paper.)

Sabelli and Saxey (2021) derive local member shears based on a full-height gusset that has continuous thickness. Similar column shears occur in traditional gussets, and similar methods may be employed to utilize the gusset size and an optimal stress distribution to eliminate the need for local reinforcement.

The Uniform Force Method (UFM) (AISC, 2017) is a commonly employed method of analyzing traditional gussets. The UFM utilizes the gusset dimensions 2α and 2β, as well as the beam and column eccentricities e_b and e_c. Note that 2α and 2β are the dimensions of a virtual gusset, and the actual gusset dimensions ($2\overline{\alpha}$ and $2\overline{\beta}$) may be different, as shown in Figure 4 and discussed later. The method presented here is a supplement to the UFM, showing necessary dimensions to eliminate the need for web reinforcement. For additional guidance and background on the application of the UFM readers should consult the AISC *Manual* (AISC, 2017) and AISC Design Guide 29 (Muir and Thornton, 2014).

Force Equations

The UFM defines vertical and horizontal forces at the gusset interfaces with the column and the beam. The column vertical force corresponding to the brace axial force, P, is given by AISC *Manual* Equation 13-2:

$$V_c = \frac{\beta}{r} P \tag{2}$$

where

r = gusset centroid offset from workpoint per AISC *Manual* Equation 13-6:

$$= \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \tag{3}$$

and

P = brace axial force, kips

e_b = eccentricity from beam flange to beam centerline, equal to half the beam depth, in.

e_c = eccentricity from column flange to column centerline, equal to half the column depth, in.

α = distance from column face to centroid of Uniform Force Method force acting on beam flange, in.

β = distance from beam flange to centroid of Uniform Force Method force acting on column face, in.

The column horizontal force is given by AISC *Manual* Equation 13-3:

$$H_c = \frac{e_c}{r} P \tag{4}$$

The beam vertical force is given by AISC *Manual* Equation 13-4:

$$V_b = \frac{e_b}{r} P \tag{5}$$

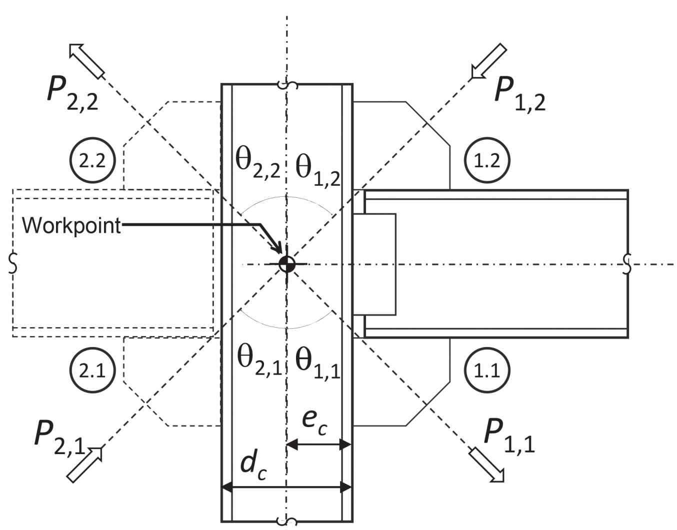


Fig. 3. Diagram with numbering and sign conventions.

The beam horizontal force is given by AISC *Manual* Equation 13-5:

$$H_b = \frac{\alpha}{r} P \tag{6}$$

For both edges of the virtual gusset to be centered on the forces acting on the beam and column flanges, the virtual dimensions α and β must conform to the following relationship (AISC *Manual* Equation 13-1):

$$\alpha - \beta \tan\theta = e_b \tan\theta - e_c \tag{7}$$

where

θ = brace angle from vertical, deg

Equation 7 may also be expressed as:

$$\beta = \frac{\alpha + e_c}{\tan\theta} - e_b \tag{8}$$

Equation 3 may be combined with Equation 7 and Equation 8 thus:

$$\begin{aligned} r &= \frac{e_b + \beta}{\cos\theta} \\ &= \frac{e_c + \alpha}{\sin\theta} \end{aligned} \tag{9}$$

These relationships may also be expressed as:

$$\alpha = r\sin\theta - e_c \tag{10}$$

$$\beta = r\cos\theta - e_b \tag{11}$$

These virtual dimensions locate the centroids of the forces acting on the beam flange and column flange. As presented here, the UFM gusset forces are constrained to conform to the proportioning relationships defined by Equations 7 through 11, regardless of the actual gusset proportioning. Specifically, Equations 9, 10, and 11 are used to convert the force equations into functions of *r*, which can then be selected to optimize the connection. Thus, although three dimensions (*r,* α, and β) are used in the following equations, they are constrained to each other and represent a single variable in the design.

In the UFM procedure [both as defined by Thornton (1991) and as applied here], these gusset forces act on the beam and column at the points indicated on Figure 4 and are proportioned such that their force vectors pass through "control points" at the beam centerline at the column face (for beam forces H_b and V_b) and at the column centerline at the beam top or bottom elevation (for column forces H_c and V_c).

The virtual dimensions α and β do not necessarily correspond to the centroid of the gusset welds or bolted joints.

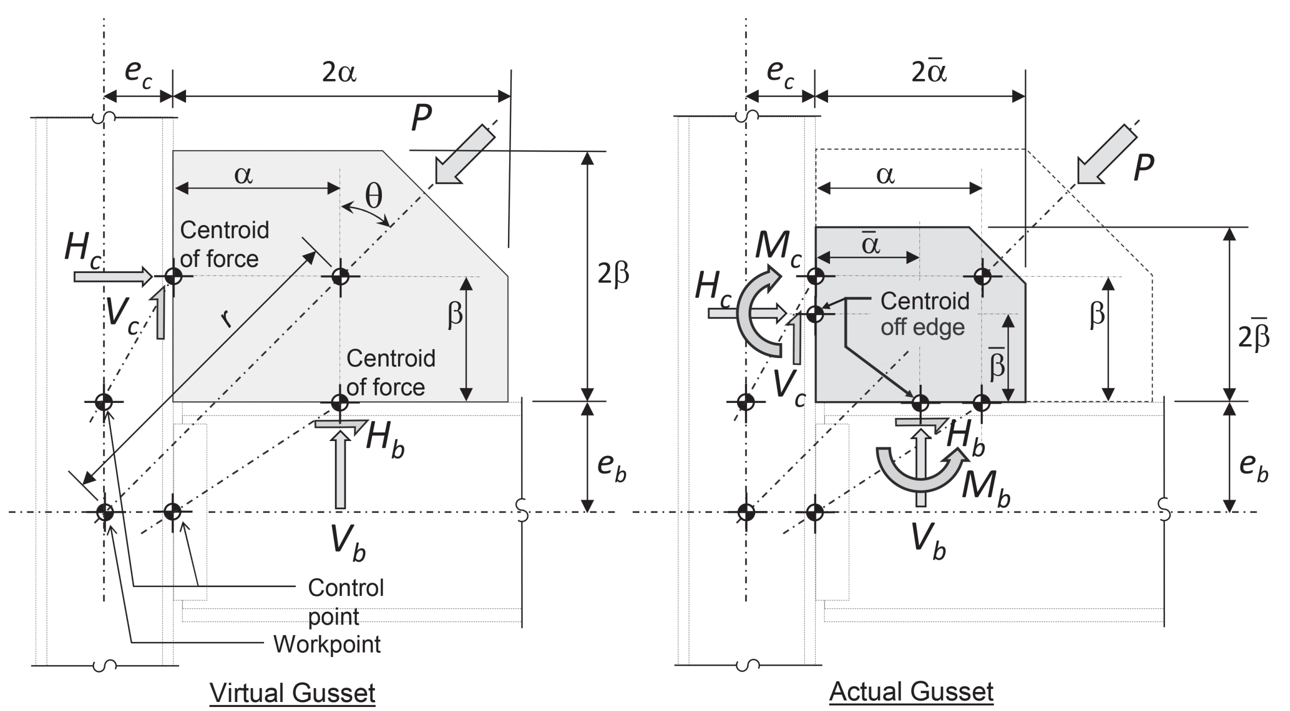


Fig. 4. Uniform Force Method dimensions.

They locate the centroids of the shear and normal forces at the beam or column flange, respectively. At those locations, there is effectively no moment. However, if that location is not also the centroid of the joint (i.e., when the gusset dimension $\bar{\alpha} \neq \alpha$ or $\bar{\beta} \neq \beta$), there is a resulting moment due to the eccentricity. The AISC *Manual* provides methods of determining the relationship between the ideal gusset dimensions (2α and 2β) and actual dimensions ($2\bar{\alpha}$ and $2\bar{\beta}$) and for determining the resulting moments (AISC, 2017). For efficient design, it is convenient to determine the minimum virtual dimensions α and β to control the transverse loading on the column and beam so as to avoid overloading these members in shear and to subsequently select smaller gusset dimensions $2\bar{\alpha}$ and $2\bar{\beta}$ independently (i.e., not constrained to each other) and design the gusset and its interfaces for the resulting combination of normal force, shear force, and moment.

Figure 4 shows the UFM forces acting on the gusset, along with the dimensions used in the UFM. (Forces acting on the member are of opposite sign.) The diagram on the left shows a gusset with virtual dimensions 2α and 2β; forces on the column act at the centroids of the interfaces with the virtual gusset with no moment. The diagram on the right shows the smaller gusset of dimensions $2\bar{\alpha}$ and $2\bar{\beta}$; forces acting at the centroids of the actual gusset edges include moments due to the difference between the virtual and actual gusset dimensions.

In the following derivations, the relationship between α and β defined by Equation 7 is maintained. This simplifies the method, while still permitting independent selection of actual dimensions $2\bar{\alpha}$ and $2\bar{\beta}$ that are not constrained by this relationship.

Using Equations 7 through 11 with Equations 2 through 6, the force equations can be presented in terms of the virtual dimension, r:

$$V_c = \left(\cos\theta - \frac{e_b}{r}\right)P \tag{12}$$

$$H_c = \frac{e_c}{r}P \tag{13}$$

$$V_b = \frac{e_b}{r}P \tag{14}$$

$$H_b = \left(\sin\theta - \frac{e_c}{r}\right)P \tag{15}$$

The larger the virtual dimension, r, the larger the portion of the brace force that is resisted in shear at the interfaces with the column flange and beam flange (V_c and H_b, Equations 12 and 15), and the less is resisted in the normal forces that cause member shear (H_c and V_b, Equations 13 and 14, which are used to evaluate sections C1 and B1, respectively). Subsequent equations solve for the minimum virtual dimension, r, based on those forces in comparison to the member shear strength. From that dimension, the virtual dimensions α and β are defined by Equations 10 and 11. Once those virtual dimensions are computed, the actual gusset dimensions $2\bar{\alpha}$ and $2\bar{\beta}$ are selected. There is not a unique relationship between the force distribution (represented by the virtual dimensions r, α, and β) and the gusset geometry (i.e., $2\bar{\alpha}$ and $2\bar{\beta}$); a range of dimensions $2\bar{\alpha}$ and $2\bar{\beta}$ may be used with the same virtual dimensions α and β, and vice versa. The engineer may optimize the design by selecting values of α and β to produce forces that do not overload the beam (or connection) and column in shear at the B1 and C1 sections and subsequently select among a range of possible gusset dimensions $2\bar{\alpha}$ and $2\bar{\beta}$ that are consistent with those forces and do not overload the beam and column in shear at the B2 and C2 sections.

GUSSETS RESISTING UFM BRACE FORCES ONLY

This section derives closed-form solutions for minimum virtual and actual gusset dimensions based on UFM gusset forces if no other forces are required to be transmitted by the gussets. The subsequent sections develop the (more complicated) equations that include other forces that the designer may elect to assign to the gusset: bypass forces to transfer more force to the column than would result from the application of the UFM and haunch forces for beam-end moments.

Determination of Column Effective Shear Strength

Column and beam shears are readily obtained using the UFM; they are H_c and V_b, respectively, combined with any shear in the member from other sources. (If the moment at the interface is large, the resulting effect on local shear may increase the member shear, as is discussed later.)

$$H_c \leq V_{efCol} \tag{16}$$

where

V_{efCol} = effective column shear strength

The column effective shear strength is reduced by additional shear demands that must be simultaneously resisted by the column. For rigid beam-to-column connections, the moment at the column face due to lateral loads entails a panel-zone shear in the column web and a corresponding shear in the column segments above and below the connection. Typically, the panel-zone shear is not critical as its direction is opposite that of the connection shear induced by the braces. The column shear outside of the panel zone, however, needs to be considered in determining the total shear demand:

$$V_{efCol} = \phi V_n - V_{col} \tag{17}$$

where

V_{col} = column shear due to moment-frame behavior, kips

V_n = nominal member shear strength, kips

ϕ = resistance factor

Determination of Beam and Beam-to-Column Effective Shear Strength

The shear demand from the brace connection must not exceed the beam effective shear capacity, considering this shear demand present in the beam due to lateral loads. Gravity shear, V_g, and frame shear, V_{BMF}, may further reduce the effective beam shear strength:

$$V_{efBm} = \phi V_n - V_g - V_{BMF} \tag{18}$$

where

V_{BMF} = beam shear due to moment-frame behavior, kips

V_g = beam shear due to gravity, kips

When the vertical brace force is acting upward, such as in the case of the braces on left-hand side of Figure 3 (or the case with braces on the right-hand side during the reversal of the indicated force), the gravity shear, V_g, is in the opposite direction of the brace-induced shear, and thus increases the effective shear strength of the beam; reduced-gravity load combinations are appropriate for such cases.

The shear strength of the beam connection to the column, V_{efConn}, may be less than that of the beam:

$$V_{efConn} = \phi R_n - V_g - V_{BMF} \le V_{efBm} \tag{19}$$

where

R_n = nominal strength (of beam-to-column connection), kips

For convenience, a strength ratio, U_C, is defined:

$$U_C = \frac{V_{efConn}}{V_{efBm}} \le 1 \tag{20}$$

Thus,

$$V_{efConn} = U_C V_{efBm} \tag{21}$$

Note that the factor U_c varies with the load combinations that correspond to the shears V_g and V_{BMF}.

Apportionment of Shear Strength for Multiple Gussets

For connections with gussets both above and below (e.g., $P_{1,1}$ and $P_{1,2}$), the column shear must be checked for both separately. For connections with gussets both left and right of the column (e.g., $P_{1,1}$ and $P_{2,1}$), the column forces $H_{c1,1}$ and $H_{c2,1}$ of these gussets are additive, and only a portion of the column shear capacity may be utilized to resist each of these forces. The effective shear capacity of the column, V_{efCol}, must be apportioned between the braces to the left and right of the column (e.g., $P_{1,1}$ and $P_{2,1}$, as shown in Figure 3). In this method, it is apportioned considering their horizontal components as follows:

$$V_{efC1,1} = V_{efCol} \frac{P_{1,1}\sin\theta}{P_{1,1}\sin\theta + P_{2,1}\sin\theta} \tag{22}$$

$$V_{efC2,1} = V_{efCol} \frac{P_{2,1}\sin\theta}{P_{1,1}\sin\theta + P_{2,1}\sin\theta} \tag{23}$$

This apportionment is arbitrary and may be modified based on the demands $H_{c1,1}$ and $H_{c2,1}$. Apportionment as shown in Equations 22 and 23, however, permits independent design of the two gussets.

The condition at the beam is evaluated similarly for connections with both a brace above the beam and another below the beam; only a portion of the beam shear capacity may be utilized to resist each UFM vertical beam force, V_b. The effective shear capacity of the beam, V_{efBm}, must be apportioned between the braces above and below the beam (see Figure 3):

$$V_{efB1,1} = V_{efBm} \frac{P_{1,1}\cos\theta}{P_{1,1}\cos\theta + P_{1,2}\cos\theta} \tag{24}$$

$$V_{efB1,2} = V_{efBm} \frac{P_{1,2}\cos\theta}{P_{1,1}\cos\theta + P_{1,2}\cos\theta} \tag{25}$$

This apportionment applies to both the beam shear strength and the connection shear strength. As with the apportionment of column shear strength (Equations 22 and 23), this apportionment is arbitrary and may be modified based on the demands ($V_{b1,1}$ and $V_{b1,2}$ in this case).

Determination of Minimum Virtual Dimensions

The virtual dimension, r, determines the shear at the column section C1 and the beam section B1, and thus the minimum value for this dimension is established based on the effective shear strength at those sections.

The horizontal force must not exceed the column effective shear strength. Combining Equations 13 and 16 gives:

$$r \ge r_{minCol} = \frac{e_c P}{V_{efC}} \tag{26}$$

where

r_{minCol} = minimum dimension, r, based on column shear yielding, in.

The condition at the beam is evaluated similarly. The shear is set equal to the beam shear capacity (deducting shear due to other sources), and the minimum dimension, r, is derived based on the effective shear capacity of the beam-to-column connection (considering shear demands on the beam not due to the brace force).

The vertical force on the beam due to the brace force from the UFM analysis is set equal to the effective beam-connection shear strength apportioned to that gusset:

$$V_b \le U_C V_{efB} \tag{27}$$

Per Equation 19, the connection effective shear strength, V_{efConn}, may be limited by the effective beam shear strength, V_{efBm}.

Combining Equation 14 and Equation 27 gives:

$$r \ge r_{minBm} = \frac{e_b P}{U_C V_{efB}} \tag{28}$$

where

r_{minBm} = minimum dimension, r, based on beam shear yielding, in.

The maximum length, r, from the check of the column and the beam controls the design of the gusset (or the need for reinforcement):

$$r \ge \max\left(r_{minCol}, r_{minBm}\right) \tag{29}$$

The value of r selected is used in subsequent equations for determination of UFM forces. The corresponding virtual gusset dimensions α and β (determined using Eqs. 10 and 11) are used in the determination of the minimum actual dimensions $\bar{\alpha}$ and $\bar{\beta}$, respectively. Note that by selecting one of these two values (r_{minCol} or r_{minBm}), the design method constrains the corresponding evaluation of member shear (section C1 or B1) to indicate a demand-to-capacity ratio of 1.0. While larger values of the virtual dimension, r, may be used, selection of the lowest value tends to minimize the member shears. It is possible that greater economy in the gusset can be achieved with a larger dimension, r, for designs that can accommodate higher member shear.

