The Indirect Analysis Method of Design for Stability: An Amplifier to Address Member Inelasticity, Member Imperfections, and Uncertainty in Member Stiffness

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ABSTRACT

Design for stability requires multiple considerations, including geometric second-order effects (P-Δ and P-δ effects) and the effects on structure response of member inelasticity, member imperfections, and uncertainty in member stiffness. The "indirect analysis method" provides a simple amplifier approach to addressing these effects such that effective length factors may be taken as 1.0. The indirect analysis method meets the stability design requirements of the AISC *Specification* and can reduce analysis and design effort for many typical building structures.

Keywords: stability, direct analysis method, effective length, first-order analysis.

The indirect analysis method (IAM) provides a simple amplifier approach to addressing the effects on structure response of member inelasticity (including the effects of residual stress), member imperfections, and uncertainty in member stiffness. [Together the effects of these three factors (inelasticity, member imperfection, and stiffness uncertainty) are referred to here as *stiffness-reduction effects*.] The IAM permits second-order analysis with full stiffness properties; stiffness-reduction effects are addressed by an amplifier (B_3) applied to lateral loads prior to analysis or to their effects afterward. Verification studies confirm design forces consistent with the AISC *Specification* direct analysis method over a broad range of multiple parameters. The indirect analysis method can also be used as the basis of an improved first-order analysis method.

CURRENT METHODS OF DESIGN FOR STABILITY IN THE AISC *SPECIFICATION*

The AISC *Specification for Structural Steel Buildings*, hereafter referred to as the AISC *Specification*, defines three methods of design for stability (AISC, 2016): the direct analysis method (the DM, presented in Chapter C), the effective-length method (the ELM, presented in Appendix 7, Section 7.2), and the first-order analysis method (the FOM, presented in Appendix 7, Section 7.3). Each method includes coordinated analysis and design requirements that result in structures with sufficient strength to prevent instability, considering the structures' stiffness with respect to lateral displacement in the presence of vertical loads. Each method addresses stiffness-reduction effects differently, and each method requires either a second-order analysis (DM and ELM) or determination of the second-order effect (FOM). Second-order analysis may be either explicit or approximate, such as by means of the B_1 and B_2 factors [where B_1 and B_2 are the second-order effect multipliers for P-Δ and P-δ effects, respectively, defined in AISC *Specification* Appendix 8 (AISC, 2016)].

The DM addresses stiffness-reduction effects in the analysis, requiring a second-order analysis using reduced member stiffness. This produces the internal member forces that are expected to result from the application of the external forces, considering stiffness-reduction effects on the response of the structure. Stiffness-reduction effects in the DM are addressed both globally and member by member. Globally, all stiffness terms are reduced using a factor of 0.8. Additionally, the member flexural stiffness reduction for moment frame columns includes a term, τ_b, that relates to the member axial force, and thus at least one analysis iteration is required to determine the axial force corresponding to the loading condition. (Each load combination could entail its own particular set of member stiffness reductions, although grouping is typically done to reduce design effort.) Nevertheless, because the reduced stiffness of the DM model is not intended to be used for calculating drifts, a minimum of two models is typically required for building design: one for strength verification and another for drift determination [and period determination for seismic loads (ASCE, 2016)]. (Drift may be a service limit state, as is often the case for wind, or a safety limit state, as is the

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case for seismic; in either case, full stiffness properties are used, although for different reasons.) With the DM, stability is adequately addressed without the need for an effective length factor, K, greater than 1.0. The DM defines notional lateral loads to represent initial system imperfections due to out-of-plumbness. These notional loads are required for gravity-only load combinations; they are also required for combinations that include other lateral loads for which the ratio of second-order drift to first-order drift exceeds 1.7 in the reduced-stiffness DM analysis.

The ELM addresses stiffness-reduction effects by requiring reduced compression strength (implemented by effective length factors in the design of columns) and addresses P-Δ effects by requiring a second-order analysis. The flexural demands imposed by columns that are near their stability limit on the column connections and on beams are not directly addressed in the ELM. This method allows for effective length factors to be taken as 1.0 for braced-frame columns and for moment-frame columns when $B_2 \leq 1.1$; determination of effective length factors for moment-frame columns is required when $B_2 > 1.1$. Several methods exist to account for the effect of leaning columns in determining effective length factors, with minor variation in the results (Geschwindner, 2002). The ELM is limited to cases in which the second-order effect does not exceed 1.5; gravity-only load combinations require notional loads. (The limit of 1.5 for a full-stiffness ELM model is equivalent to 1.7 for a reduced-stiffness DM analysis.)

In the FOM, stiffness-reduction and P-Δ effects are addressed together in the analysis by means of an additional lateral force in a first-order analysis. This force is derived in AISC Design Guide 28, Appendix B: "Development of the First-Order Analysis Method" (Griffis and White, 2013). To permit this simplification, limits are placed on both axial force in moment-frame columns and on second-order effects. FOM additional forces assume that second-order effects are at the maximum level permitted: $B_2 = 1.5$. Similar to the DM, the FOM allows for effective length factors to be taken as 1.0. Application of the additional lateral forces in the FOM for strength design does not guarantee that second-order effects on drift have been adequately addressed when consideration of these is required, such as by ASCE/SEI 7, Section 12.8.7 (ASCE, 2016), unless these forces are included in the drift analysis. Inclusion of such forces in the drift analysis is problematic for two reasons: Drift is typically checked in a separate analysis not subject to the same minima as the analysis for strength design; only geometric second-order effects, not stiffness-reduction effects, are required in drift determination (ASCE, 2016). While the FOM does not require a second-order analysis, its use requires that the second-order effect not exceed 1.5. Confirmation of this requires either a second-order analysis or calculation of the B_2 factor, and thus only modest additional effort would be required to incorporate second-order effects into the analysis.