Similar designs are performed for each gusset, determining the minimum dimension, r, for beam or connection shear yielding and column shear yielding and apportioning the effective shear strength, considering the gusset on the opposite side of the beam or column for the respective gusset design.

Selection of Actual Gusset Dimensions: Column Interface

As discussed earlier, the minimum virtual gusset dimensions can be established based on the shear strengths at sections C1 and B1, and the minimum actual gusset dimensions can be established based on the virtual dimensions and the shear strengths at sections C2 and B2.

The required gusset dimension $2\bar{\beta}$ at the column is determined based on the combined effects of the force, H_c, corresponding to the virtual gusset dimension, r, and, if $\bar{\beta} \ne \beta$ (as is typical), a moment, M_c, on the gusset-column interface:

$$M_c = H_c(\beta - \bar{\beta}) \tag{30}$$

The larger the deviation of the actual dimension $\bar{\beta}$ from the virtual dimension β, the larger the moment that must be resisted at the gusset interface.

As shown in Figure 5, the column shear at section C2 at the gusset mid-height combines the shear required to transmit the moment with half of the interface horizontal force:

$$V_{mid} = \frac{H_c}{2} + \frac{M_c}{\bar{\beta}} \tag{31}$$
$$= H_c\left(\frac{\beta}{\bar{\beta}} - \frac{1}{2}\right)$$

This shear must be less than the column effective (apportioned) shear capacity from Equation 17:

$$\left|V_{mid}\right| \le V_{efC} \tag{32}$$

The minimum dimension $\bar{\beta}$ may be determined by combining Equations 31 and 32:

$$\bar{\beta} \ge \frac{\beta}{\left|\frac{V_{efC}}{H_c}\right| + \frac{1}{2}} \qquad \text{for } r \ge r_{minCol} \tag{33}$$

The minimum gusset dimension obtained using Equation 33 should be compared to the gusset size required for the brace-to-gusset connection.

If the virtual dimension, r, is equal to r_{minCol}, Equation 33 can be combined with Equations 4 and 26 and simplified to:

$$\bar{\beta} \ge \frac{2}{3}\beta \qquad \text{for } r = r_{minCol} \tag{34}$$

If r is greater than r_{minCol}, Equation 34 will result in a larger value than Equation 33, which gives the true minimum length; nevertheless, Equation 34 may be used for convenience.

Figure 5 shows column shear diagrams in the column due to the effects of the horizontal force, H_c (distributed over the gusset height), and the moment, M_c. (The effects of the vertical force, V_c, are not shown because they do not contribute to column shear.) Two cases are shown. In the upper set of diagrams, the virtual column dimension, r, is selected such that the column shear is exactly equal to the column-effective shear strength, and thus $r = r_{minCol}$. In this case, the column shear at section C1 at the bottom of the gusset, H_c, will equal the effective shear strength, V_{efC}. As such, the shear at section C2 at the midheight, V_{mid}, must not exceed the shear at the bottom, and the moment (and the eccentricity causing the moment) must consequently be limited. In the lower set of diagrams, the column shear at section C1 at the bottom of the gusset, H_c, is less than the effective shear strength, V_{efC}. As such, the shear at section C2 at the gusset mid-height can be larger than the shear at section C1, and a greater moment and eccentricity can be tolerated; a larger eccentricity results from selecting a smaller dimension $\bar{\beta}$.

If $\bar{\beta} > \frac{2}{3}\beta$, the maximum shear occurs at section C1. Conversely, if $\bar{\beta} < \frac{2}{3}\beta$, the maximum shear occurs at section C2. (If $\bar{\beta} = \frac{2}{3}\beta$, the maximum shear occurs for the entire bottom half of the gusset height between sections C1 and C2; this case is not shown in the figure.)

Note that Equation 34 can be used to determine the required shear strength of the column based on a selected gusset size $2\bar{\beta}$ such as might be determined based on the brace-to-gusset connection. Assuming r_{minCol} controls, Equations 9, 13, 16, and 34 can be combined to estimate the required shear strength:

$$V_{efC} \geq \frac{e_c \cos\theta P}{\frac{3}{2}\bar{\beta} + e_b} \tag{35}$$

This value can also be used to design web reinforcement for an already selected column and gusset dimension $2\bar{\beta}$.

Selection of Actual Gusset Dimensions: Beam Interface

The required gusset dimension $2\bar{\alpha}$ at the beam is determined similarly, based on the force distribution corresponding to the virtual gusset dimension α. Similar to the condition at the column, if $\bar{\alpha} \neq \alpha$, there is a moment, M_b, on the gusset-beam interface:

$$M_b = V_b(\alpha - \bar{\alpha}) \tag{36}$$

The shear at section B2 at the gusset mid-length combines the shear required to transmit the moment with half of the interface vertical force:

$$\begin{aligned} V_{mid} &= \frac{V_b}{2} + \frac{M_b}{\bar{\alpha}} \\ &= V_b\left(\frac{\alpha}{\bar{\alpha}} - \frac{1}{2}\right) \end{aligned} \tag{37}$$

This shear must be less than the beam shear capacity:

$$|V_{mid}| \leq V_{efB} \tag{38}$$

Note that as this check is performed away from the connection, the full (apportioned) beam shear effective strength, V_{efB}, is used without the reduction related to connection effective shear strength, U_C.

The minimum dimension, $\bar{\alpha}$, may be determined by combining Equations 37 and 38:

$$\bar{\alpha} \geq \frac{\alpha}{\left|\frac{V_{efB}}{V_b}\right| + \frac{1}{2}} \quad \text{for } r \geq r_{minBm} \text{ or } V_{efBm} > V_{efConn} \tag{39}$$

The minimum gusset dimension obtained using Equation 39 should be compared to the gusset size required for the brace-to-gusset connection.

If the virtual dimension, r, is equal to r_{minBm}, Equation 39 can be combined with Equation 27 and simplified to:

$$\bar{\alpha} \geq \frac{\alpha}{\frac{1}{U_C} + \frac{1}{2}} \quad \text{for } r = r_{minBm} \text{ and } V_{efBm} > V_{efConn} \tag{40}$$

If $V_{efBm} = V_{efConn}$, Equation 39 can be further simplified to:

$$\bar{\alpha} \geq \frac{2}{3}\alpha \quad \text{for } r = r_{minBm} \text{ and } V_{efBm} = V_{efConn} \tag{41}$$

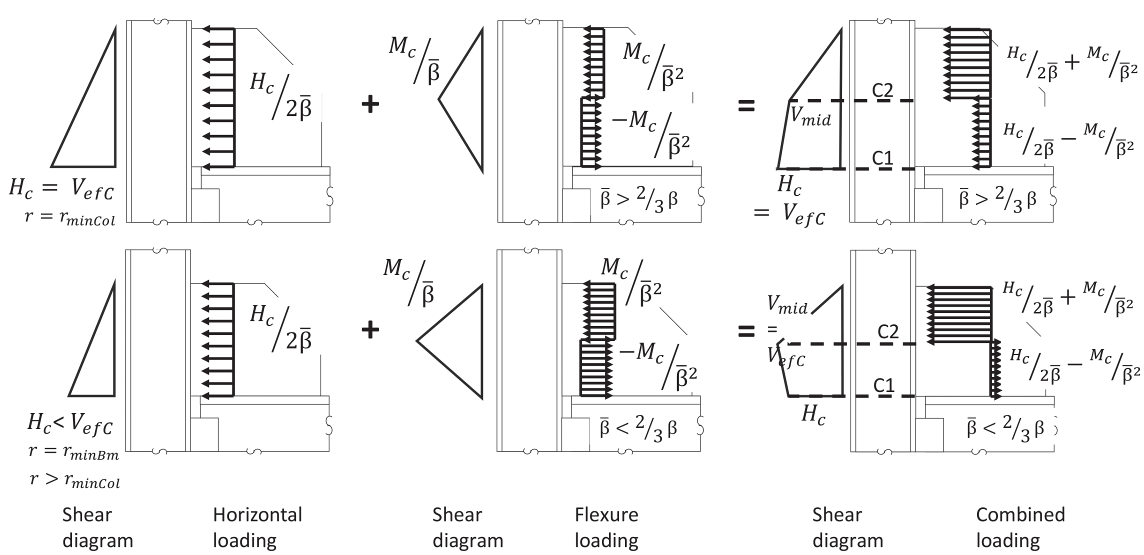


Fig. 5. Shear in column due to horizontal force and moment.

If r is greater than r_{minBm}, Equations 40 and 41 will result in larger values than Equation 39, which gives the true minimum length.

Figure 6 shows beam shear diagrams and vertical loading for two cases. In the upper set of diagrams, the connection effective shear strength, V_{efConn}, is equal to the beam shear effective strength, V_{efBm}. The limit on shear demand at section B1 at the connection, V_b, and the shear demand on section B2 at the gusset mid-length are thus the same. (This occurs when the maximum eccentricity is defined by Equation 41 for cases in which $r = r_{minBm}$ and $V_{efBm} = V_{efConn}$.) In the lower set of diagrams, a connection weaker than the beam is shown ($U_c < 1.0$), and thus a higher shear than V_b can be resisted at the gusset mid-length. (A shear at section B2 higher than V_b results from selection of $\bar{\alpha} < ⅔\alpha$. If $\bar{\alpha} > ⅔\alpha$, the shear at section B2 is less than V_b; this case is not shown in Figure 6 but is similar to the upper diagrams in Figure 5.)

Similar to Equation 34, Equation 41 can be used to determine the required shear strength of the connection based on a selected gusset size $2\bar{\alpha}$:

$$U_C V_{efB} \geq \frac{e_b \sin\theta P}{\frac{3}{2}\bar{\alpha} + e_c} \tag{42}$$

If the beam shear strength is equal to the connection shear strength (that is, if $U_C = 1.0$), Equation 42 also defines the required beam shear strength. If the connection shear strength is the limiting factor (that is, if $U_C < 1.0$), the required beam shear strength is determined from Equations 5, 28, and 40:

$$V_{efB} \geq \frac{e_b P \sin\theta}{\bar{\alpha}\left(1 + \frac{U_C}{2}\right) + U_C e_c} \tag{43}$$

This value can also be used to design beam-web reinforcement for a given beam and desired gusset dimension $2\bar{\alpha}$.

Selection of Gusset Thickness

The gusset and its connections must be evaluated for the combined effects of moment, horizontal, and vertical forces at both the column interface (M_c, H_c, and V_c), and the beam interface (M_b, H_b, and V_b). The von Mises yield criterion may be used to determine the minimum gusset thickness, t_g, considering forces at the interface with the column:

$$t_g \geq \sqrt{\left(\frac{V_c}{\phi 0.6 F_y 2\bar{\beta}}\right)^2 + \left[\frac{|H_c|}{\phi F_y 2\bar{\beta}} + \frac{|M_c|}{\phi F_y (2\bar{\beta})^2/4}\right]^2} \tag{44}$$
$$= \frac{1}{\phi F_y 2\bar{\beta}}\sqrt{\left(\frac{V_c}{0.6}\right)^2 + \left(|H_c| + \frac{2|M_c|}{\bar{\beta}}\right)^2}$$

where

F_y = material specified minimum yield stress, ksi

The minimum thickness at the beam interface is similarly determined:

$$t_g \geq \sqrt{\left(\frac{H_b}{\phi 0.6 F_y 2\bar{\alpha}}\right)^2 + \left[\frac{|V_b|}{\phi F_y 2\bar{\alpha}} + \frac{|M_b|}{\phi F_y (2\bar{\alpha})^2/4}\right]^2} \tag{45}$$
$$= \frac{1}{\phi F_y 2\bar{\alpha}}\sqrt{\left(\frac{H_b}{0.6}\right)^2 + \left(|V_b| + \frac{2|M_b|}{\bar{\alpha}}\right)^2}$$

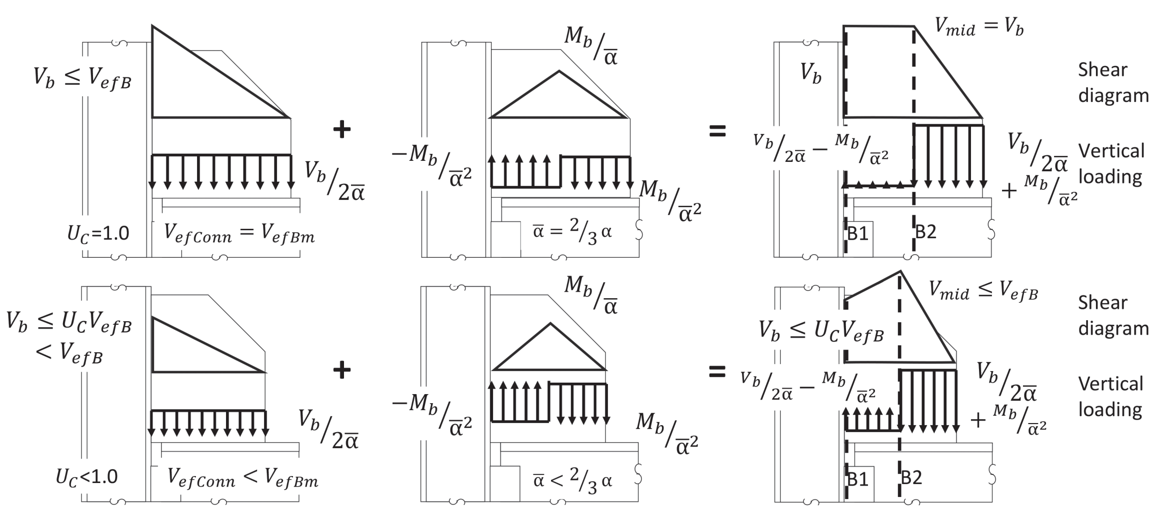


Fig. 6. Shear in beam due to vertical force and moment.

The larger thickness from Equations 44 and 45 should be used. If the required gusset thickness is excessive, a larger value of $\bar{\beta}$ or $\bar{\alpha}$ (as appropriate) may be selected. [If the length $\bar{\beta}$ or $\bar{\alpha}$ required for a certain gusset thickness is desired, the required gusset length is the root of a fourth-power polynomial; various solution methods are available, including trial-and error and computer solvers. A closed-form solution may also be derived using Ferrari's formula (Euler, 1765).]

Normal Forces on Column and Beam

The column and beam should each be evaluated for web local yielding and web crippling for a required strength, R_u, due to the combined effects of normal force and moment over a bearing length, N, of $\bar{\beta}$ or $\bar{\alpha}$, respectively:

$$R_u = V_{mid} = H_c\left(\frac{\beta}{\bar{\beta}} - \frac{1}{2}\right) \tag{46}$$

$$R_u = V_{mid} = V_b\left(\frac{\alpha}{\bar{\alpha}} - \frac{1}{2}\right) \tag{47}$$

GUSSETS RESISTING ADDITIONAL FORCES

The preceding derivations are based on the gussets being designed using the UFM to transmit the brace force to the beam and column. The methods developed (including selection of dimensions to preclude the need for member shear reinforcement) can be extended to gussets designed to resist other forces as well. In the following section, forces corresponding to two refinements are developed: gussets designed with a "bypass" method to transfer more of the brace force to the column (and less to the beam) than would result from the application of the UFM, and gussets acting as moment-transferring haunches. Equations for minimum virtual and actual gusset dimensions considering the total gusset forces (UFM, bypass, and haunch forces) are derived.

Force Transfer from Beam to Column Using Bypass Method

Gussets proportioned in the manner described in the previous section will have sufficient strength and will not cause local yielding in the beam and column webs. Nevertheless, such gussets may be overly large or otherwise undesirable, or such a design might entail reinforcing the web of one of the members (typically the beam). Additionally, the shear yielding of the web of one of the members does not constitute a complete plastic mechanism; both webs must yield to allow this to occur.

In fact, a complete mechanism requires extensive shear yielding of both member webs over the entire gusset region and diagonal elongation of the gusset in the direction transverse to the brace axis. Such a mechanism engages member shear and flexure outside of the connection region. When, as is typical, such forces are not considered in the member design, the design of the connection, to be consistent, should not depend on such forces. In such cases, the complete mechanism is limited by the capacity of the beam and column to withstand flexural forces not considered in the design in conjunction with the axial forces and other actions that were considered. The mechanism may nevertheless represent significant reserve strength over that of the design.

Often there is a great deal of economy to be achieved by taking advantage of the additional capacity of the column to relieve the beam of some of the force. The UFM provides a method of transferring some of the vertical force: "Special Case #2" (AISC, 2017). An alternative method is presented here in which the brace force is divided into two components: a "UFM force" that is delivered to the column and beam per the method described above and a "bypass force" that is delivered only to the column. The apportionment is achieved by selecting a factor, λ, between 0.0 and 1.0. The UFM portion of the force is λP and is addressed using the UFM as described previously (with modified effective column shear strength). The remaining bypass force is $(1–\lambda)P$ and is assigned to the column, which must resist the additional vertical and horizontal forces, as well as the corresponding moment. Note that for the design approach anticipated (in which $\bar{\beta} < \beta$), this moment is in the opposite direction from the UFM column moment. The factor, λ, may be selected to reduce the required length of gusset on the beam or to eliminate the overstress in the beam that would necessitate a web doubler.

The resulting column forces due to the bypass force are:

$$H_{cBP} = (1-\lambda)P\sin\theta \tag{48}$$

$$V_{cBP} = (1-\lambda)P\cos\theta = \frac{H_{cBP}}{\tan\theta} \tag{49}$$

$$M_{cBP} = H_{cBP}(e_b + \bar{\beta}) - V_{cBP}e_c \tag{50}$$

where

λ = brace force apportionment factor

Figure 7 shows these forces acting on the gussets.

If the beam strength is the limiting factor in sizing the virtual gusset ($r = r_{minBm}$ and $V_{efConn} = V_{efBm}$), the factor, λ, required to avoid beam web doublers may be obtained by modifying Equation 42:

$$\lambda \le \frac{V_{efB}}{e_b \sin\theta P}\left[\frac{3}{2}\bar{\alpha} + e_c\right] \tag{51}$$

If the beam strength is not the limiting factor, the λ factor required to avoid beam web doublers may be obtained by modifying Equation 43:

$$\lambda \le \frac{V_{efB}}{e_b \sin\theta P}\left[\bar{\alpha}\left(1+\frac{U_C}{2}\right)+U_C e_c\right] \tag{52}$$

If this bypass-force method is used, Equations 26 and 28 should be modified as follows:

$$r_{minCol} = \frac{\lambda P e_c}{V_{efC} - H_{cBP}} \tag{53}$$

$$r_{minBm} = \frac{\lambda P e_b}{U_C V_{efB}} \tag{54}$$

Similarly, the equations for minimum values of $\bar{\beta}$ and $\bar{\alpha}$ must be modified to address bypass forces; Equations 33 and 39 are not valid for such cases. Modified equations (Equations 77 and 78) considering total forces and moments (including haunch forces) are presented in a subsequent section.

Equations 53 and 54 may be combined with Equation 48 to facilitate member selection. The result is an equation for λ:

$$\lambda = 1 - \frac{V_{efC} - \dfrac{e_c}{e_b} U_C V_{efB}}{P \sin\theta} \tag{55}$$

This can be combined with Equation 52 to derive the required relationship between horizontal gusset length and column and beam connection strength:

$$\bar{\alpha} \ge \frac{e_b P \sin\theta - V_{efC} e_b}{V_{efB}\left(1+\dfrac{U_C}{2}\right)} \tag{56}$$

The bypass method can be used to design connections that satisfy this relationship to preclude the need for reinforcement.

In some circumstances, it may be convenient to transfer force from the column to the beam (rather than from the beam to the column). In such circumstances the engineer may either derive similar bypass equations for the beam or may use a value of λ greater than 1.0, increasing the beam UFM forces and reducing the total column forces.

Use of Gussets as Haunches for Moment Transfer

It is possible to transfer the beam moment utilizing the gusset as a haunch as shown in Figure 8. The use of gussets as haunches will affect the force transfer at the gusset edges, and these forces must be combined with those related to the brace axial force. The resulting equations for the minimum virtual gusset dimension, r, to preclude shear yielding (determined in a subsequent section), and the corresponding virtual dimensions α and β, are necessarily more complicated. For convenience, the method presented utilizes the beam moment at the column face, M_f, rather than at a location based on the (as yet undetermined) gusset horizontal dimension. Thus, if the moment is based on beam flexural strength (as might be the case for seismic design) projection of that moment based on an assumed gusset dimension and corresponding beam shear is necessary.

$$M_f = M_{BM} + V_{BMF}\left(2\bar{\alpha}\right) \tag{57}$$

where

M_{BM} = moment at a beam section aligned with the gusset edge face, kip-in.

The designer has some discretion in selecting the height of the force couple utilized. To permit independent design

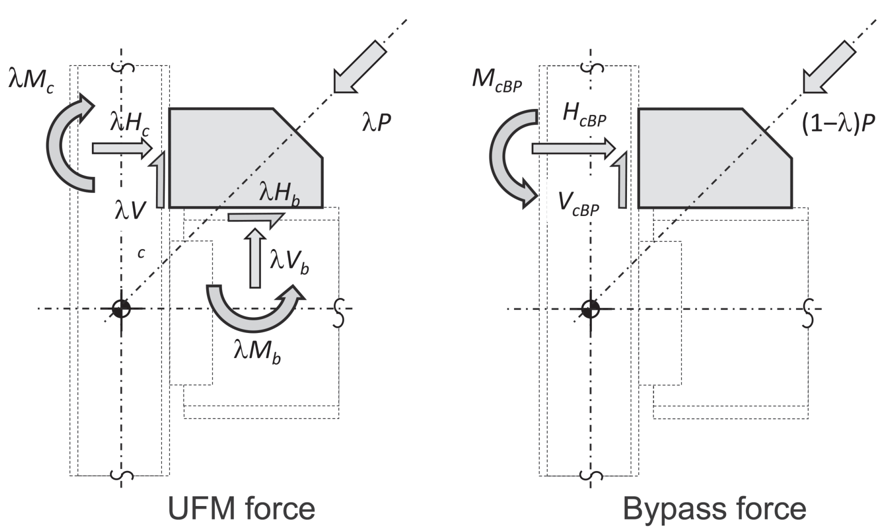


Fig. 7. Force proportioning.

of the gusset (considering the possibility of no gusset on the opposite beam flange or a gusset of yet-to-be-determined size), the gusset is assigned one-half of the beam moment to be resisted by a force couple over a vertical distance from the beam centerline to the virtual gusset centroid defined by β. (Other values of this depth may be used, but this value provides for efficient gusset designs using simpler equations in determining the appropriate dimension, β.)

As shown in Figure 8, at conditions with a gusset, the horizontal force that forms part of this moment-resisting force couple may be resisted by the beam flange connection to the column, the gusset, or a combination of the two.

The value of the moment is ½M_f, assuming that the moment is resisted by two force couples: one involving the top flange or gusset and the web connection and the other involving the bottom flange or gusset and the web connection. If the connection involves only the top, for example, and there is no bottom gusset and no bottom-flange connection to the column, the value of the moment assigned to the top flange or gusset and beam web force couple is the full M_f.

The moment is apportioned between a beam-connection moment, M_{BC}, and a haunch moment, M_H, using a distribution factor ρ (between 0.0 and 1.0); thus:

$$M_{BC_{i,j}} = \frac{\rho_{i,j} M_{f_i}}{2} \tag{58}$$

where

$$M_{H_{i,j}} = (1-\rho_{i,j})\frac{M_{f_i}}{2} \tag{59}$$

ρ = beam moment apportionment factor (between beam flange and haunch force in gusset)

Thus, the total moment from beam "i," M_{fi}, is divided into four (potential) parts:

$$M_{f_i} = M_{BC_{i,1}} + M_{BC_{i,2}} + M_{H_{i,1}} + M_{H_{i,2}} \tag{60}$$

It is expected, however, that in most cases, ρ will be selected as either 1.0 or 0.0 for each flange of each beam as is shown in Figure 8, and thus two of the components of Equation 60 will be of zero value.

For simplicity, the beam-connection moment is assigned to the beam flange as a force, P_f:

$$P_{f_{i,j}} = \frac{M_{BC_{i,j}}}{\frac{1}{2} h_{o_i}} = \frac{\rho_{i,j} M_{f_i}}{h_{o_i}} \tag{61}$$

where

h_o = distance between beam flange centroids

The haunch moment results in a horizontal force, H_{cH} (gusset-designation subscripts omitted):

$$H_{cH} = \frac{M_H}{\frac{d_b}{2}+\beta} = \frac{(1-\rho)M_f}{2(e_b+\beta)} \tag{62}$$

where

d_b = beam depth, in.

To satisfy equilibrium, there is an opposite horizontal force, H_{bH}, at the gusset interface with the beam:

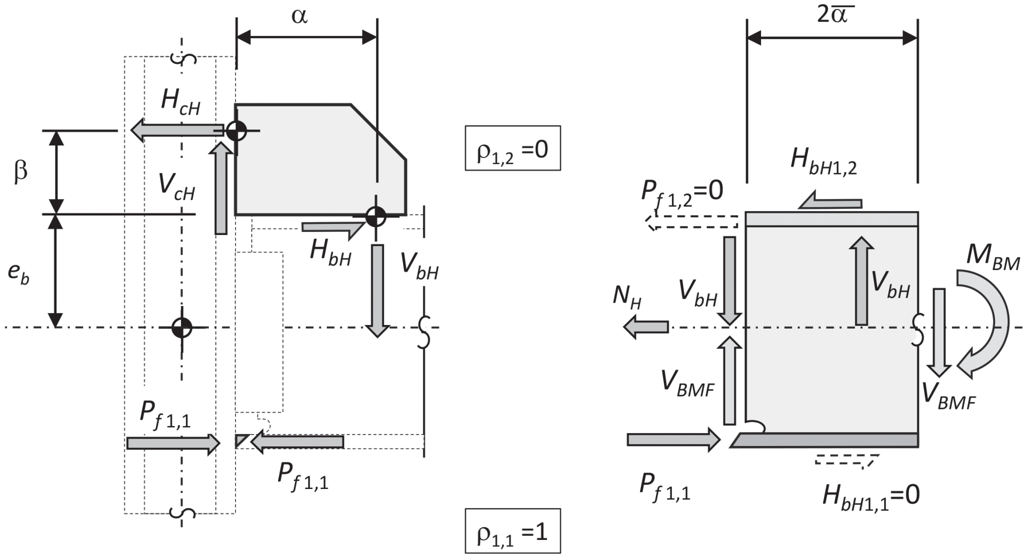


Fig. 8. Gusset plates acting as haunches.

$$H_{bH} = H_{cH} \tag{63}$$

The eccentricity between H_{cH} and H_{bH} is β, and there is thus a moment, βH_{cH}, that must be resolved. A counteracting force couple is determined, with a vertical force, V_{cH}, at the column flange and an opposite vertical force, V_{bH}, which is assigned to the location α for simplicity:

$$V_{bH} = V_{cH} \tag{64}$$

$$= \frac{\beta H_{cH}}{\alpha}$$

At the gusset centroid, H_{cH} causes a moment, M_{cH}, due to the eccentricity between β and $\bar{\beta}$:

$$M_{cH} = (\beta - \bar{\beta}) H_{cH} \tag{65}$$

The moment due to the vertical force at the beam interface, M_{bH}, is:

$$M_{bH} = (\alpha - \bar{\alpha}) V_{bH} \tag{66}$$

The horizontal force acting at the beam midheight may not completely cancel with the horizontal force from the remaining half of the moment addressed at the opposite flange, potentially necessitating a small additional force, N_H, to be resisted at the beam-to-column connection. As shown in Figure 8, the horizontal force acting on the beam at one flange ($P_{f1,2} + H_{bH1,2}$) has a corresponding force at the opposite flange resisted by a combination of flange and gusset forces ($P_{f1,1} + H_{bH1,1}$). Due to potentially different values of ρ used in Equations 58 and 59, and potentially different moment arms used to determine these forces in Equation 62, the horizontal forces at opposite flanges may not be equal, and the difference must be resisted in the beam web-to-column connection to satisfy equilibrium. This force, N_H, is:

$$N_H = P_{f_{1,2}} + H_{bH_{1,2}} - \left[P_{f_{1,1}} + H_{bH_{1,1}}\right] \tag{67}$$

The haunch method may be used in conjunction with the UFM with or without applying the bypass method.

Total Gusset Forces: UFM, Bypass, and Haunch Forces

Figure 9 shows these forces acting on the gusset edges. Forces shown are gusset forces; member forces are equal and opposite. Note that the beam UFM forces are reduced by the factor λ. In most cases, the haunch forces can have a beneficial effect; the designer should consider whether it is appropriate to neglect that term or take a reduced value to address uncertainty in the level of moment. For seismic design, the potential for cyclic inelastic drift may result in moments in the opposite direction of those shown, which could be a more critical case. Determination of appropriate combinations of design forces is outside of the scope of this study. At a minimum, connections should have sufficient strength to resist maximum brace forces and maximum beam forces both combined with each other (with consistent directions of forces as shown) and separately.

The direction of the UFM, haunch, and bypass column forces is shown in Figure 9. The resulting total column forces (considering the direction of each component) are determined:

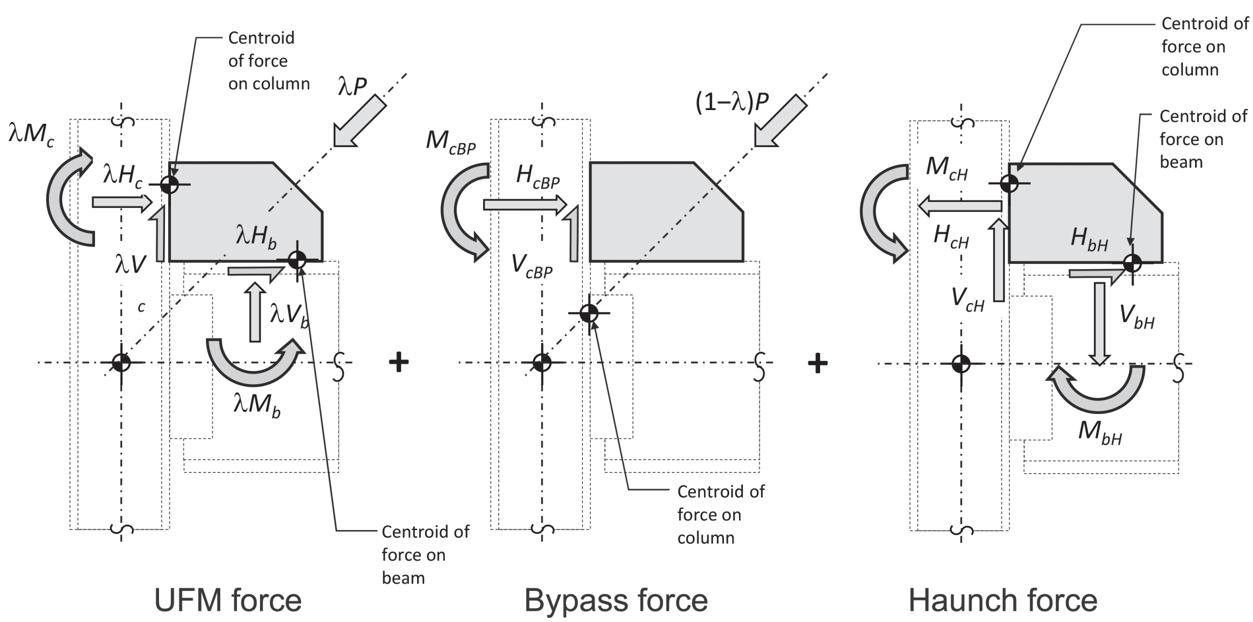


Fig. 9. Total gusset forces.

$$H_{cTot} = \lambda H_c + H_{cBP} - H_{cH} \tag{68}$$

$$V_{cTot} = \lambda V_c + V_{cBP} + V_{cH} \tag{69}$$

$$M_{cTot} = \lambda M_c - M_{cBP} - M_{cH} \tag{70}$$

The shear in the column panel zone is:

$$V_{PZ} = H_{cTot} - P_f \tag{71}$$

The total beam forces (including the UFM, haunch, and bypass forces) are:

$$H_{bTot} = \lambda H_b + H_{bH} \tag{72}$$

$$V_{bTot} = \lambda V_b - V_{bH} \tag{73}$$

$$M_{bTot} = \lambda M_b - M_{bH} \tag{74}$$

These equations are used to select the minimum virtual dimension, r, considering the shear loading of the column, H_{cTot}, at the beam flange and the shear loading of the beam, V_{bTot}, at the column flange. Once this is selected, the actual dimensions $2\bar{\alpha}$ and $2\bar{\beta}$ may be selected considering the shear loading of the column and beam at the mid-length of the gusset as it is affected by the moments (M_{cTot} and M_{bTot}) that are functions of these dimensions. The interface moments are then computed based on the selected dimensions and are used in conjunction with the vertical and horizontal forces to design the gusset and its connections to the beam and column.

Figure 10 shows a free-body diagram of the gusset. The centroid of the UFM column force, λH_c, is at the location defined by the virtual dimension, β, which is typically greater than $\bar{\beta}$, causing a moment, λM_c, (when evaluated at the centroid of the gusset-column interface.). The haunch force, H_{cH}, (which acts in the direction opposite to λH_c) is applied at the same elevation, and the bypass force, H_{cBP}, (which acts in the same direction as λH_c) is located on the brace centerline where it crosses the column face; these two forces cause moments in the opposite direction of M_c.

Minimum Virtual Gusset Dimensions Considering Haunch and Bypass Forces

In previous sections, Equations 26 and 28 provide the minimum virtual dimensions, r, for cases with UFM forces only; Equations 53 and 54 address combined UFM and bypass forces. If haunch forces are included in the gusset design, equations for minimum virtual dimensions, r, become more complicated (but nevertheless solvable). A satisfactory value for this virtual dimension can be determined by trial and error, such that the beam and column are not overloaded in shear by V_{bTot} and H_{cTot}, respectively. Alternatively, the following methods can be used.

Combining Equations 16 and 68 results in a quadratic equation for the minimum virtual dimension, r, considering column web shear, r_{minCol}:

$$r_{minCol}^{2}\left[(V_{efC} - H_{cBP})\cos\theta\right] + r_{minCol} \left[(V_{efC} - H_{cBP})(e_c - e_b) + (1-\rho)\frac{M_f}{2} - \lambda P e_c \cos\theta\right] + \left[\lambda P e_c (e_b - e_c)\right] \geq 0 \tag{75}$$

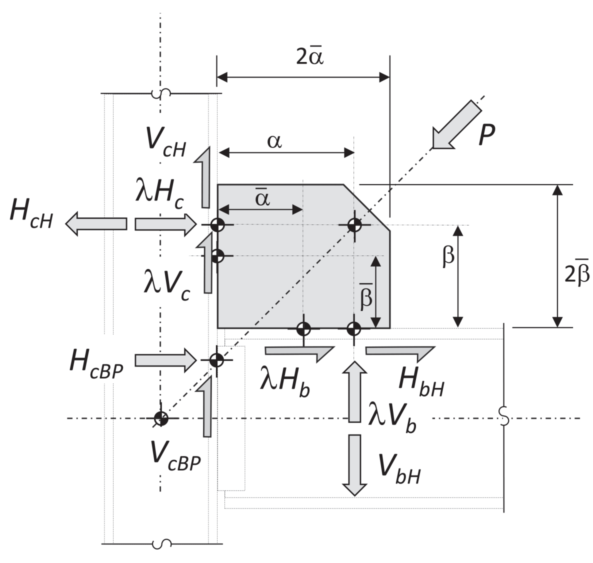


Fig. 10. Free-body diagram of gusset.