THE INDIRECT ANALYSIS METHOD

The IAM is a simplified version of the DM, with stiffness-reduction effects addressed either globally or story by story rather than member by member. The IAM requires only relatively easy-to-implement means (second-order analysis and an amplification factor applied to lateral loads or to their effects) and does not require more cumbersome methods (such as determination of effective length factors greater than 1.0 or multiple models). The IAM incorporates both second-order analysis (either explicit second-order analysis or first-order analysis amplified by B_1 and B_2 factors) and an amplification factor to address stiffness reduction (B_3). This amplification factor is derived to amplify the second-order analysis forces such that the results obtained match those from the DM. Thus, the B_3 factor effectively performs the same function as the $0.8\tau_b$ stiffness reduction in the DM, permitting the use of effective length factors, K, of 1.0 in the IAM. Use of a global or story value of τ_b may make the IAM more conservative than the DM for cases with a wide range of moment-frame column axial force ratios.

General Stability Design Requirements

The AISC *Specification* requires five considerations for any admissible stability design method. These are listed in the following, along with the description of how each is addressed in the IAM. The approach to each consideration closely matches that for the DM, with the difference that items 3–5 utilize the B_3 factor, which incorporates $0.8\tau_b$, to address stiffness-reduction effects.

1. *Consider all deformations.* This requirement is addressed by proper modeling.
2. *Consider second-order effects (including P-Δ and P-δ effects).* This requirement is addressed by performing a second-order analysis, either explicit or through application of B_1 and B_2 factors.
3. *Consider geometric imperfections.* This requirement is addressed by application of notional loads (for the effect of structural imperfections on the structural response), application of the B_3 factor (for the effect of member imperfections on the structural response), and application of member strength formulae with $L_c = L$ (for the effect of member imperfections on member strength).
4. *Consider stiffness reduction due to inelasticity.* This requirement is addressed by application of the B_3 factor (for the effect on the structural response) and application of member strength formulae with $L_c = L$ (for the effect on member strength).

5. *Consider uncertainty in strength and stiffness.* This requirement is addressed by application of the B_3 factor (for the effect on the structural response) and application of member available strength formulae with $L_c = L$ (for the effect on member strength).

General Design Guidance

Although the investigation of stability effects tends to address systems near instability, examination of such cases should not be taken as endorsement of designing systems near instability. The limits proposed for the IAM are based solely on the verification of the method. As systems approach those limits, they become increasingly sensitive to the accuracy of loading assumptions, modeling, and analysis. Many engineers—including the author—apply lower limits on second-order and stiffness-reduction effects to their designs. Furthermore, it should be noted that for seismic design, ASCE/SEI 7, Section 12.8.7, limits the stability coefficient, θ, to 0.25 (or lower, for many systems). According to the ASCE/SEI 7 commentary (ASCE, 2016), the stability coefficient (using symbols from the AISC *Specification*) can be expressed as:

$$\theta = \frac{\alpha P_{story}\Delta_H}{HL} \tag{1}$$

where

H = total story shear, kips

L = story height, in.

P_{story} = total vertical load supported by the story, kips

α = ASD/LRFD force level adjustment factor, equal to 1.0 (LRFD) or 1.6 (ASD)

Δ_H = first-order interstory drift due to lateral forces, in.

Thus a limit on the stability coefficient, θ, is a limit on geometric stability (P-Δ) effects.

DERIVATION OF THE INDIRECT ANALYSIS METHOD

The IAM is derived from the DM, with the simplification that stiffness reductions, rather than being applied member by member, are computed story by story and applied either story by story or globally. This approach permits quantifying the effect and applying it as a factor in an analysis using full stiffness properties rather than requiring a modified model for stability design.

Following the method presented in Design Guide 28 (Griffis and White, 2013), the IAM is derived in the following based on a dual-purpose second-order amplifier, $\overline{B}_2$, that addresses both geometric nonlinearity (P-Δ effects) and the effects of stiffness reduction. [The $\overline{B}_2$ amplifier was developed as $\overline{B}_{lt}$ in White et al. (2007)]. Note that use of this amplifier to determine displacements is equivalent to establishing a reduced stiffness due to geometric effects combined with the effects of inelasticity, member imperfection, and stiffness uncertainty. The amplification has a nonlinear relationship to the destabilizing gravity load. However, the resulting stiffness defines a linear relationship between lateral load and displacement.

The second-order shear (with notional loads) is set equal to first-order shear with additional forces required for design for stability:

$$\overline{B}_2\left(H + \Sigma N\right) = H + \overline{H}_{P\Delta} \tag{2}$$

where

$\overline{B}_2$ = second-order amplifier using the stiffness reduction $EI^* = 0.8\tau_b EI$

$\overline{H}_{P\Delta}$ = P-Δ shear forces [from Equation B-10b in Griffis and White (2013)], kips

ΣN = notional loads to address out-of-plumbness (at or above the level under consideration), kips

and

E = modulus of elasticity, ksi

I = moment of inertia, in.^{4}

τ_b = stiffness-reduction parameter defined in AISC *Specification* Section C2, Equations C2-2a and C2-2b

The AISC *Specification* definition of τ_b can be expressed as:

$$\tau_b = 4\frac{\alpha P_r}{P_{ns}}\left(1 - \frac{\alpha P_r}{P_{ns}}\right) \le 1.0 \tag{3}$$

where

P_{ns} = cross-section compressive strength, kips

= $F_y A_g$ (for columns without slender elements), kips

= $F_y A_e$ (for columns with slender elements), kips

P_r = required axial compressive strength using LRFD or ASD load combinations, kips

and

A_e = effective area, in.^{2}

A_g = gross area of member, in.^{2}

F_y = specified minimum yield stress, ksi

Griffis and White (2013) use an equality similar to Equation 2 to establish that FOM forces correspond to DM forces within the limits of FOM application. The displacement-magnification term $\overline{B}_2$ in the Design Guide 28 formulation [Equation B-4 in Griffis and White (2013)] incorporates a stiffness-reduction factor of 0.8 for the FOM:

$$\overline{B}_2 = \frac{1}{1 - Q/0.8} \tag{4}$$

where Q is a stability index, related to the stability coefficient θ; thus:

$$Q = \theta / R_M \tag{5}$$

and R_M is a coefficient to account for member P-δ influence on structure P-Δ.