Combining Equations 28 and 73 results in a quadratic equation for the minimum virtual dimension, r, considering beam web shear:

$$r_{minBm}^{2}\left[\sin\theta U_c V_{efBm}\right]+r_{minBm}\left[M_H-e_c U_c V_{efBm}-\lambda P e_b \sin\theta\right]$$
$$+\left[\lambda P e_b e_c - M_H \frac{e_b}{\cos\theta}\right]\geq 0 \tag{76}$$

The largest of the roots from Equations 75 and 76 is used. Excessively large values of the minimum dimension, r, may indicate that more of the moment, M_f, should be transferred by the beam flanges. Note also that both the effective column shear strength, V_{efCol}, in Equation 75 and the effective beam shear strength, V_{efB}, in Equation 76 include terms that, similar to M_f, are reflective of the moment-frame behavior, and the use of consistent loads is recommended. As discussed in the design examples, consideration of cases without these flexural forces may be appropriate to address the range of possible conditions.

Equation 76 presupposes a value of λ. If the design considers both the condition with moment and the condition with moment equal to zero (for which haunch forces are likewise zero), Equation 52 may be used to select a value of λ for the latter condition. As illustrated in the design example, the value of λ determined for the zero-moment condition may be sufficient for all conditions; otherwise, trial and error may be used.

The maximum virtual dimension, r, from the check of the column and the beam from Equations 75 and 76 controls the design of the gusset (or the need for reinforcement), and therefore, the larger value should be used for the virtual dimension, r, in subsequent equations per Equation 29. As the haunch forces causing shear in the beam and column are opposite those from the Uniform Force Method for the typical direction of moment, member shear may not always be a governing consideration. Thus, selection of the minimum permissible value of the virtual dimension, r, may not be the most economical choice in all cases; the gusset thickness could be optimized in some cases by selecting a higher value of r. The authors have not developed equations to solve for this optimum value; trial and error may be used. Note that if the design also considers a case without the moment, that latter case typically is governed by beam shear and governs the required gusset thickness.

Minimum Actual Gusset Dimensions Considering Haunch and Bypass Forces

The minimum gusset dimension, $2\bar{\beta}$, is governed by the shear in the column at the gusset mid-height; $\bar{\beta}$ is defined by combining Equations 16, 31, 68, and 70:

$$\bar{\beta}\geq\left(\beta-\frac{\alpha}{\tan\theta}\frac{H_{cBP}}{H_{cTot}}\right)\frac{1}{\left|\dfrac{V_{efC}}{H_{cTot}}\right|+\dfrac{1}{2}} \tag{77}$$

The minimum gusset dimension, $\bar{\alpha}$, is likewise determined by combining Equations 27, 37, 73, and 74:

$$\bar{\alpha}\geq\frac{\alpha}{\left|\dfrac{V_{efB}}{V_{bTot}}\right|+\dfrac{1}{2}} \tag{78}$$

Note that this is based on the beam shear at the gusset mid-length, and thus the connection factor, U_C, is not used.

Designers may wish to begin trials using Equations 77 and 78 with $\lambda = 1.0$ (i.e., with no bypass force). If the gusset dimension $2\bar{\alpha}$ is excessive, the value of λ may be decreased and the dimension $\bar{\beta}$ may be selected using Equation 77.

Beam-Web-to-Column Connection

The sum of the horizontal forces at the beam web-to-column connection, H_{bW}, is:

$$H_{bW}=N_B+H_{bTot1,2}-H_{bTot1,1}+N_H \tag{79}$$

where

N_B = beam axial force delivered to the connection, kips

Note that the horizontal forces, H_b, from the gussets above and below counteract each other. For connections with only one gusset, the required strength of this connection will be significantly larger; this may be an important consideration in determining how much force to divert using the bypass method, which increases H_b.

The sum of the vertical forces at the beam web-to-column connection, V_{bW}, is:

$$V_{bW}=V_g+V_{BMF}+V_{bTot1,2}+V_{bTot1,1} \tag{80}$$

The forces in the beam flange are given in Equation 61. These web and flange forces are shared with the gusset plate, and the designer should avoid connecting the beam to column in a manner that is not compatible with the connection of the gusset to column such as a welded gusset connection and bolted beam-web connection.

DESIGN OF WELDS

At a minimum, the welds connecting the gusset plates must be capable of transferring the forces at the joint determined for the design of the gusset. In some cases, such welds may be insufficient to develop the full strength of the gusset plate. As the design methods presented here utilize the lower-bound theorem, the ductility of such weak-weld/strong-gusset joints must be demonstrated. It is important

<table>
<caption>Table 1. Summary of Equations</caption>
<tr><th>Design Quantity</th><th>UFM Method</th><th>Bypass Method</th><th>Haunch Method</th></tr>
<tr><td>λ</td><td>—</td><td>(52) or (55)</td><td>—</td></tr>
<tr><td>V_{efCol}</td><td colspan="3">(17)</td></tr>
<tr><td>Required V_{efC}</td><td>(35)</td><td colspan="2">(35) [λP for P] or (55)</td></tr>
<tr><td>V_{efBm}, V_{efConn}</td><td colspan="3">(18), (19)</td></tr>
<tr><td>U_C</td><td colspan="3">(20)</td></tr>
<tr><td>$U_C V_{efB}$ [below; above]</td><td colspan="3">(24), (25)</td></tr>
<tr><td>Required $U_C V_{efB}$</td><td>(42)</td><td colspan="2">(42) [λP for P]</td></tr>
<tr><td>Required V_{efB}</td><td>(43)</td><td colspan="2">(43) [λP for P]</td></tr>
<tr><td>r_{minCol}</td><td>(26)</td><td>(53)</td><td>(75)</td></tr>
<tr><td>r_{minBm}</td><td>(28)</td><td>(54)</td><td>(76)</td></tr>
<tr><td>r</td><td colspan="3">(29)</td></tr>
<tr><td>α, β</td><td colspan="3">(10), (11)</td></tr>
<tr><td>V_c, H_c, V_b, H_b</td><td colspan="3">(2), (4), (5), (6)</td></tr>
<tr><td>H_{cBP}, V_{cBP}</td><td>—</td><td>(48), (49)</td><td>—</td></tr>
<tr><td>H_{cH}, H_{bH}</td><td>—</td><td>—</td><td>(62), (63)</td></tr>
<tr><td>V_{cH}, V_{bH}</td><td>—</td><td>—</td><td>(64)</td></tr>
<tr><td>H_{cTot}, V_{cTot}</td><td>—</td><td colspan="2">(68), (69)</td></tr>
<tr><td>H_{bTot}, V_{bTot}</td><td>—</td><td colspan="2">(72), (73)</td></tr>
<tr><td>Required $\bar{\beta}$</td><td>(33)</td><td colspan="2">(77)</td></tr>
<tr><td>Required $\bar{\alpha}$</td><td>(40)</td><td>(78) or (56)</td><td>(78)</td></tr>
<tr><td>M_c</td><td colspan="3">(30)</td></tr>
<tr><td>M_{cBP}</td><td>—</td><td>(50)</td><td>—</td></tr>
<tr><td>M_{cH}</td><td>—</td><td>—</td><td>(65)</td></tr>
<tr><td>M_{cTot}</td><td>—</td><td colspan="2">(70)</td></tr>
<tr><td>M_b</td><td colspan="3">(36)</td></tr>
<tr><td>M_{bH}</td><td>—</td><td>—</td><td>(66)</td></tr>
<tr><td>M_{bTot}</td><td>—</td><td colspan="2">(74)</td></tr>
<tr><td>V_{mid} (column, beam)</td><td colspan="3">(31), (37)</td></tr>
<tr><td>t_g</td><td colspan="3">Maximum of (44) and (45)</td></tr>
</table>

to consider that the redistributions required to engage other elements necessitates that separate joints in the connection deform together. For example, the limitation of the gusset-to-beam forces in the bypass method may assign more force to the beam-to-column connection, and the deformation demands on the two weld groups are not independent. In the absence of a nonlinear or mechanism analysis demonstrating that weld deformations are within their rupture limit, design of the weld for the gusset plate strength ensures the ductility of the joint by favoring inelastic deformation of the gusset or the web of the member over rupture of the weld.

SUMMARY OF EQUATIONS

The preceding discussion includes many intermediate equations necessary for derivation of the design equations. Additionally, some equations have multiple forms: simpler equations for the UFM and more terms with the introduction of bypass and haunch forces. Table 1 presents the equation numbers to be used for the determination of design quantities for the UFM, bypass, and haunch methods. (The haunch method equations include bypass forces.) Certain equations developed for the UFM are modified by substituting λP for P; this is indicated in the table.

Table 2. Brace Forces			
	P (kips)	$P_{cos}\theta$ (kips)	$P_{sin}\theta$ (kips)
$P_{1,1}$	550	352	423
$P_{1,2}$	460	295	353

DESIGN EXAMPLES

The connection shown in Figure 11 will be designed using the methods developed in this study.

The design is performed three times:

- Preliminary design (wind loads). The gusset is designed using UFM method without bypass forces.
- Redesign with bypass forces (wind loads). The gusset is redesigned using UFM and bypass to eliminate web reinforcement.
- Seismic design including haunch forces (and bypass forces).

The design forces are presented in Table 2. To facilitate subsequent calculations, the horizontal and vertical components of the brace forces are determined and presented in the table. (P denotes the axial force in the brace.) The angle from vertical, θ, is 50.2°.

To facilitate comparison, the same brace axial forces are used for both the wind-load and seismic-load designs.

The beam and column forces delivered to the connection are shown in Table 3.

The seismic forces correspond to the formation of plastic hinges at each end of the beam, represented by the symbol E_{cl}. The plastic-hinge moment is the maximum flexure that the beam can deliver and is thus not combined with the gravity moment, although the corresponding shear is additive to the gravity shear. The plastic-hinge moment is 1.1 times the beam expected

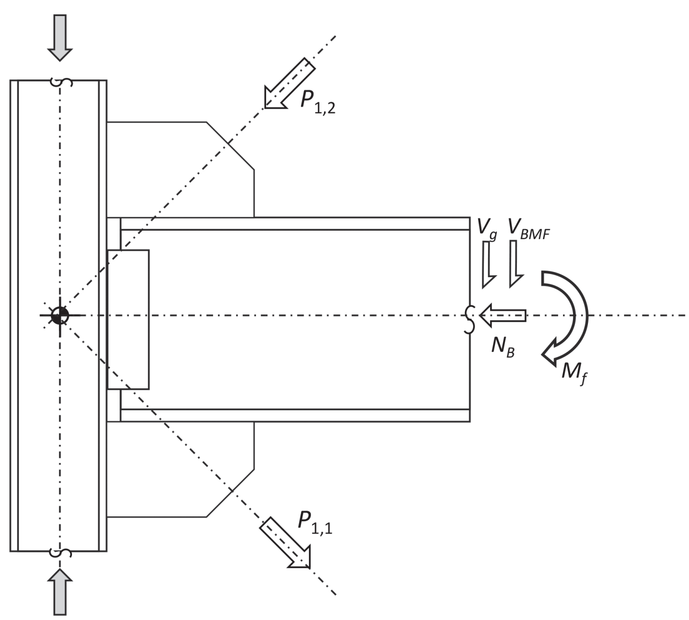


Fig. 11. Design example connection.

Table 3. Beam and Column Forces			
Load Case or Combination		Load	Force
Gravity	$1.2D + L$	V_g	10 kips
		M_f	100 kip-in.
Wind	$1.0W$	V_{BMF}	7.1 kips
		M_f	not used
		V_{col}	17.1 kips
Seismic	E_{cl}	V_{BMF}	44.9 kips
		M_f	7770 kip-in.
		V_{col}	108 kips
Gravity + wind	$1.2D + L + 1.0W$	$V_g + V_{BMF}$	17.1 kips
		M_f	not used
Gravity + seismic	$1.2D + L + 1.0E_{cl}$	$V_g + V_{BMF}$	54.9 kips
	$0D + 0L + E_{cl}$	M_f	7770 kip-in.

Table 4. Member Sizes and Grades		
Member	Size	Grade
Column	W14×120	50 ksi
Beam	W18×55	50 ksi
Gussets	—	50 ksi

beam flexural strength, as required for the design of Special Concentrically Braced Frames and Buckling-Restrained Braced Frames in the AISC *Seismic Provisions* (AISC, 2016) Sections F2.6b(b) and F4.6b(b), respectively.

The member sizes and materials are shown in Table 4. Members are ASTM A992 material, and plate is ASTM A572 material.

The connection strength is based on a 15-in.-deep portion of the beam complete-joint-penetration (CJP) welded to the column flange:

$$\phi R_n \geq 176 \text{ kips}$$

Based on the brace-to-gusset connection (not shown in this example), minimum dimensions for the gusset plate are 23 in. wide and 17 in. high. The dimensions used for calculation allow for an extra inch for weld termination. The minimum gusset interface centroids are:

$$\bar{\beta} \geq 8 \text{ in.}$$

$$\bar{\alpha} \geq 11 \text{ in.}$$

1. Preliminary Design: Design of Top Gusset (with No Bypass or Haunch Force)

The following example shows the design of the top gusset to resist the force $P_{1,2}$. The design considers the effect of the force $P_{1,1}$ on the beam, but otherwise the two gusset designs are independent of each other.

For preliminary design, the gusset will be proportioned to resist brace forces only, assuming the beam moment is transferred to the column by the flanges (i.e., $\rho = 1.0$). Similarly, the bypass-force method will not be used. Both of these methods are addressed in a gusset redesign.

Column Effective Shear Strength

The effective column shear strength is reduced considering the column shear.

$$V_{col} = 17.1 \text{ kips}$$

$$\begin{aligned} V_{efCol} &= \phi V_n - V_{col} \\ &= 257 \text{ kips} - 17.1 \text{ kips} \\ &= 240 \text{ kips} \end{aligned} \tag{17}$$

Using Equation 35, the minimum shear strength is:

$$\begin{aligned} V_{efCol} &= V_{efC} \\ &\geq \frac{e_c \cos\theta P}{\frac{3}{2}\bar{\beta} + e_b} \\ &= \frac{(7.25 \text{ in.})0.640(460 \text{ kips})}{\frac{3}{2}(8.0 \text{ in.}) + (9.05 \text{ in.})} \\ &= 101 \text{ kips} \end{aligned} \tag{35}$$

Beam Effective Shear Strength

The effective beam shear strength is limited by the strength of the connection and is reduced considering the beam shear:

$$\begin{aligned} V_{efBm} &= \phi V_n - V_g - V_{BMF} \\ &= 212 \text{ kips} - 10.0 \text{ kips} - 7.1 \text{ kips} \\ &= 195 \text{ kips} \end{aligned} \tag{18}$$

$$\begin{aligned} V_{efConn} &= \phi R_n - V_g - V_{BMF} \\ &= 176 \text{ kips} - 10.0 \text{ kips} - 7.1 \text{ kips} \\ &= 158 \text{ kips} \end{aligned} \tag{19}$$

$$\begin{aligned} U_C &= \frac{V_{efConn}}{V_{efBm}} \\ &= \frac{158 \text{ kips}}{195 \text{ kips}} \\ &= 0.814 \end{aligned} \tag{20}$$

The apportioned effective beam shear strength that can be utilized for each gusset can be apportioned considering the vertical components of the brace forces:

$$\begin{aligned} U_C V_{efB1,1} &= U_C V_{efBm} \frac{P_{1,1}\cos\theta}{P_{1,1}\cos\theta + P_{1,2}\cos\theta} \\ &= 0.814(195 \text{ kips}) \frac{352 \text{ kips}}{352 \text{ kips} + 295 \text{ kips}} \\ &= 86.2 \text{ kips} \end{aligned} \tag{from Eq. 24}$$

$$\begin{aligned} U_C V_{efB1,2} &= U_C V_{efBm} \frac{P_{1,2}\cos\theta}{P_{1,1}\cos\theta + P_{1,2}\cos\theta} \\ &= 0.814(195 \text{ kips}) \frac{295 \text{ kips}}{352 \text{ kips} + 295 \text{ kips}} \\ &= 72.1 \text{ kips} \end{aligned} \tag{from Eq. 25}$$

Using Equation 42, the minimum apportioned effective connection shear strength is:

$$U_C V_{efB1,2} \geq \frac{e_b \sin\theta P}{\frac{3}{2}\bar{\alpha} + e_c} \qquad \text{(from Eq. 42)}$$

$$= \frac{(9.05 \text{ in.})(0.768)(460 \text{ kips})}{\frac{3}{2}(11.0 \text{ in.}) + (7.25 \text{ in.})}$$

$$= 135 \text{ kips} > 72.1 \text{ kips}$$

Reinforcement is therefore required.

The beam shear strength is apportioned:

$$V_{efB1,2} = V_{efBm} \frac{P_{1,2} \cos}{P_{1,1} \cos\theta + P_{1,2} \cos\theta} \tag{25}$$

$$= (195 \text{ kips})(0.455)$$

$$= 88.7 \text{ kips}$$

The minimum apportioned effective beam shear strength is:

$$V_{efB} \geq \frac{e_b P \sin\theta}{\bar{\alpha}\left(1 + \frac{U_C}{2}\right) + U_C e_c} \tag{43}$$

$$= \frac{(9.05 \text{ in.})(460 \text{ kips})(0.768)}{(11.0 \text{ in.})\left(1 + \frac{0.814}{2}\right) + 0.814(7.25 \text{ in.})}$$

$$= 150 \text{ kips} > 88.7 \text{ kips}$$

Because $V_{efB} > V_{efB1,2}$, reinforcement is required.

If the proposed gusset length is to be used, both the connection strength and the beam strength are insufficient. Based on the degree of this deficiency, it is anticipated that an unreinforced beam will require an excessively long gusset. (In fact, calculations were performed, and the required horizontal length exceeds 42 in.) Instead, the web will be reinforced with a ⅜-in. doubler, 15 in. deep (matching the available web depth of the connection) and extending horizontally to match the gusset length. The reinforced strength for both the connection and the beam is determined and apportioned as before. The additional strength is:

$$\phi R_n \geq \phi 0.6 F_y dt$$

$$= (1.0)(0.6)(50 \text{ ksi})(15 \text{ in.})(\tfrac{3}{8} \text{ in.})$$

$$= 169 \text{ kips}$$

The beam strength is:

$$\phi V_n = 212 \text{ kips} + 169 \text{ kips}$$

$$= 381 \text{ kips}$$

The connection strength is:

$$\phi V_n = 176 \text{ kips} + 169 \text{ kips}$$

$$= 344 \text{ kips}$$

The effective strengths are recalculated:

$$V_{efBm} = \phi V_n - V_g - V_{BMF} \tag{18}$$
$$= 381 \text{ kips} - 10.0 \text{ kips} - 7.1 \text{ kips}$$
$$= 363 \text{ kips}$$

$$V_{efConn} = \phi R_n - V_g - V_{BMF} \tag{19}$$
$$= 344 \text{ kips} - 10.0 \text{ kips} - 7.1 \text{ kips}$$
$$= 327 \text{ kips}$$

With this reinforcement, the value of U_c is:

$$U_C = \frac{V_{efConn}}{V_{efBm}} = 0.900 \tag{20}$$

$$U_C V_{efB1,2} = 0.455(327 \text{ kips})$$
$$= 149 \text{ kips} > 135 \text{ kips} \quad \textbf{o.k.}$$

With this additional beam web reinforcement, the effective beam and effective connection capacities will be increased.

$$V_{efB1,2} = 0.455(363 \text{ kips})$$
$$= 166 \text{ kips} > 150 \text{ kips} \quad \textbf{o.k.}$$

Minimum Virtual Gusset Dimensions

The virtual gusset dimensions are determined based on the effective column and beam shear strengths calculated earlier.

$$r_{minCol} = \frac{e_c P}{V_{efC}} \tag{26}$$
$$= \frac{(7.25 \text{ in})(460 \text{ kips})}{240 \text{ kips}}$$
$$= 13.9 \text{ in.}$$

$$r_{minBm} = \frac{e_b P}{U_C V_{efB1,2}} \qquad \text{(from Eq. 28)}$$
$$= \frac{(9.05 \text{ in})(460 \text{ kips})}{0.900(166 \text{ kips})}$$
$$= 27.9 \text{ in.}$$

$$r \geq \max(r_{minCol}, r_{minBm}) \tag{29}$$
$$\geq \max(13.9 \text{ in.}, 27.9 \text{ in.})$$
$$= 27.9 \text{ in.}$$

This value will be used for subsequent calculations. The corresponding virtual dimensions are:

$$\alpha = r \sin\theta - e_c \tag{10}$$
$$= (27.9 \text{ in.})(0.768) - 7.25 \text{ in.}$$
$$= 14.2 \text{ in.}$$

$$\beta = r \cos\theta - e_b \tag{11}$$
$$= (27.9 \text{ in.})(0.640) - 9.05 \text{ in.}$$
$$= 8.80 \text{ in.}$$

Gusset Forces and Member Shear Checks (at Connection)

The virtual gusset dimensions are used to determine the forces acting at the centroids of the virtual gusset interfaces with the beam and the column:

$$\begin{aligned} V_c &= \frac{\beta}{r}P \\ &= \left(\frac{8.80 \text{ in.}}{27.9 \text{ in.}}\right)(460 \text{ kips}) \\ &= 146 \text{ kips} \end{aligned} \tag{2}$$

$$\begin{aligned} H_c &= \frac{e_c}{r}P \\ &= \left(\frac{7.25 \text{ in.}}{27.9 \text{ in.}}\right)(460 \text{ kips}) \\ &= 119 \text{ kips} < V_{efC} \quad \textbf{o.k.} \end{aligned} \tag{4}$$

$$\begin{aligned} V_b &= \frac{e_b}{r}P \\ &= \left(\frac{9.05 \text{ in.}}{27.9 \text{ in.}}\right)(460 \text{ kips}) \\ &= 149 \text{ kips} < U_C V_{efB1,2} \quad \textbf{o.k.} \end{aligned} \tag{5}$$

$$\begin{aligned} H_b &= \frac{\alpha}{r}P \\ &= \left(\frac{14.2 \text{ in.}}{27.9 \text{ in.}}\right)(460 \text{ kips}) \\ &= 234 \text{ kips} \end{aligned} \tag{6}$$

Minimum Actual Gusset Dimensions and Member Shear Checks (at Mid-Length of Gusset)

The value of r_{minBm} controls; therefore, $r = r_{minBm}$ and $r > r_{minCol}$. The dimensions for $\bar{\alpha}$ and $\bar{\beta}$ are calculated as follows:

$$\begin{aligned} \bar{\beta} &\geq \frac{\beta}{\left|\dfrac{V_{efC}}{H_c}\right| + \dfrac{1}{2}} \\ &= \frac{8.80 \text{ in.}}{\left|\dfrac{240 \text{ kips}}{119 \text{ kips}}\right| + \dfrac{1}{2}} \\ &= 3.50 \text{ in.} \end{aligned} \tag{33}$$

$$\begin{aligned} \bar{\alpha} &\geq \frac{\alpha}{\dfrac{1}{U_C} + \dfrac{1}{2}} \\ &= \frac{14.2 \text{ in.}}{\dfrac{1}{0.900} + \dfrac{1}{2}} \\ &= 8.81 \text{ in.} \end{aligned} \tag{40}$$

The following values will be used:

$$\bar{\beta} = 8 \text{ in.}$$

$$\bar{\alpha} = 11 \text{ in.}$$

Both of these values exceed two-thirds of the corresponding virtual dimensions:

$$\frac{\bar{\beta}}{\beta} = \frac{8 \text{ in.}}{8.80 \text{ in.}}$$
$$= 0.909 > \tfrac{2}{3}$$

$$\frac{\bar{\alpha}}{\alpha} = \frac{11 \text{ in.}}{14.2 \text{ in.}}$$
$$= 0.775 > \tfrac{2}{3}$$

As such, the maximum member shear will be at sections C1 and B1 rather than C2 and B2. For completeness, the evaluation is shown next.

The moments due to the eccentricities between the actual dimensions and the virtual dimensions are:

$$M_c = H_c\left(\beta - \bar{\beta}\right) \tag{30}$$
$$= 119 \text{ kips}(8.80 \text{ in.} - 8 \text{ in.})$$
$$= 100 \text{ kip-in.}$$

$$M_b = V_b\left(\alpha - \bar{\alpha}\right) \tag{36}$$
$$= 149 \text{ kips}(14.2 \text{ in.} - 11 \text{ in.})$$
$$= 479 \text{ kip-in.}$$

Check column shear:

$$V_{mid} = \frac{H_c}{2} + \frac{M_c}{\bar{\beta}} \tag{31}$$
$$= \frac{119 \text{ kips}}{2} + \frac{100 \text{ kip-in.}}{8 \text{ in.}}$$
$$= 72.8 \text{ kips} < V_{efCol} \quad \textbf{o.k.}$$

Check beam shear:

$$V_{mid} = \frac{V_b}{2} + \frac{M_b}{\bar{\alpha}} \tag{37}$$
$$= \frac{149 \text{ kips}}{2} + \frac{479 \text{ kip-in.}}{11 \text{ in.}}$$
$$= 118 \text{ kips} < V_{efBm} \quad \textbf{o.k.}$$

Gusset Thickness

The gusset thickness is selected considering the combined forces on the gusset-to-column interface and the gusset-to-beam interface. Considering the column side, the minimum thickness is:

$$t_g \geq \frac{1}{\phi F_y 2\bar{\beta}}\sqrt{\left(\frac{V_c}{0.6}\right)^2 + \left(\left|H_c\right| + \frac{2\left|M_c\right|}{\bar{\beta}}\right)^2} \tag{44}$$
$$= \frac{1}{0.90(50 \text{ ksi})2(8 \text{ in.})}\sqrt{\left(\frac{146 \text{ kips}}{0.6}\right)^2 + \left(\left|119 \text{ kips}\right| + \frac{2\left|100 \text{ kip-in.}\right|}{8 \text{ in.}}\right)^2}$$
$$= 0.390 \text{ in.}$$

Considering the beam side, the minimum thickness is:

$$t_g \geq \frac{1}{\phi F_y 2\bar{\alpha}} \sqrt{\left(\frac{H_b}{0.6}\right)^2 + \left(|V_b| + \frac{2|M_b|}{\bar{\alpha}}\right)^2} \tag{45}$$

$$= \frac{1}{0.90(50\text{ ksi})2(11\text{ in.})} \sqrt{\left(\frac{234\text{ kips}}{0.6}\right)^2 + \left(|149\text{ kips}| + \frac{2|479\text{ kip-in.}|}{11\text{ in.}}\right)^2}$$

$$= 0.460\text{ in.}$$

A ½- in.-thick gusset is required, with overall dimensions of 23 in. wide by 17 in. high (providing extra length for weld termination). The gusset design is shown in Figure 12.

2. Redesign with Bypass Forces

The connection in this example is redesigned using the bypass method in order to avoid the need for the beam-web doubler. Values for V_{efB} and U_c correspond to the unreinforced condition.

The transfer factor λ is obtained from Equation 52:

$$\lambda \leq \frac{V_{efB}}{e_b \sin\theta P}\left[\bar{\alpha}\left(1+\frac{U_C}{2}\right) + U_C e_c\right] \tag{52}$$

$$= \frac{88.6\text{ kips}}{(9.05\text{ in.})0.768(460\text{ kips})}\left[(11\text{ in.})\left(1+\frac{0.814}{2}\right) + 0.814(7.25\text{ in.})\right]$$

$$= 0.592$$

Bypass forces are calculated:

$$H_{cBP} = (1-\lambda)P\sin\theta \tag{48}$$

$$= (1-0.592)(460\text{ kips})0.768$$

$$= 144\text{ kips}$$

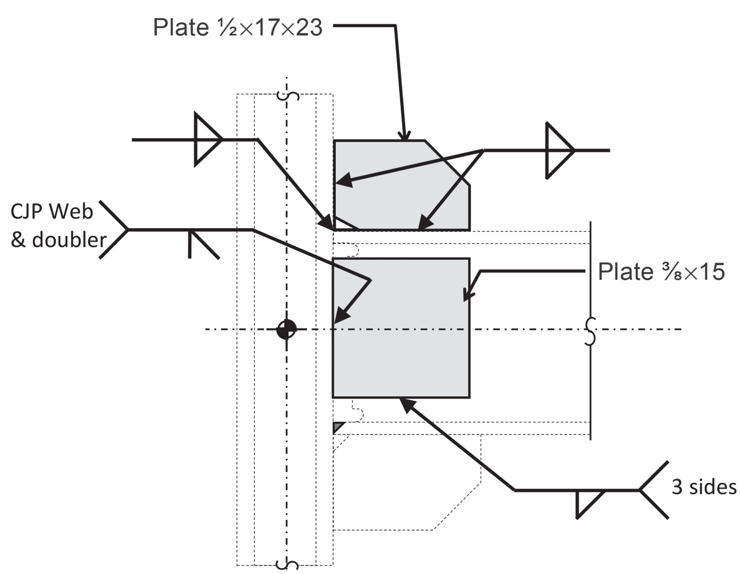


Fig. 12. Preliminary gusset design.

$$V_{cBP} = \frac{H_{cBP}}{\tan\theta} \tag{49}$$
$$= \frac{144 \text{ kips}}{1.20}$$
$$= 120 \text{ kips}$$

Minimum Virtual Gusset Dimensions

The virtual gusset dimensions are determined:

$$r_{minCol} = \frac{\lambda P e_c}{V_{efC} - H_{cBP}} \tag{53}$$
$$= \frac{(0.592)(460 \text{ kips})(7.25 \text{ in.})}{(240 \text{ kips}) - (144 \text{ kips})}$$
$$= 20.7 \text{ in.}$$

$$r_{minBm} = \frac{\lambda P e_b}{U_C V_{efB}} \tag{54}$$
$$= \frac{(0.592)(460 \text{ kips})(9.05 \text{ in.})}{0.814(88.6 \text{ kips})}$$
$$= 34.2 \text{ in.}$$

$$r \geq \max(r_{minCol}, r_{minBm}) \tag{29}$$
$$= \max(20.7 \text{ in.}, 34.2 \text{ in.})$$
$$= 34.2 \text{ in.}$$

This value will be used for subsequent calculations. The corresponding virtual dimensions

$$\alpha = r\sin\theta - e_c \tag{10}$$
$$= (34.2 \text{ in.})(0.768) - 7.25 \text{ in,}$$
$$= 19.0 \text{ in.}$$

$$\beta = r\cos\theta - e_b \tag{11}$$
$$= (34.2 \text{ in.})(0.640) - 9.05 \text{ in.}$$
$$= 12.8 \text{ in.}$$

Gusset Forces and Member Shear Checks (at Connection)

The virtual gusset dimensions are used to determine the forces acting at the centroids of the virtual gusset interfaces with the beam and the column:

$$\lambda V_c = \frac{\beta}{r}\lambda P \tag*{(from Eq. 2)}$$
$$= \left(\frac{12.8 \text{ in.}}{34.2 \text{ in.}}\right)(0.592)(460 \text{ kips})$$
$$= 102 \text{ kips}$$

$$\lambda H_c = \frac{e_c}{r}\lambda P \qquad \text{(from Eq. 4)}$$

$$= \left(\frac{7.25 \text{ in.}}{34.2 \text{ in.}}\right)(0.592)(460 \text{ kips})$$

$$= 57.8 \text{ kips}$$

$$\lambda V_b = \frac{e_b}{r}\lambda P \qquad \text{(from Eq. 5)}$$

$$= \left(\frac{9.05 \text{ in.}}{34.2 \text{ in.}}\right)(0.592)(460 \text{ kips})$$

$$= 72.1 \text{ kips}$$

$$\lambda H_b = \frac{\alpha}{r}\lambda P \qquad \text{(from Eq. 6)}$$

$$= \left(\frac{19.0 \text{ in.}}{34.2 \text{ in.}}\right)(0.592)(460 \text{ kips})$$

$$= 151 \text{ kips}$$

Determination of Total Forces and Member Shear Checks (at Connection)

The total vertical and horizontal forces are used in the determination of minimum required gusset dimensions. (Moments cannot be computed until the actual gusset dimensions are set.) The forces on the column edge of the gusset are:

$$H_{cTot} = \lambda H_c + H_{cBP} \qquad \text{(from Eq. 68)}$$

$$= 57.8 \text{ kips} + 144 \text{ kips}$$

$$= 202 \text{ kips} < V_{efC} \quad \textbf{o.k.}$$

$$V_{cTot} = \lambda V_c + V_{cBP} \qquad \text{(from Eq. 69)}$$

$$= 102 \text{ kips} + 120 \text{ kips}$$

$$= 222 \text{ kips}$$

The forces on the beam edge of the gusset are:

$$H_{bTot} = \lambda H_b \qquad \text{(from Eq. 72)}$$

$$= 151 \text{ kips}$$

$$V_{bTot} = \lambda V_b \qquad \text{(from Eq. 73)}$$

$$= 72.1 \text{ kips} < U_C V_{efB} \quad \textbf{o.k.}$$

Determination of Minimum Actual Gusset Dimensions

$$\bar{\beta} \geq \left(\beta - \frac{\alpha}{\tan\theta}\frac{H_{cBP}}{H_{cTot}}\right)\frac{1}{\left|\dfrac{V_{efC}}{H_{cTot}}\right| + \dfrac{1}{2}} \tag{77}$$

$$= \frac{12.8 \text{ in.} - \left(\dfrac{19.0 \text{ in.}}{1.20}\right)\left(\dfrac{144 \text{ kips}}{202 \text{ kips}}\right)}{\left|\dfrac{240 \text{ kips}}{202 \text{ kips}}\right| + \dfrac{1}{2}}$$

$$= 0.900 \text{ in.}$$

$$\bar{\alpha} \geq \frac{\alpha}{\left|\dfrac{V_{efB}}{V_{bTot}}\right| + \dfrac{1}{2}} \tag{78}$$

$$= \frac{19.0 \text{ in.}}{\left|\dfrac{88.6 \text{ kips}}{72.1 \text{ kips}}\right| + \dfrac{1}{2}}$$

$$= 11.0 \text{ in.}$$

The following values will be used:

$$\bar{\beta} = 8 \text{ in.}$$

$$\bar{\alpha} = 11 \text{ in.}$$

Determination of Moments at Interfaces and Member Shear Checks (at Mid-Length of Gusset)

The moments on the column edge of the gusset are:

$$\lambda M_c = \lambda H_c(\beta - \bar{\beta}) \qquad \text{(from Eq. 30)}$$

$$= (57.8 \text{ kips})(12.8 \text{ in.} - 8 \text{ in.})$$

$$= 279 \text{ kip-in.}$$

$$M_{cBP} = H_{cBP}(e_b + \bar{\beta}) - V_{cBP}e_c \tag{50}$$

$$= (144 \text{ kips})(9.05 \text{ in.} + 8 \text{ in.}) - (120 \text{ kips})(7.25 \text{ in.})$$

$$= 1{,}590 \text{ kip-in.}$$

$$M_{cTot} = \lambda M_c - M_{cBP} \qquad \text{(from Eq. 70)}$$

$$= 279 \text{ kip-in.} - 1{,}590 \text{ kip-in.}$$

$$= -1{,}310 \text{ kip-in.}$$

The moments on the beam edge of the gusset are:

$$M_{bTot} = \lambda M_b \qquad \text{(from Eq. 74)}$$

$$= \lambda V_b(\alpha - \bar{\alpha}) \qquad \text{(from Eq. 36)}$$

$$= (72.1 \text{ kips})(19.0 \text{ in.} - 11 \text{ in.})$$

$$= 577 \text{ kip-in.}$$

Check column shear:

$$V_{mid} = \frac{H_{cTot}}{2} + \frac{M_{cTot}}{\bar{\beta}} \qquad \text{(from Eq. 31)}$$

$$= \frac{202 \text{ kips}}{2} + \frac{-1{,}310 \text{ kip-in.}}{8 \text{ in.}}$$

$$= 62.6 \text{ kips} < V_{efC} \quad \textbf{o.k.}$$

Check beam shear:

$$V_{mid} = \frac{V_{bTot}}{2} + \frac{M_{bTot}}{\bar{\alpha}} \qquad \text{(from Eq. 37)}$$

$$= \frac{72.1 \text{ kips}}{2} + \frac{577 \text{ kip-in.}}{11 \text{ in.}}$$

$$= 88.5 \text{ kips} < V_{efB} \quad \textbf{o.k.}$$

Note that Equations 31 and 37 are modified to utilize total forces (H_{cTot}, etc.).