The factor of 0.8 in Equation 4 represents the stiffness reduction of the DM, with the limitation that in the FOM, axial stress may not be permitted to reach a level sufficient to affect flexural stiffness of moment-frame beams and columns. A more general form of Equation 4 can be written to incorporate the axial-force-dependent flexural-stiffness-reduction parameter, τ_b:

$$\overline{B}_2 = \frac{1}{1 - Q / 0.8\tau_b} = \frac{1}{1 - \theta / 0.8\tau_b R_M} \tag{6}$$

Incorporation of τ_b into $\overline{B}_2$ effectively applies this parameter uniformly to all members and all actions (axial as well as flexure). This extensive application of τ_b to the entire lateral load effect potentially overestimates its influence but permits the derivation of a single factor on the lateral load to account for the effect of high axial stress on the system stiffness.

The stability index, Q, can also be expressed thus:

$$Q = \frac{\alpha P_{story} \Delta_H}{R_M H L} \tag{7}$$

The stability index Q is thus more complete than the stability coefficient θ, including the stiffness reduction related to moment-frame column compressive force, R_M. The stability coefficient, θ, however, is used in the ASCE 7 standard (ASCE, 2016) and is therefore used in this paper (in conjunction with R_M). The terms that define the stability index Q in Equation 7 can also be used to define the B_2 amplifier:

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story} \Delta_H}{R_M H L}} \tag{8}$$

Thus, the index Q can be expressed as a function of B_2:

$$Q = 1 - \frac{1}{B_2} \tag{9}$$

For development of the IAM, the geometric effects and stiffness-reduction effects that comprise the factor $\overline{B}_2$ are distinguished:

$$\overline{B}_2 = B_2 B_3 \tag{10}$$

where

B_3 = IAM amplification factor to account for stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty

Thus, B_2 addresses geometric second-order effects, and B_3 addresses stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty. This separation permits the use of typical second-order analysis using nominal stiffness properties with a separate factor, B_3, to account for the effects of the stiffness reduction inherent in the DM. By separating B_2 and B_3, the IAM does not affect established second-order analysis procedures that use full-stiffness member properties, including those incorporated into analysis software. [Use of the B_2 amplifier is valid only for systems with vertical columns (AISC, 2016), and thus the IAM (which utilizes B_2) is similarly limited.]

Combining Equation 6 and Equation 10 gives:

$$B_2 B_3 = \frac{1}{1 - \theta / 0.8\tau_b R_M} \tag{11}$$

The second-order amplifier B_2 can also be written in similar terms (similar to Equation B-3 in Griffis and White, 2013):

$$B_2 = \frac{1}{1 - \theta / R_M} \tag{12}$$

Equation 11 and Equation 12 are combined to determine B_3:

$$B_3 = \frac{1 - \theta / R_M}{1 - \theta / 0.8\tau_b R_M} \tag{13}$$

The factor B_3 is a function of the stability coefficient, θ, the P-δ factor, R_M, and the flexural-stiffness-reduction parameter, τ_b. Limits on these values can be set in advance of design such that the factor B_3 may be determined and incorporated into the analysis from the first iteration. (B_3 is also fundamentally an expression of the 0.8 stiffness-modifier term, but this parameter is fixed.)

Equation 13 can be written in terms of B_2 and τ_b utilizing Equation 9 and Equation 5:

$$B_3 = \frac{0.8\tau_b}{1 - (1 - 0.8\tau_b) B_2} \tag{14}$$

Equation 13 or 14 may be used to amplify the results of a second-order analysis. If $\alpha P_r / P_{ns} \leq 0.5$, $\tau_b = 1.0$, and Equation 14 reduces to:

$$B_3 = \frac{4}{5 - B_2} \tag{15}$$

USE OF THE SECOND-ORDER EFFECT DETERMINED FROM ANALYSIS

If an explicit second-order analysis is performed, a second-order amplification term may be determined from the analysis results and used in lieu of B_2 from AISC *Specification* Equation A8-6 to calculate the stiffness-reduction amplifier.

This amplification term (shown here as B_2') is determined using the ratio of second-order displacements to first-order displacements from the analysis:

$$B_2' = \frac{\Delta_2}{\Delta_1} \tag{16}$$

where

Δ_1 = first-order story drift using full stiffness properties, in.

Δ_2 = second-order story drift using full stiffness properties, in.

The corresponding stiffness-reduction amplifier, B_3', is obtained by combining Equation 14 and Equation 16 (with $B_2 = B_2'$):

$$B_3' = \frac{0.8\tau_b}{1-[1-0.8\tau_b]\Delta_2/\Delta_1} \tag{17}$$

Notional Loads

The IAM notional loads are the same as are required for the DM. These notional loads are:

$$N_i = \frac{\Delta_o}{L} Y_i \tag{18}$$

where

N_i = notional load at level i, kips

Y_i = gravity load applied at level i, kips

Δ_o = nominal initial out-of-plumbness, in.

The initial out-of-plumbness, Δ_o, is taken as 0.002 times the story height in the AISC *Specification* (AISC, 2016). Notional loads are applied story by story, and thus accumulate as the vertical load supported, P_{story}, accumulates.

Consistent with the DM, these notional loads are proposed to be applied for gravity-only load combinations for cases with $B_2 \le 1.5$ and for all load combinations with $B_2 > 1.5$. This results in a minor error for load combinations that include lateral loads and have $B_2 \le 1.5$; this too is consistent with the DM.