Determination of Required Gusset Thickness

Considering the column side, the minimum thickness is:

$$t_g \ge \frac{1}{\phi F_y 2\bar{\beta}}\sqrt{\left(\frac{V_{cTot}}{0.6}\right)^2 + \left(\left|H_{cTot}\right| + \frac{2\left|M_{cTot}\right|}{\bar{\beta}}\right)^2} \qquad \text{(from Eq. 44)}$$

$$= \frac{1}{0.90(50\text{ ksi})2(8\text{ in.})}\sqrt{\left(\frac{222\text{ kips}}{0.6}\right)^2 + \left(\left|202\text{ kips}\right| + \frac{2\left|-1{,}310\text{ kip-in.}\right|}{8\text{ in.}}\right)^2}$$

$$= 0.897\text{ in.}$$

Considering the beam side, the minimum thickness is:

$$t_g \ge \frac{1}{\phi F_y 2\bar{\alpha}}\sqrt{\left(\frac{H_{bTot}}{0.6}\right)^2 + \left(\left|V_{bTot}\right| + \frac{2\left|M_{bTot}\right|}{\bar{\alpha}}\right)^2} \qquad \text{(from Eq. 45)}$$

$$= \frac{1}{0.90(50\text{ ksi})2(11\text{ in.})}\sqrt{\left(\frac{151\text{ kips}}{0.6}\right)^2 + \left(\left|72.11\text{ kips}\right| + \frac{2\left|577\text{ kip-in.}\right|}{11\text{ in.}}\right)^2}$$

$$= 0.311\text{ in.}$$

The minimum thickness is taken as the maximum of Equations 44 and 45. Thus, a plate 1 in. × 17 in. high × 23 in. wide can be used. The web doubler in the beam is eliminated (as are the CJP welds of the doubler and beam web to the column) at the cost of a thicker gusset (and larger gusset-to-column welds). Alternatively, if a thinner gusset is desired, the dimension $\bar{\beta}$ could be increased beyond the original target value. Fillet welds are based on the strength of the gusset plate. The gusset design is shown in Figure 13.

3. Design Considering Haunch Force (with Bypass Method)

In this example, a design of the connection will be performed considering requirements for buckling restrained braced frames. (As discussed earlier, inelastic demands on the gusset plate due to brace buckling, as would be expected for the seismic response of other braced-frame systems, are not addressed in this design method.) The example considers seismic forces, but the method illustrated is applicable to design for wind loads as well.

For seismic design, the beam moments and shears considered correspond to the flexural yielding of the beam because the frame is expected to undergo large displacements. While the beam end moment could be resisted by the beam-to-column connection (as it was for the previous examples for wind load), there can be significant economy in utilizing the gussets as haunches and eliminating beam-flange-to-column-flange welds. In this example, the haunch method is used in conjunction with the bypass method utilized in the previous example.

The designer should consider the range of possible beam-end moments that could coincide with maximum brace forces (and vice versa), as the drift ranges for the beam and for the braces are likely to be substantially different. For this example, the range of moments is based on the beam expected moment strength in one direction (positive M_f) and zero in the opposite direction. The authors do not intend to imply that this range is adequate for all (or even most) conditions. Determination of the appropriate combinations of beam moment and brace axial forces for systems subject to inelastic drift requires further study and is outside of the scope of this paper.

Values are presented in pairs in braces (“{}”), with the first value corresponding to the inclusion of forces (both shears and moments) corresponding to moment-frame action (condition 1) and the second value without (condition 2). A separate final check is performed using only the haunch forces from beam moment with brace forces taken as zero.

Given the difference in the drift that results in yield of the braces and that corresponding to yield of the beam, it is possible for cyclic inelastic drift to result in a condition in which the value of beam moment is negative and the forces λH_c and H_{cH} (Eq. 68) are additive. Consideration of such a condition is beyond the scope of this study.

The haunch force is based on an apportionment factor $\rho = 0.0$.

$$M_f = \{7770, 0\} \text{ kip-in.}$$

$$\begin{aligned} M_H &= (1-\rho)\frac{M_{f_i}}{2} && (59) \\ &= (1-0)\frac{\{7770,\ 0\} \text{ kip-in.}}{2} \\ &= \{3880, 0\} \text{ kip-in.} \end{aligned}$$

$$V_{col} = \{108, 0\} \text{ kips}$$

$$V_{BMF} = \{44.9,\ 0\} \text{ kips}$$

$$\begin{aligned} V_{efCol} &= \phi V_n - V_{col} && (17) \\ &= 257 \text{ kips} - \{108,\ 0\} \text{ kips} \\ &= \{149,\ 257\} \text{ kips} \end{aligned}$$

$$\begin{aligned} V_{efBm} &= (\phi V_n - V_g - V_{BMF})\frac{P_{1,2}\cos\theta}{P_{1,1}\cos\theta + P_{1,2}\cos\theta} && \text{(from Eqs. 18, 25)} \\ &= (212 \text{ kips} - 10.0 \text{ kips} - \{44.9,\ 0\}\text{kips})\left(\frac{295 \text{ kips}}{352 \text{ kips} + 295 \text{ kips}}\right) \\ &= \{71.4,\ 91.9\} \text{ kips} \end{aligned}$$

$$\begin{aligned} V_{efConn} &= (\phi R_n - V_g - V_{BmF})\frac{P_{1,2}\cos\theta}{P_{1,1}\cos\theta + P_{1,2}\cos\theta} && \text{(from Eqs. 19, 25)} \\ &= (176 \text{ kips} - 10.0 \text{ kips} - \{44.9,\ 0\}\text{kips})\left(\frac{295 \text{ kips}}{352 \text{ kips} + 295 \text{ kips}}\right) \\ &= \{54.9,\ 91.9\} \text{ kips} \end{aligned}$$

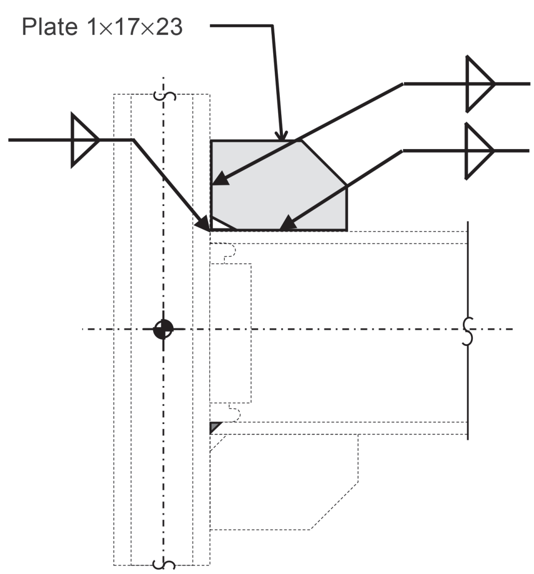


Fig. 13. Gusset design with bypass forces.

$$U_C = \frac{V_{efConn}}{V_{efBm}} \tag{20}$$
$$= \left\{\frac{57.5 \text{ kips}}{74.0 \text{ kips}}, \frac{75.4 \text{ kips}}{91.9 \text{ kips}}\right\}$$
$$= \{0.769, 0.820\}$$

The apportionment factor λ is determined for condition 2 ($M_f = 0$) using the corresponding effective beam and connection strength.

$$\lambda \le \frac{V_{efB}}{e_b \sin\theta P}\left[\bar{\alpha}\left(1+\frac{U_C}{2}\right)+e_c U_C\right] \tag{52}$$
$$= \frac{(91.9 \text{ kips})}{(9.05 \text{ in.})(0.768)(460 \text{ kips})}\left[(11 \text{ in.})\left(1+\frac{0.820}{2}\right)+(0.820)(7.25 \text{ in.})\right]$$
$$= 0.617$$

This apportionment factor based on condition 2 will be used for both conditions 1 and 2.

$$H_{cBP} = (1-\lambda)P\sin\theta \tag{48}$$
$$= (1-0.617)(460 \text{ kips})0.768$$
$$= 135 \text{ kips}$$

Determination of Minimum Virtual Gusset Dimensions

The minimum required gusset dimension, r, is determined for each case:

$$r_{minCol}{}^2\left[(V_{efC}-H_{cBP})\cos\theta\right]+r_{minCol}\left[(V_{efC}-H_{cBP})(e_c-e_b)+(1-\rho)\frac{M_f}{2}-\lambda P e_c\cos\theta\right]+\left[\lambda P e_c(e_b-e_c)\right]\ge 0 \tag{75}$$
$$r_{minCol}{}^2\left[(\{163, 257\}-135)0.640\right]$$
$$+r_{minCol}\left[(\{163, 257\} \text{ kips}-135 \text{ kips})(7.25 \text{ in.}-9.05 \text{ in.})+\frac{\{7765, 0\} \text{ kip-in.}}{2}-0.617(460 \text{ kips})(7.25 \text{ in.})(0.640)\right]$$
$$+\left[0.617(460 \text{ kips})(7.25 \text{ in.})(9.05 \text{ in.}-7.25 \text{ in.})\right]\ge 0$$

This is solved by the quadratic formula:

$$r_{minCol} = \{-1.50, 17.0\} \text{ in.}$$

$$r_{minBm}{}^2\left[\sin\theta U_c V_{efBm}\right]+r_{minBm}\left[M_H-e_c U_c V_{efBm}-PFe_b\sin\theta\right]+\left[\lambda P e_b e_c-M_H\frac{e_b}{\cos\theta}\right]\ge 0 \tag{76}$$
$$r_{minBm}{}^2\left[0.768\{54.9, 91.9\} \text{ kips}\right]$$
$$+r_{minBm}\left[\{3880, 0\} \text{ kip-in.}-(7.25 \text{ in.})\{54.9, 91.9\} \text{ kips}-0.617(460 \text{ kips})(9.05 \text{ in.})0.768\right]$$
$$+\left[0.617(460 \text{ kips})(9.05 \text{ in.})(7.25 \text{ in.})-\{3880, 0\} \text{ kip-in.}\left(\frac{9.05 \text{ in.}}{0.640}\right)\right]\ge 0$$

This is solved by the quadratic formula:

$$r_{minBm} = \{16.4, 34.1\} \text{ in.}$$

$$r \ge \max(r_{minCol}, r_{minBm}) \tag{29}$$
$$= \{\max(-1.50, 16.4), \max(17.0, 34.1)\} \text{ in.}$$
$$= \{16.4, 34.1\} \text{ in.}$$

As discussed in the derivation of Equations 75 and 76, for cases in which the member shear is not a governing consideration, selection of a higher value of the virtual dimension, r, could result in lower required gusset thickness. By trial and error, a value of the virtual dimension $r = 18.9$ in. was found to optimize the required gusset thickness for condition 1. (The same gusset-edge forces could also be obtained with $\lambda = 1.0$ and $r = 35.2$.) However, because this example also considers the zero-moment condition 2, no reduction results from such optimization, and such trial-and-error methods are generally not beneficial.

$$\begin{aligned}\alpha &= r\sin\theta - e_c \\ &= \{18.9,\ 34.1\}\text{ in.}(0.768) - 7.25\text{ in.} \\ &= \{7.20,\ 18.9\}\text{ in.}\end{aligned} \tag{10}$$

$$\begin{aligned}\beta &= r\cos\theta - e_b \\ &= \{18.9,\ 34.1\}\text{ in.}(0.640) - 9.05\text{ in.} \\ &= \{3.00,\ 12.8\}\text{ in.}\end{aligned} \tag{11}$$

Determination of Forces at Interfaces

The minimum actual gusset dimensions $\bar{\beta}$ and $\bar{\alpha}$ are functions of the UFM, bypass, and haunch forces:

$$\begin{aligned}\lambda V_c &= \lambda\frac{\beta}{r}P \\ &= 0.617\frac{\{3.00,\ 12.8\}\text{ in.}}{\{18.9,\ 34.1\}\text{ in.}}(460\text{ kips}) \\ &= \{45.5,\ 106\}\text{ kips}\end{aligned} \tag{from Eq. 2}$$

$$\begin{aligned}\lambda H_c &= \lambda\frac{e_c}{r}P \\ &= 0.617\frac{7.25\text{ in.}}{\{18.9,\ 34.1\}\text{ in.}}(460\text{ kips}) \\ &= \{109,\ 60.4\}\text{ kips}\end{aligned} \tag{from Eq. 4}$$

$$\begin{aligned}\lambda V_b &= \lambda\frac{e_b}{r}P \\ &= 0.617\frac{9.05\text{ in.}}{\{18.9,\ 34.1\}\text{ in.}}(460\text{ kips}) \\ &= \{136,\ 75.4\}\text{ kips}\end{aligned} \tag{from Eq. 5}$$

$$\begin{aligned}\lambda H_b &= \lambda\frac{\alpha}{r}P \\ &= 0.617\frac{\{7.20,\ 18.9\}\text{ in.}}{\{18.9,\ 34.1\}\text{ in.}}(460\text{ kips}) \\ &= \{109,\ 158\}\text{ kips}\end{aligned} \tag{from Eq. 6}$$

$$\begin{aligned}V_{cBP} &= \frac{H_{cBP}}{\tan\theta} \\ &= \frac{135\text{ kips}}{1.20} \\ &= 113\text{ kips}\end{aligned} \tag{49}$$

$$H_{cH} = H_{bH} \qquad \text{(from Eqs. 62, 63)}$$
$$= \frac{M_H}{e_b + \beta}$$
$$= \frac{\{3880,\ 0\}\text{ kip-in.}}{9.05\text{ in.} + \{3.00,\ 12.8\}\text{ in.}}$$
$$= \{322,\ 0\}\text{ kips}$$

$$V_{bH} = V_{cH} \tag{64}$$
$$= H_{cH}\frac{\beta}{\alpha}$$
$$= \{322,\ 0\}\text{ kips}\frac{\{3.00,\ 12.8\}\text{ in.}}{\{7.20,\ 18.9\}\text{ in.}}$$
$$= \{134,\ 0\}\text{ kips}$$

Determination of Total Forces and Member Shear Checks (at Connection)

The total vertical and horizontal forces are used in the determination of minimum required gusset dimensions. (Moments cannot be computed until the actual gusset dimensions are set.) The forces on the column edge of the gusset are:

$$H_{cTot} = \lambda H_c + H_{cBP} - H_{cH} \tag{68}$$
$$= \{109,\ 60.4\}\text{kips} + 135\text{ kips} - \{322,\ 0\}\text{kips}$$
$$= \{-77.1,\ 196\}\text{kips} < V_{efC} = V_{efCol} \quad \textbf{o.k.}$$

$$V_{cTot} = \lambda V_c + V_{cBP} + V_{cH} \tag{69}$$
$$= \{45.5,\ 106\}\text{ kips} + 113\text{ kips} - \{134,\ 0\}\text{ kips}$$
$$= \{293,\ 219\}\text{ kips}$$

The forces on the beam edge of the gusset are:

$$H_{bTot} = \lambda H_b + H_{bH} \tag{72}$$
$$= \{109,\ 158\}\text{ kips} + \{322,\ 0\}\text{ kips}$$
$$= \{431,\ 158\}\text{ kips}$$

$$V_{bTot} = \lambda V_b - V_{bH} \tag{73}$$
$$= \{136,\ 75.4\}\text{kips} - \{134,\ 0\}\text{ kips}$$
$$= \{1.90,\ 75.4\}\text{kips} < U_C V_{efB} \quad \textbf{o.k.}$$

Determination of Minimum Actual Gusset Dimensions

$$\bar{\beta} \ge \frac{\beta H_{cTot} - H_{cBP}\left(\beta + e_b - \dfrac{e_c}{\tan\theta}\right)}{V_{efC} + \dfrac{1}{2}H_{cTot}} \tag{77}$$
$$= \frac{\left(\{3.00,\ 12.8\}\text{in.}\right)\left(\{-77.1,\ 196\}\text{kips}\right) - 135\text{ kips}\left(\{3.00,\ 12.8\}\text{in.} + 9.05\text{ in.} - \dfrac{7.25\text{ in.}}{1.20}\right)}{\{149,\ 257\}\text{kips} + \dfrac{1}{2}\{-77.1,\ 196\}\text{ kips}}$$
$$= \{5.60,\ 1.00\}\text{ in.}$$

$$\bar{\alpha} \geq \frac{\alpha}{\left|\frac{V_{efB}}{V_{bTot}}\right| + \frac{1}{2}} \tag{78}$$

$$= \frac{\{7.20,\ 18.9\}\ \text{in.}}{\left|\frac{\{71.4,\ 91.9\}\ \text{kips}}{\{1.90,\ 75.4\}\ \text{kips}}\right| + \frac{1}{2}}$$

$$= \{0.200,\ 11.0\}\ \text{in.}$$

The following values will be used for the actual gusset:

$$\bar{\beta} = 8\ \text{in.}$$

$$\bar{\alpha} = 11\ \text{in.}$$

It should be noted that condition 2 (which does not include the beam moment) is the governing condition. This is because the beam shear is the governing consideration in selecting the gusset size, and beam shear induced by the haunch forces counteracts that due to the brace force. This highlights the need to consider the minimum value of the beam moment in design. It also shows that in some cases the gussets can provide flexural resistance at essentially no cost.

Determination of Moments at Interfaces and Member Shear Checks (at Mid-Length of Gusset)

The moments on the column edge of the gusset are:

$$\lambda M_c = \lambda H_c (\beta - \bar{\beta}) \qquad \text{(from Eq. 30)}$$

$$= \{109,\ 60.4\}\ \text{kips}(\{3.00,\ 12.8\}\ \text{in.} - 8\ \text{in.})$$

$$= \{-543,\ 288\}\ \text{kip-in.}$$

$$M_{cBP} = H_{cBP}(e_b + \bar{\beta}) - V_{cBP} e_c \tag{50}$$

$$= 135\ \text{kips}(7.25\ \text{in.} + 8\ \text{in.}) - 113\ \text{kips}(7.25\ \text{in.})$$

$$= \{1490,\ 1490\}\ \text{kip-in.}$$

$$M_{cH} = (\beta - \bar{\beta}) H_{cH} \tag{65}$$

$$= (\{3.00,\ 12.8\}\ \text{in.} - 8\ \text{in.})\{32.6,\ 0\}\ \text{kips}$$

$$= \{-1600,\ 0\}\ \text{kip-in.}$$

$$M_{cTot} = \lambda M_c - M_{cBP} - M_{cH} \tag{70}$$

$$= \{-543,\ 288\}\ \text{kip-in.} - \{1490,\ 1490\}\ \text{kip-in.} - \{-1600,\ 0\}\ \text{kip-in.}$$

$$= \{-433,\ -1200\}\ \text{kip-in.}$$

The moments on the beam edge of the gusset are:

$$\lambda M_b = \lambda V_b (\alpha - \bar{\alpha}) \qquad \text{(from Eq. 36)}$$

$$= \{136,\ 75.4\}\ \text{kips}(\{7.2,\ 18.9\}\ \text{in.} - 11\ \text{in.})$$

$$= \{-512,\ 597\}\ \text{kip-in.}$$

$$M_{bH} = (\alpha - \bar{\alpha}) V_{bH} \tag{66}$$

$$= (\{7.20,\ 18.9\}\ \text{in.} - 11\ \text{in.})\{134,\ 0\}\ \text{kips}$$

$$= \{-505,\ 0\}\ \text{kip-in.}$$

$$M_{bTot} = \lambda M_b - M_{bH} \tag{74}$$

$$= \{-512,\ 597\}\ \text{kip-in.} - \{-505,\ 0\}\ \text{kip-in.}$$

$$= \{-7.00,\ 597\}\ \text{kip-in.}$$

Check column shear:

$$V_{mid} = \frac{H_{cTot}}{2} + \frac{M_{cTot}}{\bar{\beta}} \qquad \text{(from Eq. 31)}$$

$$= \frac{\{-77.1,\ 196\}\text{ kips}}{2} + \frac{\{-433, -1200\}\text{ kip-in.}}{8\text{ in.}}$$

$$= \{-92.6,\ -52.4\}\text{ kips} < V_{efC} \quad \textbf{o.k.}$$

Check beam shear:

$$V_{mid} = \frac{V_{bTot}}{2} + \frac{M_{bTot}}{\bar{\alpha}} \qquad \text{(from Eq. 37)}$$

$$= \frac{\{1.90,\ 75.4\}\text{ kips}}{2} + \frac{\{-7.00,\ 597\}\text{ kip-in.}}{11\text{ in.}}$$

$$= \{0.300,\ 91.9\}\text{ kips} < V_{efB} \quad \textbf{o.k.}$$

Determination of Required Gusset Thickness

Considering the column side, the minimum thickness is:

$$t_g \geq \frac{1}{\phi F_y 2\bar{\beta}} \sqrt{\left(\frac{V_{cTot}}{0.6}\right)^2 + \left(\left|H_{cTot}\right| + \frac{2\left|M_{cTot}\right|}{\bar{\beta}}\right)^2} \qquad \text{(from Eq. 44)}$$

$$= \frac{1}{0.90(50\text{ ksi})2(8\text{ in.})} \sqrt{\left(\frac{\{293,\ 219\}\text{ kips}}{0.6}\right)^2 + \left(\left|\{-77.1,\ 196\}\text{ kips}\right| + \frac{2\left|\{-433, -1200\}\text{ kip-in.}\right|}{8\text{ in.}}\right)^2}$$

$$= \{0.725,\ 0.856\}\text{ in.}$$

Considering the beam side, the minimum thickness is:

$$t_g \geq \frac{1}{\phi F_y 2\bar{\alpha}} \sqrt{\left(\frac{H_{bTot}}{0.6}\right)^2 + \left(\left|V_{bTot}\right| + \frac{2\left|M_{bTot}\right|}{\bar{\alpha}}\right)^2} \qquad \text{(from Eq. 45)}$$

$$= \frac{1}{0.90(50\text{ ksi})2(11\text{ in.})} \sqrt{\left(\frac{\{431,\ 158\}\text{ kips}}{0.6}\right)^2 + \left(\left|\{1.90,\ 75.4\}\text{ kips}\right| + \frac{2\left|\{-7.00,\ 597\}\text{ kip-in.}\right|}{11\text{ in.}}\right)^2}$$

$$= \{0.725,\ 0.324\}\text{ in.}$$

The minimum thickness is taken as the maximum of Equations 44 and 45.

Evaluation of Connection for Moment Only

As discussed earlier, the connection should have sufficient strength to resist the required moment without the offsetting effects of the brace forces.

The effective column shear strength in this case is increased due to the column shear (similar to column shear in a moment frame being in the opposite direction from the beam flange force and thus reducing the panel-zone shear demand).

$$V_{efCol} = \phi V_n + V_{col}$$

$$= 257\text{ kips} + 108\text{ kips}$$

$$= 365\text{ kips}$$

Similarly, the shear in the beam due to moment-frame behavior, V_{BMF}, is in the opposite direction from the shear induced by the haunch force, effectively increasing the effective beam shear:

$$V_{efBm} = (\phi V_n - V_g + V_{BMF})\frac{P_{1,2}\cos\theta}{P_{1,1}\cos\theta + P_{1,2}\cos\theta} \quad \text{(from Eqs. 18, 25)}$$
$$= (212 \text{ kips} - 10.0 \text{ kips} + 44.9 \text{ kips})(0.455)$$
$$= 112 \text{ kips}$$

The forces on the column edge of the gusset are:

$$H_{cTot} = H_{cH} \quad \text{(from Eq. 68)}$$
$$= 322 \text{ kips} < V_{efCol} \quad \textbf{o.k.}$$

$$V_{cTot} = V_{cH} \quad \text{(from Eq. 69)}$$
$$= 134 \text{ kips}$$

The forces on the beam edge of the gusset are:

$$H_{bTot} = H_{bH} \quad \text{(from Eq. 72)}$$
$$= 322 \text{ kips}$$

$$V_{bTot} = V_{bH} \quad \text{(from Eq. 73)}$$
$$= 134 \text{ kips}$$

Because $V_{bTot} > U_C V_{efB}$, reinforcement is required.

The moments on the column edge of the gusset are:

$$M_{cTot} = M_{cH} \quad \text{(from Eq. 70)}$$
$$= 1{,}600 \text{ kip-in.}$$

The moments on the beam edge of the gusset are:

$$M_{bTot} = M_{bH} \quad \text{(from Eq. 74)}$$
$$= 505 \text{ kip-in.}$$

Check column shear:

$$V_{mid} = \frac{H_{cTot}}{2} + \frac{M_{cTot}}{\bar{\beta}} \quad \text{(from Eq. 31)}$$
$$= \frac{322 \text{ kips}}{2} + \frac{1{,}600 \text{ kip-in.}}{8 \text{ in.}}$$
$$= 361 \text{ kips} < V_{efC} \quad \textbf{o.k.}$$

Check beam shear:

$$V_{mid} = \frac{V_{bTot}}{2} + \frac{M_{bTot}}{\bar{\alpha}} \quad \text{(from Eq. 37)}$$
$$= \frac{134 \text{ kips}}{2} + \frac{505 \text{ kip-in.}}{11 \text{ in.}}$$
$$= 113 \text{ kips} \approx V_{efB} \quad \textbf{o.k.}$$

Check gusset on column edge:

$$t_g \geq \frac{1}{\phi F_y 2\bar{\beta}}\sqrt{\left(\frac{V_{cTot}}{0.6}\right)^2 + \left(\left|H_{cTot}\right| + \frac{2\left|M_{cTot}\right|}{\bar{\beta}}\right)^2} \qquad \text{(from Eq. 44)}$$

$$= \frac{1}{0.90(50\text{ ksi})2(8\text{ in.})}\sqrt{\left(\frac{134\text{ kips}}{0.6}\right)^2 + \left(\left|322\text{ kips}\right| + \frac{2\left|1{,}600\text{ kip-in.}\right|}{8\text{ in.}}\right)^2}$$

$$= 1.05\text{ in.}$$

Considering the beam side, the minimum thickness is:

$$t_g \geq \frac{1}{\phi F_y 2\bar{\alpha}}\sqrt{\left(\frac{H_{bTot}}{0.6}\right)^2 + \left(\left|V_{bTot}\right| + \frac{2\left|M_{bTot}\right|}{\bar{\alpha}}\right)^2} \qquad \text{(from Eq. 45)}$$

$$= \frac{1}{0.90(50\text{ ksi})2(11\text{ in.})}\sqrt{\left(\frac{322\text{ kips}}{0.6}\right)^2 + \left(\left|134\text{ kips}\right| + \frac{2\left|505\text{ kip-in.}\right|}{11\text{ in.}}\right)^2}$$

$$= 0.588\text{ in.}$$

The use of the gusset plates as haunches, in the absence of the offsetting brace forces, requires increasing the gusset thickness and providing modest beam reinforcement. Alternatively, the gusset could be enlarged. For wind loading, it may be possible to rely on the offsetting effects of the beam moment and brace forces to be in phase; for seismic loading consideration of out-of-phase behavior is prudent.

The design is shown in Figure 14. Note that beam-flange-to-column-flange welds are not required due to the entire beam-end moment being resisted by the gussets. As with the previous example, fillet welds are based on the strength of the gusset plate.

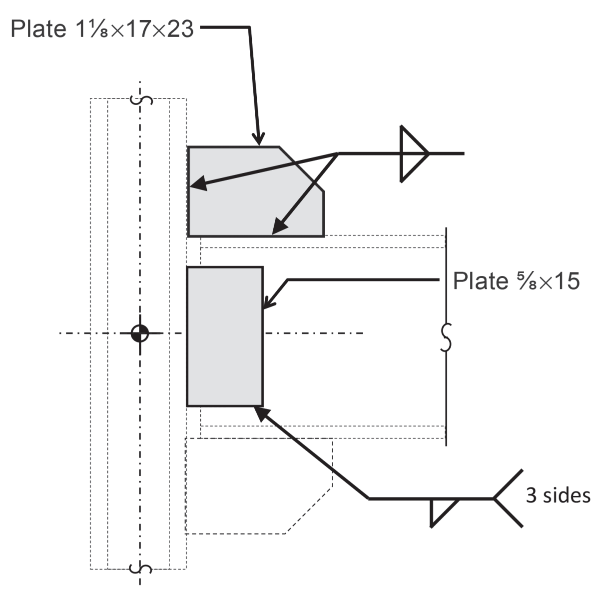


Fig. 14. Gusset design considering haunch forces.

VALIDATION

In order to demonstrate that the design methods presented in this study produce adequate designs, finite element analyses were conducted on two models representing the design in Example 3. The analyses corresponded to both the condition with the full expected beam moment (condition 1) and the condition with zero moment (condition 2), which governed the design.

The two analysis models were constructed using Abaqus software. Flanges, webs, and gussets were meshed with shell elements in the connection region. Columns were meshed from the connection region to the column mid-height. Beams were meshed from the connection region to the beam quarter-length for condition 1. Frame members were used for columns and beams outside these regions. The models use an expected material strength of 55 ksi for all elements, moderately higher than the design strength employed in the design model (ϕF_y = 45 ksi). Consistent with the design, beam flanges are not connected to the column flange and are separated by a small gap. A web-connection plate is joined to the beam web with a vertical line of tie elements and two 2-in. returns.

The analysis for condition 1 is displacement controlled, with the displacements at the two levels constrained to a proportion of 2:1. Brace axial post-elastic stiffness in the model is tuned such that the brace axial forces used in design from Table 2 develop at a drift of 2.46%, which is the drift that results in the design moment from beam plastic hinging from Table 3. Thus, brace and beam forces transmitted to the connection area closely match those used in Example 3.

For condition 2, no lateral drift is imposed, and the analysis is force controlled. The design forces (brace axial forces from Table 2 and beam shear from Table 3) are imposed on a constrained section of the beam at the connection boundary. Figure 15 shows diagrams of the models for conditions 1 and 2.

Figure 16 shows the von Mises stresses and equivalent plastic strains in the connection region, with gusset interface forces determined by stress integration. The analysis for condition 1 shows the formation of a beam plastic hinge close to the edge of the gusset. With the exception of the plastic hinge region, the stress in the beam web is low, consistent with the low forces indicated at section B2 in the design calculations (Equation 37, which indicates 1.2 kips of beam shear from the top gusset).

The analysis confirms that the connection can resist the design forces, including the beam plastic-hinge moment.

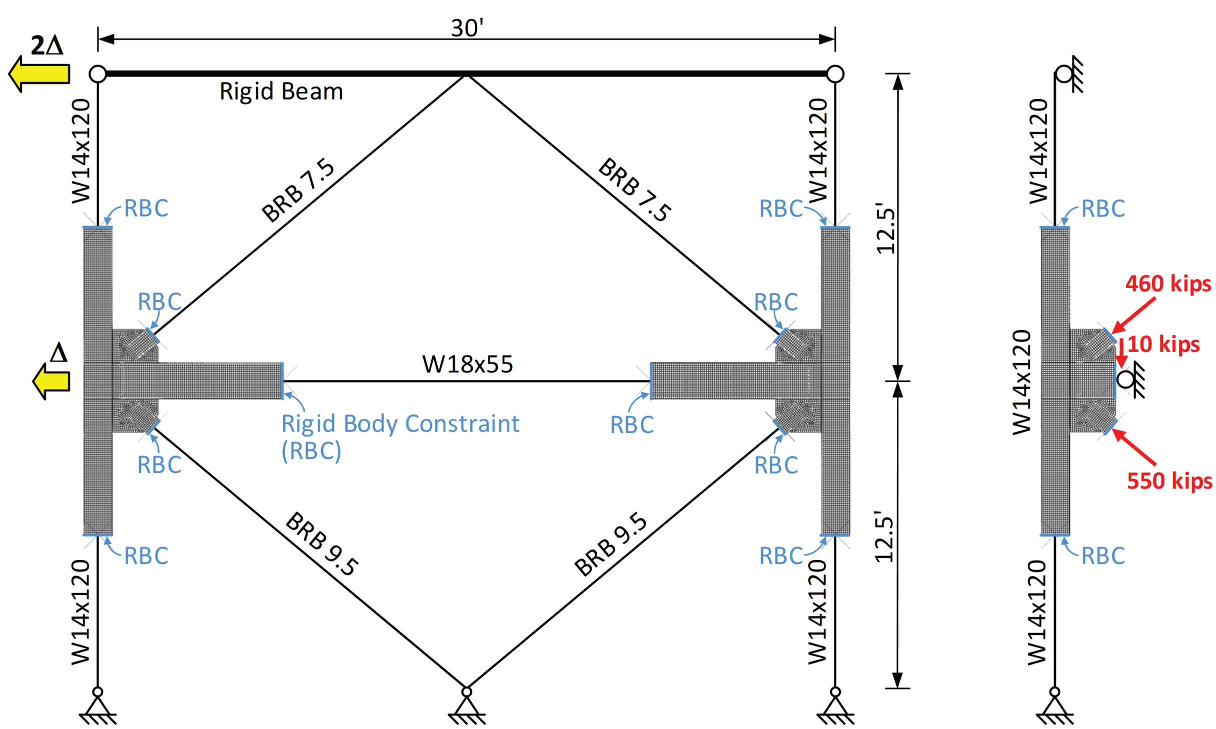


Fig. 15. Diagrams of finite element models for condition 1 (a) and condition 2 (b).