As the IAM has been derived to be consistent with the DM, the alternative of using an additional notional load equal to 0.001 times the gravity load and $\tau_b = 1.0$ in lieu of $\tau_b < 1.0$ may be used [AISC *Specification*, C2.3(c)].

Values of B_3 and Proposed Range of Applicability of the Indirect Analysis Method

Table 1 shows the factor B_3 for different combinations of B_2 and the flexural stiffness-reduction parameter, τ_b, using Equation 14. Values of B_3 are presented corresponding to the values of B_2 ranging from 1.00 to 5.00 and $\alpha P_r/P_{ns}$ ranging from 0 to 1.00. B_2 = 1.0 corresponds to a system that has no gravity load (or is infinitely stiff). B_2 = 5.00 and $\alpha P_r/P_{ns}$ = 1.00 conditions are unstable regardless of other factors. Shading indicates bands of similar values. Linear interpolation may be used with this table to obtain a liberal estimate of B_3.

The B_3 amplifier increases disproportionately with increasing axial-load ratios $\alpha P_r/P_{ns}$ and second-order amplifier B_2. At higher ranges of these parameters, the calculation results in negative or zero values, indicating insufficient stiffness to prevent instability regardless of strength; these cases are shown in black. High values of B_3 indicate high sensitivity to loading and modeling assumptions. In practice, a limit on the permissible B_3 value may be used to prevent large sensitivity to small approximations, errors, or uncertainties in loading or modeling.

The IAM is not proposed for $B_2 > 2.0$ due to this sensitivity. For $B_2 > 1.5$, a second-order analysis that includes the effect of P-δ on P-Δ is necessary (Nair, 2009). (While use of a B_2 amplifier that includes R_M complies with this requirement, an explicit second-order analysis reduces error.) Similarly, the IAM is not proposed for $\alpha P_r/P_{ns} > 0.7$ for moment-frame columns.

The proposed IAM may be used if the following conditions are met:

- $\alpha P_r/P_{ns} \le 0.7$ for moment-frame columns.
- Columns are vertical.
- A second-order analysis is performed (or the factors B_1 and B_2 are applied).
- Notional loads are applied for $B_2 > 1.5$ and for gravity-only load combinations.
- B_3 is taken as the largest value at or above the level under consideration.
- The value of τ_b is determined (or confirmed) using the results of a second-order analysis that is amplified using the factor B_3.
- The largest displacement of the story is used to determine both B_2 and B_3.

For torsional loading (i.e., lateral loading that causes the diaphragm to rotate), Equations 16 and 17 may be used with results from a three-dimensional analysis, or values of B_2 and B_3 may be computed based on maximum drifts. Notional load patterns should include a torsion-inducing pattern.

FIRST-ORDER ANALYSIS

Engineers accustomed to performing a first-order analysis may prefer to use amplifiers to address both second-order effects (B_2) and the stiffness-reduction (B_3). This product, obtained by multiplying Equation 14 by B_2, may be applied to the lateral loads or to their effects determined using a first-order analysis:

$$B_2B_3 = \frac{0.8\tau_b}{1/B_2 - (1 - 0.8\tau_b)} \tag{19}$$

This product is also a useful indication of the magnitude of stability effects on the design. Such an approach is a version of the IAM (rather than the FOM), and the limits discussed in the previous section apply. If $\alpha P_r/P_{ns} \le 0.5$, $\tau_b = 1.0$, and Equation 19 reduces to:

$$B_2B_3 = \frac{4}{5/B_2 - 1} \tag{20}$$

This equation can be used as an amplification factor on lateral loads with greater economy compared to the FOM, with results corresponding to those that would be obtained from the DM. At $B_2 = 1.5$ this equation results in $B_2B_3 = 1.71$, and for gravity-only load combinations, the amplified notional load (B_2B_3N) is $1.71 \times 0.002P_{story} = 0.00343P_{story}$, 82% of what is required by the FOM in the AISC *Specification*. [This difference is due to the conservative amplification of the initial out-of-plumbness, Δ_o, in the FOM by dividing by 0.8, which simplifies the FOM derivation (Equation B-9b in Griffis and White, 2013).] The advantage provided by the IAM is greater at lower values of B_2 because the FOM forces are based on $B_2 = 1.5$ regardless of the actual second-order effect.

Table 2 shows the product of the factors B_2 and B_3 for different combinations of B_2 and flexural stiffness-reduction parameter, τ_b. The table utilizes Equation 19 (B_2B_3). Similar to Table 1, shading in Table 2 marks bands of similar values. Linear interpolation may be used with this table to obtain a liberal estimate of the product B_2B_3.

APPLICATION

Similar to the DM, the IAM addresses the stiffness-reduction effects in the analysis rather than in the design strength equations (as the ELM does). As discussed in the derivation, the B_2 and B_3 amplifiers represent a reduced linear lateral stiffness. The IAM amplification may thus be applied to loads prior to the second-order analysis or to their effects afterward, prior to design, or (conservatively) to demand-to-capacity ratios. Simpler methods of application are necessarily more conservative. Methods of application include:

- *Demand-to-capacity ratio limit.* The simplest method is to use the inverse of the factor B_3 as a limit on permissible demand-to-capacity ratios. This method effectively amplifies the gravity load effect, which introduces arbitrary conservatism; this may be acceptable for cases where B_3 is small. For example, in seismic design per ASCE/SEI 7 (ASCE, 2016), limits on the stability coefficient, θ, for special moment frames are such that if $\alpha P_r/P_{ns} \le 0.5$, the product B_2B_3 cannot exceed 1.15; for this value, demand-to-capacity ratios not exceeding 0.866 for a first-order analysis are acceptable without more refined amplification and combination of loads.
- *Amplification of loads.* Alternatively, the value of B_3 can be selected in advance of design by limiting $\alpha P_r/P_{ns}$ and taking B_2 as the lesser of that corresponding to the limit on the stability coefficient, θ, and that corresponding to the drift limit. This value of B_3 may be applied as a supplementary lateral load factor. The factor B_3 may be used to amplify the entire set of results from a lateral analysis, utilizing the largest value of B_3 for all stories. Because B_3 is based on the smallest value of τ_b for all moment-frame columns in a story, this may extend the effect of a single low value of τ_b dramatically, but in typical practice, moment-frame columns rarely have axial force above $0.5P_{ns}$, and the axial force must be substantially greater than this value to have a significant effect on stiffness. If some $\alpha P_r/P_{ns}$ ratios exceed 0.5 (and member selection is governed by strength rather than the stiffness required to meet drift limits), story-by-story application of B_3 may be preferable.
- *Story-by-story amplification of load effects.* Use of a single value of B_3 on the entire lateral load will be conservative for stories where the second-order effect is relatively low. B_3 may be applied to the load effect story by story instead of as a global factor. However, use of the B_3 factor as a story-specific factor on the applied force (rather than on the load effect) is potentially unconservative because the amplification of overturning effects is less than the shear amplification at levels with a higher value of B_3 than the levels above (which is typical if drifts are uniform), and thus this method is not recommended until this potential discrepancy is studied. Story-by-story application of the product B_2B_3 on the lateral load effect is straightforward with superposition of load effects from a first-order analysis. Story-by-story application of factor B_3 with the results of a second-order analysis may be done by developing a model with geometric stiffness based on the vertical loads; analyses using this geometric-stiffness model are suitable for linear superposition (Wilson and Habibullah, 1987) and are thus suitable for application of the B_3 factor either to lateral loads or lateral-load effects.

The proposed procedure for IAM application is:

1. Establish limit on drift and corresponding vertical and lateral loads.
2. Establish vertical and lateral loads for strength evaluations.

Table 1. Amplification Factors B_3 for IAM

$\alpha P_r/P_{ns}$	≤0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1.00
τ_b	1	0.99	0.96	0.91	0.84	0.75	0.64	0.51	0.36	0.19	0.00
B_2	B_3										
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.05	1.01	1.01	1.02	1.02	1.03	1.03	1.05	1.08	1.14	1.39	
1.10	1.03	1.03	1.03	1.04	1.05	1.07	1.11	1.17	1.33	2.26	
1.15	1.04	1.04	1.05	1.06	1.08	1.11	1.17	1.28	1.59	6.13	
1.20	1.05	1.06	1.06	1.08	1.11	1.15	1.24	1.41	1.98		
1.25	1.07	1.07	1.08	1.10	1.14	1.20	1.31	1.57	2.62		
1.30	1.08	1.09	1.10	1.13	1.17	1.25	1.40	1.77	3.87		
1.35	1.10	1.10	1.12	1.15	1.21	1.30	1.50	2.03	7.42		
1.40	1.11	1.12	1.14	1.18	1.24	1.36	1.62	2.38	90.0		
1.45	1.13	1.13	1.16	1.20	1.28	1.43	1.75	2.88			
1.50	1.14	1.15	1.18	1.23	1.32	1.50	1.91	3.64			
1.55	1.16	1.17	1.20	1.26	1.37	1.58	2.10	4.95			
1.60	1.18	1.19	1.22	1.29	1.41	1.67	2.34	7.73			
1.65	1.19	1.21	1.24	1.32	1.46	1.76	2.63	17.6			
1.70	1.21	1.23	1.27	1.35	1.52	1.88	3.00				
1.75	1.23	1.25	1.29	1.39	1.58	2.00	3.51				
1.80	1.25	1.27	1.32	1.43	1.64	2.14	4.21				
1.85	1.27	1.29	1.35	1.47	1.71	2.31	5.27				
1.90	1.29	1.31	1.37	1.51	1.78	2.50	7.03				
1.95	1.31	1.33	1.40	1.55	1.86	2.73	10.6				
2.00	1.33	1.36	1.43	1.60	1.95	3.00	21.3				
2.20	1.43	1.46	1.57	1.81	2.41	5.00					
2.40	1.54	1.58	1.73	2.10	3.16	15.0					
2.60	1.67	1.72	1.94	2.49	4.57						
2.80	1.82	1.90	2.19	3.05	8.24				**Unstable**		
3.00	2.00	2.11	2.53	3.96	42.0						
3.50	2.67	2.91	4.09	15.2							
4.00	4.00	4.71	10.7								
4.50	8.00	12.4									
5.00											

3. Size members to meet drift limit.
4. Check stability coefficient limit θ, if required (as it is for seismic design).
5. Determine second-order effect B_2 (Equation 12) for the applicable load combinations.
6. Calculate preliminary value of B_3 (based on $\alpha P_r/P_{ns} \le 0.5$ using Equation 15).
7. Apply additional notional load of $0.001P_{story}$ to preclude the need to check $\alpha P_r/P_{ns}$. (This is optional; engineers may consider the check on axial load ratio to be less cumbersome than implementing the notional load. Also, this notional-load approach may result in a more conservative evaluation.)
8. Perform second-order analysis with lateral loads amplified by the largest value of B_3.