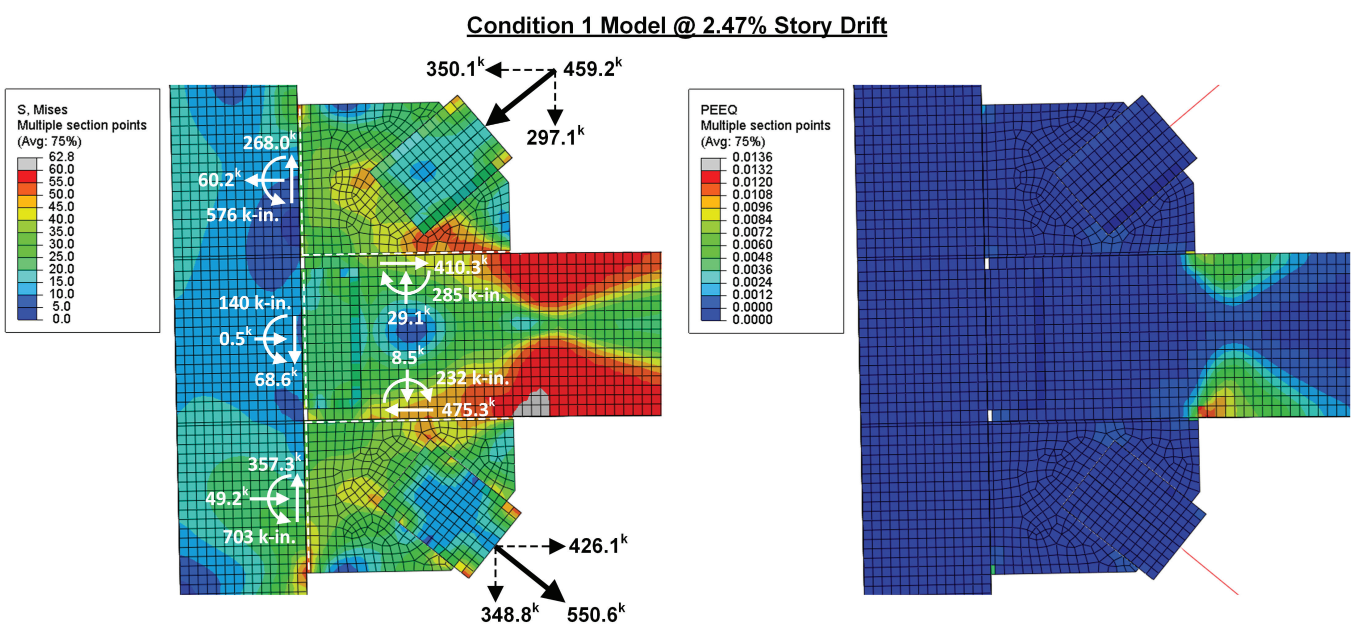
This supports the validity of the haunch method, which utilizes the gussets to transmit the beam moment to the column (rather than beam flange welds). As the plastic strain is limited to the beam plastic hinge, the analysis confirms that the design resistance can be achieved without excessive ductility demand in any part of the gusset connection.

While an accurate design model is not required by the lower-bound theorem, the forces at the interfaces are reasonably close to those in the design example: the difference between the design and analysis model values is small compared to the vertical or horizontal component of the brace force, as shown in Tables 5 and 6.

Table 5 shows that the apportionment of vertical and horizontal forces between the beam and column is roughly consistent between the design and the analysis. Table 6 includes normal, shear, and flexural force at the gusset edge at the column and at the gusset edge at the beam, for both the example calculation and from the finite element analysis. For both, stresses are calculated using the conventional plastic method utilizing the plastic section modulus and computing a vector sum of normal and shear stress. (Stresses listed for the finite element analysis are computed from total forces on the gusset edges; the stresses are not those directly reported from the analysis. These stresses are only presented as a means of quantifying the combined effect of shear, normal, and flexural force at the gusset edge.) The comparison shows that design gusset-edge stresses so calculated also correspond roughly. Nevertheless, the difference is large enough in the case of the vertical force to the beam to warrant caution in reliance on small values obtained from the difference of large forces, and the peak design stress is below the peak stress shown in Figure 16, suggesting that providing for the full gusset yield strength is prudent. Note that the edge forces from the finite element analysis are computed in the deformed condition, and thus are slightly out of alignment with the design calculations.

It should be noted, however, that the values determined by the haunch design method are moderately influenced by some of the simplifying assumptions. First, the value of the virtual dimension, *r*, was taken as the optimum value using trial and error; the method allows for other values to be used. Second, the haunch forces are set to the same locations as the UFM forces, as shown in Figure 10. Third, the apportionment factor, λ, employed for condition 1 is for convenience set equal to that for condition 2. Changes to any of these assumptions would result in somewhat different design forces, and possibly greater difference between calculated and analyzed values.

In contrast to condition 1, the finite element analysis of condition 2 (Figure 17) shows high beam shear in the connection region with substantial yielding both in the beam web (section B2) and in the connection plate (approximately at section B1). In the design method (with the virtual

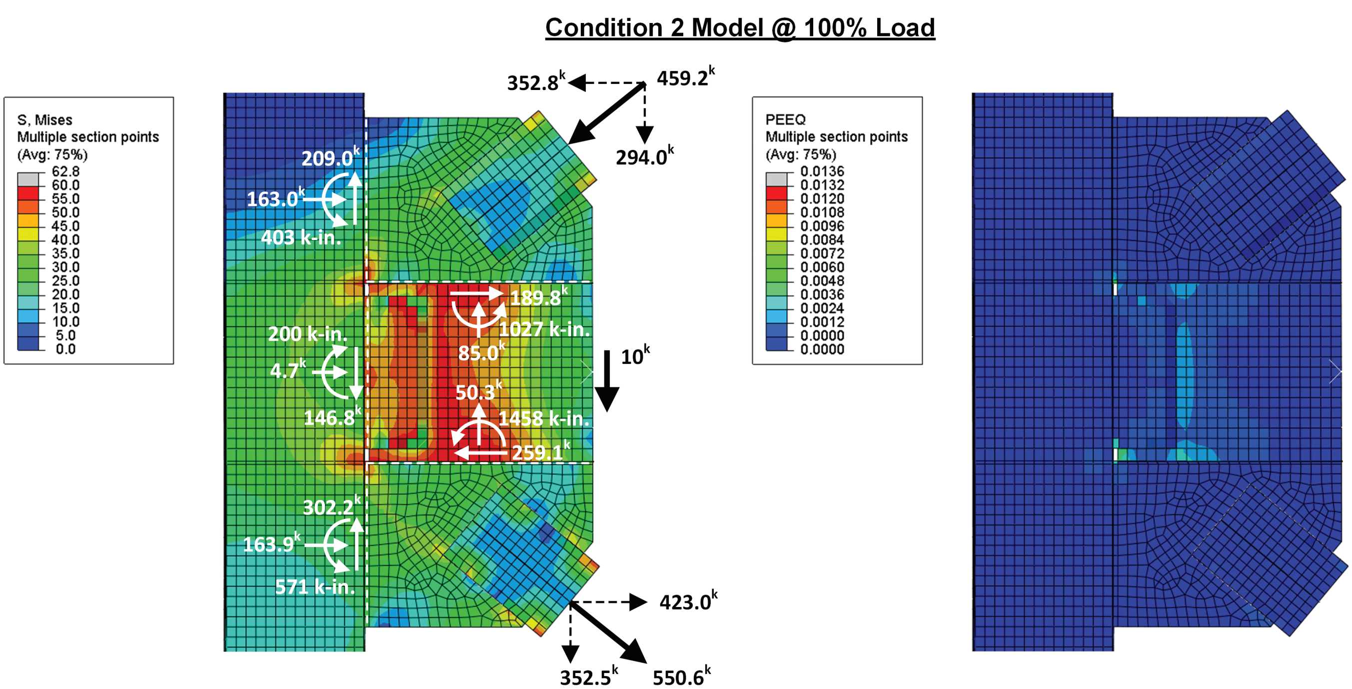


(a) Von Mises stress contour *(b) Equivalent plastic stain contour*

Fig. 16. Finite element analysis results of Example 3, condition 1.

Table 5. Comparison of Analysis and Design Vertical and Horizontal Forces for Condition 1					
Horizontal Forces	**H_{cTot} (kips)**	**H_{bTot} (kips)**	**Sum (kips)**	**% Column**	**% Beam**
Example 3 calculation	–77.1	431	353	–22%	122%
Finite element analysis	–60.2	410	350	–17%	117%
Vertical Forces	**V_{cTot} (kips)**	**V_{bTot} (kips)**	**Sum (kips)**	**% Column**	**% Beam**
Example 3 calculation	292	2.70	295	99%	1%
Finite element analysis	268	29.1	297	90%	10%

Table 6. Comparison of Analysis and Design Gusset-Edge Forces for Condition 1							
Forces at Column	**H_{cTot} (kips)**	**V_{cTot} (kips)**	**M_{cTot} (kip-in.)**	**Normal Stress (ksi)**	**Shear Stress (ksi)**	**Resultant Stress (ksi)**	**Angle (deg)**
Example 3 calculation	–77.1	292	–459	13.7	20.8	24.9	33.3
Finite element analysis	–60.2	268	–576				
Stress calculated from finite element analysis forces				14.6	19.1	24.1	37.3
Forces at Beam	**V_{bTot} (kips)**	**H_{bTot} (kips)**	**M_{bot} (kip-in.)**	**Normal Stress (ksi)**	**Shear Stress (ksi)**	**Resultant Stress (ksi)**	**Angle (deg)**
Example 3 calculation	2.70	431	0.0	0.100	22.4	22.4	0.40
Finite element analysis	29.1	410	–285				
Stress calculated from finite element analysis forces				4.20	21.3	21.7	11.2



(a) Von Mises stress contour *(b) Equivalent plastic stain conto*

Fig. 17. Finite element analysis results of Example 3, condition 2.

Table 7. Comparison of Analysis and Design Vertical and Horizontal Forces for Condition 2					
Horizontal Forces	**H_{cTot} (kips)**	**H_{bTot} (kips)**	**Sum (kips)**	**% Column**	**% Beam**
Example 3 calculation	196	158	354	55%	45%
Calculation with $\phi F_y = 55$ ksi	159	194	353	45%	55%
Finite element analysis	163	190	353	46%	54%
Vertical Forces	**V_{cTot} (kips)**	**V_{bTot} (kips)**	**Sum (kips)**	**% Column**	**% Beam**
Example 3 calculation	219	75.4	294	74%	26%
Calculation with $\phi F_y = 55$ ksi	201	93.1	294	68%	32%
Finite element analysis	209	85.0	294	71%	29%

Table 8. Comparison of Analysis and Design Gusset-Edge Forces for Condition 2							
Forces at Column	**H_{cTot} (kips)**	**V_{cTot} (kips)**	**M_{cTot} (kip-in.)**	**Normal Stress (ksi)**	**Shear Stress (ksi)**	**Resultant Stress (ksi)**	**Angle (deg)**
Example 3 calculation	196	219	–1200	35.4	15.7	38.7	66.2
Calculation with $\phi F_y = 55$ ksi	159	201	–583	21.8	14.4	26.1	56.6
Finite element analysis	140	181	–403				
Stress calculated from finite element analysis forces				17.2	13.0	21.5	53.0
Forces at Beam	**V_{bTot} (kips)**	**H_{bTot} (kips)**	**M_{bot} (kip-in.)**	**Normal Stress (ksi)**	**Shear Stress (ksi)**	**Resultant Stress (ksi)**	**Angle (deg)**
Example 3 calculation	75.4	158	597	9.6	8.2	12.6	49.4
Calculation with $\phi F_y = 55$ ksi	93.1	194	731	11.7	10.1	15.5	49.4
Finite element analysis	107	203	1030				
Stress calculated from finite element analysis forces				15.3	10.5	18.6	55.4

dimension, r, set equal to r_{minBm}), section B1 is constrained to have a demand-to-capacity ratio of 1.0. Section B2 (using Equation 37) has a calculated demand of 91.9 kips, equal to the effective capacity of the beam (Equation 18). Thus, the analysis and design are consistent on a fundamental point: As the beam yields at these sections, the column can still continue to resist higher forces. Of note is that while the beam web and its connection to the column reach yield stress, the plastic strains remain low due to the redistribution of the forces in the connection to the column, consistent with the bypass design method.

The analysis confirms that the connection can support the design loads for condition 2. This supports the use of the bypass method. It should be noted that the Uniform Force Method without the bypass modification would indicate that this connection was inadequate. The plastic strains in the beam web are small, confirming that after beam-web yielding, the bypass mechanism provides sufficient strength and stiffness to preclude significant ductility demands. The analysis also confirms that condition 2 is more critical than condition 1 for this design.

The comparison of design forces from Example 3 to the analysis forces for condition 2 shows that the design model (which utilizes a specified minimum yield stress and a resistance factor) requires a greater portion of the brace force to bypass the beam. The more accurate finite element analysis shows a distribution closer to the Uniform Force method, with somewhat less force bypassing the beam than in the calculations, as shown in Tables 7 and 8. This difference is due in part to the disparity between the stresses permitted in the design model and the greater expected strength used in the analysis. For comparison, design calculations were performed using $\phi F_y = 55$ ksi (which required a bypass factor, λ, of 0.76, as compared to the value of 0.617 from the design example, corresponding to $\phi F_y = 50$ ksi). Forces corresponding to that analysis are also in Tables 7 and 8.

CONCLUSIONS AND RECOMMENDATIONS

This study provides design equations that can be used in the design of gussets in braced frames to minimize the required shear strength of the beams and columns in order to avoid the need for reinforcement. The design method allows engineers to select the proportion of the load that will contribute to beam or column shear and to redistribute the load between beam and column. Additionally, equations are provided that allow the gussets to be used to resist beam-connection moment, which potentially allows for a very economical design by eliminating the need for beam-flange-to-column-flange welds.

The authors recommend this modified implementation of the Uniform Force Method, especially for conditions in which web doublers would otherwise be required. The use of the bypass method provides additional economy, but engineers should be cognizant of the limited study performed to date. Without additional study, the authors do not recommend bypass factors λ less than 0.6. The use of the haunch method (with or without the bypass method) similarly provides for greater economy for connections with beam fixity. Based on the limited study performed to date, the authors recommend that the haunch method be implemented considering both the upper and lower bounds of beam moment (conditions 1 and 2). Further study is required to establish the lower bound of this moment, especially for systems with cyclic inelastic drift, for which the brace forces may reverse within the elastic drift range of the beam.

SYMBOLS

F_y Material specified minimum yield stress, ksi

H_b Horizontal force transferred to beam by gusset (from Uniform Force Method analysis), kips

H_{bH} Horizontal force transferred to beam due to haunch force, kips

H_{bW} Horizontal force at beam-web-to-column connection, kips

H_{bTot} Total horizontal force transferred to beam, kips

H_c Horizontal force transferred to column by gusset (from Uniform Force Method analysis), kips

H_{cBP} Horizontal force transferred to column by gusset from bypass force, kips

H_{cH} Horizontal force transferred to column by gusset to resist beam moment, kips

H_{cTot} Total horizontal force transferred to column by gusset, kips

M_{BC} Moment at column face resisted by beam-to-column connection, kip-in.

M_b Moment at gusset–beam interface (from Uniform Force Method analysis), kip-in.

M_{bH} Moment at gusset–beam interface from haunch force, kip-in.

M_{bTot} Total moment at gusset–beam interface, kip-in.

M_{BM} Moment at a beam section aligned with the gusset edge face, kip-in.

M_c Moment at gusset–column interface (from Uniform Force Method analysis), kip-in.

M_{cBP} Moment at gusset–column interface from bypass force, kip-in.

M_{cH} Moment at gusset–column interface from haunch force, kip-in.

M_{cTot} Total moment at gusset–column interface, kip-in.

M_f Moment at column face (from beam), kip-in.

M_H Moment at column face resisted by gusset, kip-in.

N Normal force on beam or column flange, kips

N_B Beam axial force delivered to the connection, kips

N_H Horizontal force at beam-to-column web connection due to haunch forces, kips

P_f Beam flange force at connection to column, kips

$P_{i,j}$ Brace axial force for brace j connecting to gusset i; sign conventions are per the figures, kips

R_n Nominal strength (of beam-to-column connection), kips

R_u Required strength, kips

U_C Ratio of connection effective strength, V_{efConn}, to beam effective strength, V_{efBm}

V_b Vertical force transferred to beam by gusset (from Uniform Force Method analysis), kips

V_{bH} Vertical force transferred to beam by gusset to resist beam moment, kips

V_{BMF} Beam shear due to moment-frame behavior, kips

V_{bW} Vertical force at beam-web-to-column connection, kips

V_g Beam shear due to gravity, kips

V_c Vertical force transferred to column by gusset (from Uniform Force Method analysis), kips

V_{cH} Vertical force transferred to column by gusset to resist beam moment, kips

V_{Col} Column shear due to moment-frame behavior, kips

V_{cBP} Vertical force transferred to column by gusset from bypass force, kips

V_{cTot} Total vertical force transferred to column by gusset, kips

V_{efBm} Effective beam shear strength, kips

$V_{efBi,j}$ Apportioned effective beam shear strength that can be used to resist the force for brace *j* connecting to gusset *i*; sign conventions are per the figures, kips

V_{efCol} Effective column shear strength, kips

$V_{efCi,j}$ Apportioned effective column shear strength that can be used to resist the force for brace *j* connecting to gusset *i*; sign conventions are per the figures, kips

V_{efConn} Effective beam-to-column connection shear strength, kips

V_{mid} Shear in beam or column at mid-length or mid-height of gusset, kips

V_n Nominal member shear strength, kips

d_b Beam depth, in.

e_b Eccentricity from beam flange to beam centerline, equal to half the beam depth, in.

e_c Eccentricity from column flange to column centerline, equal to half the column depth, in.

h_o Distance between beam flange centroids, in.

r Gusset centroid offset (dimension from workpoint to brace control point in Uniform Force Method), in.

r_{minBm} Minimum dimension *r* based on beam shear yielding, in.

r_{minCol} Minimum dimension *r* based on column shear yielding, in.

t_g Gusset thickness, in.

α Distance from column face to centroid of Uniform Force Method force acting on beam flange, in.

$\bar{\alpha}$ Half of gusset horizontal dimension (centroid of gusset–beam interface), in.

β Distance from beam flange to centroid of Uniform Force Method force acting on column face, in.

$\bar{\beta}$ Half of gusset vertical dimension (centroid of gusset–column interface), in.

ϕ Resistance factor

λ Brace force apportionment factor (between Uniform Force Method and "bypass force")

ρ Beam moment apportionment factor (between beam flange and haunch force in gusset)

θ Brace angle from vertical, deg

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