Table 2. Amplification Factors B_2B_3 for IAM

$\alpha P_r/P_{ns}$	≤0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1.00
τ_b	1	0.99	0.96	0.91	0.84	0.75	0.64	0.51	0.36	0.19	0.00
B_2	B_2B_3										
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
1.05	1.06	1.06	1.07	1.07	1.08	1.09	1.10	1.13	1.20	1.46	
1.10	1.13	1.13	1.13	1.14	1.16	1.18	1.22	1.29	1.46	2.49	
1.15	1.19	1.20	1.20	1.22	1.24	1.28	1.34	1.47	1.83	7.05	
1.20	1.26	1.27	1.28	1.30	1.33	1.38	1.48	1.69	2.37		
1.25	1.33	1.34	1.35	1.38	1.42	1.50	1.64	1.96	3.27		
1.30	1.41	1.41	1.43	1.46	1.52	1.63	1.82	2.30	5.03		
1.35	1.48	1.49	1.51	1.55	1.63	1.76	2.03	2.74	10.0		
1.40	1.56	1.56	1.59	1.65	1.74	1.91	2.26	3.34	126		
1.45	1.63	1.64	1.68	1.74	1.86	2.07	2.54	4.18			
1.50	1.71	1.73	1.77	1.84	1.98	2.25	2.87	5.46			
1.55	1.80	1.81	1.86	1.95	2.12	2.45	3.26	7.67			
1.60	1.88	1.90	1.95	2.06	2.26	2.67	3.74	12.4			
1.65	1.97	1.99	2.05	2.18	2.42	2.91	4.34	29.0			
1.70	2.06	2.08	2.16	2.30	2.58	3.19	5.11				
1.75	2.15	2.18	2.26	2.43	2.76	3.50	6.14				
1.80	2.25	2.28	2.37	2.57	2.95	3.86	7.58				
1.85	2.35	2.38	2.49	2.71	3.16	4.27	9.74				
1.90	2.45	2.49	2.61	2.86	3.39	4.75	13.4				
1.95	2.56	2.60	2.73	3.02	3.64	5.32	20.6				
2.00	2.67	2.71	2.87	3.19	3.91	6.00	42.7				
2.20	3.14	3.21	3.45	3.99	5.31	11.0					
2.40	3.69	3.80	4.16	5.03	7.58	36.0			**Unstable**		
2.60	4.33	4.48	5.03	6.46	11.9						
2.80	5.09	5.31	6.14	8.55	23.1						
3.00	6.00	6.32	7.58	11.9	126						
3.50	9.33	10.2	14.3	53.1							
4.00	16.0	18.9	42.7								
4.50	36.0	55.7									
5.00											

9. Verify that member demand-to-capacity ratios do not exceed 1.0. If $\alpha P_r/P_{ns} \leq 0.5$ was assumed (see step 7), check this ratio.

10. If the strength or stability checks indicate insufficiency, iterate using IAM or switch to DM. For IAM iteration, check $\alpha P_r/P_{ns}$ (to determine if the additional notional load can be removed), recalculate B_3, consider applying separate B_3 factors at each story, or resize members.

Steps 4, 5, and 6 may be done in advance of design by assuming a stiffness necessary to meet drift limits (including second-order effects), simplifying the design process. (The design example in Appendix C follows this procedure.)

Because many moment frames have member sizes selected to meet drift limits rather than for strength, iteration in the subsequent design for stability is not always required, and the IAM provides a relatively simple method of verifying adequate strength to maintain stability. If iteration is required and the DM is used, the initial IAM analysis provides the necessary information to determine τ_b.

Table 3. Comparison of Methods of Design for Stability

	Advantages of IAM	Disadvantages of IAM
FOM	• Lower forces. • Permits $\alpha P_r/P_{ns} \le 0.7$ instead of $\alpha P_r/P_{ns} \le 0.5$ for moment frame columns. • Permits $B_2 \le 2.0$ instead of $B_2 \le 1.5$. • Eliminates the need for additional lateral loads (for $B_2 \le 1.5$). • B_2B_3 is an indicator of the sensitivity to modeling and loading assumptions.	• Requires second-order analysis. However, the FOM is limited to $B_2 \le 1.5$ and thus requires determination of the second-order effect to permit use.
ELM	• Lower demand-to-capacity ratios. • $K = 1$ for moment-frame columns with $B_2 > 1.1$. • Permits $B_2 \le 2.0$ instead of $B_2 \le 1.5$. • Provides appropriate design forces for beams and connections (instead of addressing only column strength). • B_3 is an indicator of the sensitivity to modeling and loading assumptions.	• Requires application of amplification factor B_3. • Limited to $\alpha P_r/P_{ns} \le 0.7$ for moment frame columns.
DM	• Single calculation of B_3 per level in lieu of member-by-member application of τ_b. • Does not require one model for strength and a second model for drift and period determination. • B_3 is an indicator of the sensitivity to modeling and loading assumptions.	• Potentially higher forces, especially for torsional response and flexible diaphragms. • Requires use of largest τ_b for each level of moment frames if $\alpha P_r/P_{ns} > 0.5$. • Suitability limited to: ◦ Vertical columns. ◦ $B_2 \le 2.0$. ◦ $\alpha P_r/P_{ns} \le 0.7$ for moment frame columns.

For mixed systems (moment frames with braced frames or shearwalls), R_M determined using AISC *Specification* Equation A-8-8 is appropriate to use in calculating B_3. The axial-force ratio $\alpha P_r/P_{ns}$ is only relevant for members whose flexural resistance contributes to the lateral stiffness; braces, beams, and columns modeled with both ends pinned may have $\alpha P_r/P_{ns} > 0.5$ and not be considered in determining the value of τ_b to be used to compute B_3.

VERIFICATION

Verification studies are presented in Appendices A, B, and C. Appendix A applies the IAM to a simple frame that was studied by Carter and Geschwindner (2008) using FOM, ELM, and DM methods. That study demonstrated the advantage of the DM over the ELM and FOM in terms of lower demand-to-capacity ratios, and Appendix A shows a perfect correspondence between the IAM and the DM. Appendix B is a multi-parameter study of similar frames, comparing IAM and DM forces and design demand-to-capacity ratios. This comparison indicates an excellent correlation between design checks using DM and IAM forces over a broad range of the second-order effect (B_2), column slenderness, and stiffness-reduction parameter, τ_b. Appendix C is a design example of a two-bay, eight-story frame, demonstrating application of the method in the design process.

QUALITATIVE COMPARISON OF THE PROPOSED INDIRECT ANALYSIS METHOD WITH OTHER METHODS

Table 3 shows qualitative advantages and disadvantages of the proposed IAM with respect to the established stability design methods (DM, ELM, and FOM).

CONCLUSIONS

The proposed indirect analysis method (IAM) is a stability design method based on the DM and developed to reduce the effort required of the designer. Calculation of the factors utilized in the IAM (B_2 and B_3) may be performed in advance of design and can be used to identify cases in which a more rigorous method of analysis and design for stability is appropriate. IAM is proposed to be limited to $B_2 \le 2.0$ and $\alpha P_r/P_{ns} \le 0.7$. Above these limits, the structure is highly sensitive to second-order effects and stiffness reduction (and thus to the accuracy of loading assumptions, modeling, and analysis).

The IAM has been compared to the DM for a range of second-order effects and axial-force ratios. Comparisons indicate close correlation, with the IAM becoming less conservative for cases with high column slenderness and high P-Δ effects (B_2). Within the proposed range of applicability, the IAM approximates the DM very closely, with differences in demand-to-capacity ratio no greater than 1%.

The IAM is particularly well suited for designs governed by drift or stability-coefficient limits (such as for seismic design), allowing a simple confirmation of compliance with AISC stability-design requirements with a single factor. Thus, drift design and strength design, which are two distinct analyses, often with different loadings, may be performed using a single model without changing member properties for steel structures. For strength-governed designs, the IAM entails a modest degree of conservatism, which can be reduced by a story-by-story application of factors on the lateral-load effect.

The IAM can be used as the basis for an improved FOM with less conservatism and a better indication of the magnitude of stability-related effects on the design.

The IAM matches the DM for structures that have moment-frame columns with an axial-load ratio of $\alpha P_r/P_{ns} \leq 0.5$; it is conservative compared to the DM for structures that have moment frame columns with differing levels of axial-load ratio $\alpha P_r/P_{ns}$ above 0.5. The level of conservatism of the IAM has not been established for more complex structures, although if member selection is governed by drift, the level of conservatism in the strength evaluation is inconsequential.

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Any reader of this paper will note its debt to the work of Griffis and White. Additionally, Larry Griffis provided valuable guidance in the development of this method. Matt Eatherton was a careful early reviewer who identified that this method is not limited to seismic applications. Lou Geschwindner provided a detailed review that helped clarify how this method differs from the other stability design methods. Ron Ziemian guided the use of more exact solutions incorporated into the verification study. Larry Kruth and Leigh Arber provided encouragement and helped focus this work on providing a benefit to practicing engineers. My colleagues Ozgur Atlayan, Joseph Dowd, and Susendar Muthukumar provided detailed review and assisted in the verification and application.

ABBREVIATIONS

ASD	Allowable strength design per AISC *Specification* Chapter B
DCR	Demand-to-capacity ratio
DM	Direct analysis method of design for stability per AISC *Specification* Chapter C
ELM	Effective-length method of design for stability per AISC *Specification* Appendix 7
IAM	Indirect analysis method of design for stability
FOM	First-order analysis method of design for stability per AISC *Specification* Appendix 7
LRFD	Load and resistance factor design per AISC *Specification* Chapter B

SYMBOLS

A_e	Effective area, in.^{2} (AISC *Specification*)
A_g	Gross area of member, in.^{2} (AISC *Specification*)
B_2	Multiplier to account for P-Δ effects (AISC *Specification* Appendix 8)
$\overline{B}_2$	Second-order amplifier using the stiffness reduction $EI^* = 0.8\tau_b EI$ (Design Guide 28)
B_2'	Second-order effect determined from second-order analysis results (Equation 16)
B_3	IAM amplification factor to account for stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty (Equation 14)
B_3'	IAM amplification factor to account for stiffness reduction due to inelasticity, member imperfection, and stiffness uncertainty calculated using B_2' (Equation 17)
C_L	Factor that accounts for the reduction in column sidesway stiffness due to the presence of axial load on the column (Design Guide 28)
D_{AF}	Displacement amplification factor (Design Guide 28)
$D_{AF\text{-}DM}$	Displacement amplification factor using reduced stiffness (Appendix B)
E	Modulus of elasticity, ksi (AISC *Specification*)
F_{AF}	Force amplification factor (Design Guide 28)
$F_{AF\text{-}DM}$	Force amplification factor using reduced stiffness (Appendix B)
F_y	Specified minimum yield stress, ksi (AISC *Specification*)
H	Total story shear, kips (AISC *Specification*)
$\overline{H}_{P\Delta}$	P-Δ shear forces, kips (Design Guide 28)
I	Moment of inertia, in.^{4} (AISC *Specification*)
K	Effective length factor (AISC *Specification* Chapter E)
L	Story height, in. (AISC *Specification*)

M_{DM} Moment resulting from a second-order analysis using reduced stiffness as required by the direct analysis method, kip-in. (Appendix B)

M_{FOA} First-order analysis moment, kip-in. (Appendix B)

M_{SOA} Second-order analysis moment, kip-in. (Appendix B)

N_i Notional load at level *i*, kips (AISC *Specification*)

ΣP Sum of applied vertical forces, kips, (Design Guide 28)

P Applied vertical force, kips, (Design Guide 28)

P_A Axial force in stability column, kips (Appendix B)

P_B Axial force in leaning column, kips (Appendix B)

P_L Story lateral force required to induce a first-order unit drift, kips (Design Guide 28)

P_{e1} Elastic critical buckling strength of the member in the plane of bending, kips (AISC *Specification*)

P_{ns} Cross-section compressive strength, kips (AISC *Specification*)

P_r Required axial compressive strength, kips (AISC *Specification*)

P_{story} Total vertical load supported by the story, kips (AISC *Specification*)

Q Stability index (Design Guide 28)

R_M Coefficient to account for member P-δ influence on structure P-Δ (AISC *Specification*)

Y_i Gravity load applied at level *i*, kips (AISC *Specification*)

α ASD/LRFD force level adjustment factor, equal to 1.0 (LRFD) or 1.6 (ASD) (AISC *Specification*)

α Ratio of the smaller to the larger end moment in the moment-frame column, determined from a first-order sidesway analysis (Design Guide 28)

β_{L} Coefficient corresponding to the contribution of the moment-frame column to the first-order story sidesway stiffness (Design Guide 28)

Δ_1 First-order story drift using full stiffness properties, in.

Δ_2 Second-order story drift using full stiffness properties, in.

Δ_{DM} Second-order story drift using reduced stiffness properties, in. (Appendix B)

Δ_H First-order interstory drift due to lateral forces, in. (AISC *Specification*)

Δ_o Nominal initial out-of-plumbness, in. (Design Guide 28)

θ Stability coefficient for P-Δ effects (ASCE/SEI 7)

τ_b Flexural stiffness-reduction parameter (AISC *Specification*)

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APPENDIX A

VERIFICATION: CANTILEVER STABILITY COLUMN WITH LEANING COLUMN (EXAMPLE)

The equivalence between the IAM and the DM can be established by analyzing a well-understood example frame: the one-bay frame studied by Carter and Geschwindner (2008). The frame consists of a cantilevered stability column (left, with a rigid base connection and a pin at the top), a leaning column (right, with pins top and bottom), and a rigid beam constraining their lateral movement. This frame is shown in Figure A1. The LRFD method is employed in this example.

Carter and Geschwindner used the following values for their study:

$P_A = P_B = 200$ kips

$L = 15$ ft

$H = 20$ kips

The notional load N depends on the stability-design method employed. (No notional load is required for the IAM in this example.) The stability column is an ASTM A992 W14×90.

Carter and Geschwindner determined axial forces and moments for four different methods of design for stability:

- The effective-length method (second order).
- The first-order method.
- The direct analysis method.
- The simplified method (a version of the ELM utilizing design tables).

The IAM is applied to this frame, utilizing the results of Carter and Geschwindner's full-stiffness second-order analysis for the system and for column A. The determination of B_2 is redone here to add a significant figure. (Such precision is not required for the IAM but is applied in this example to allow comparison of results without artifacts due to rounding.) Consistent with the 2005 edition of the AISC *Specification* (AISC, 2005), Carter and Geschwindner use a value of R_M of 0.85 rather than calculating R_m using AISC *Specification* Equation A-8-8 per the 2016 edition (AISC, 2016); for purposes of comparison of the stability-design methods, the same value is used here.

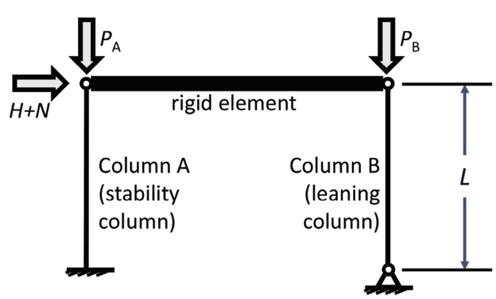

Fig. A1. Frame used for verification study.

$$\Delta_H = \frac{HL^3}{3EI}$$
$$= \frac{(20 \text{ kips})\left[(15 \text{ ft})(12 \text{ in./ft})\right]^3}{3(29{,}000 \text{ ksi})(999 \text{ in.}^4)}$$
$$= 1.34 \text{ in.}$$

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}\Delta_H}{R_M HL}} \tag{8}$$
$$= \frac{1}{1 - \frac{1.0(400 \text{ kips})(1.34 \text{ in.})}{0.85(20 \text{ kips})(180 \text{ in.})}}$$
$$= 1.21 \text{ in.}$$

$$M_{lt} = HL$$
$$= (20 \text{ kips})(15 \text{ ft})$$
$$= 300 \text{ kip-ft}$$

$$P_r = P_A$$
$$= 200 \text{ kips}$$

$$\frac{P_r}{P_c} = 0.200$$

From AISC *Manual* Table 3-10, the available flexural strength, ϕM_n, is

$$\phi M_n = 573 \text{ kip-ft}$$
$$P_{ns} = A_g F_y$$
$$= (26.5 \text{ in.}^2)(50 \text{ ksi})$$
$$= 1{,}330 \text{ kips}$$

The axial load ratio is:

$$\frac{\alpha P_r}{P_{ns}} = \frac{(1.00)(200 \text{ kips})}{1{,}330 \text{ kips}}$$
$$= 0.15$$

Because $\frac{\alpha P_r}{P_{ns}} < 0.5$, τ_b is 1.0 and the IAM amplifier is obtained from Equation 15

$$B_3 = \frac{4}{5 - B_2} \tag{15}$$
$$= \frac{4}{5 - 1.21}$$
$$= 1.06$$

Table A1. Interaction Demand-to-Capacity Ratios for Different Methods of Design for Stability	
Method	**Value**
First-order (FOM)	0.811
Second-order (ELM)	0.840
Simplified ELM	0.966
Direct analysis (DM)	0.796
Indirect analysis (IAM)	0.796

The IAM moment is:

$$\begin{aligned} M_r &= B_2 B_3 M_{lt} \\ &= (1.21)(1.06)(300 \text{ kip-ft}) \\ &= 385 \text{ kip-ft} \end{aligned}$$

Because $P_r/P_c \geq 0.2$, AISC *Specification* Equation H1-1a applies:

$$\begin{aligned} \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_r}{\phi M_n}\right) &= 0.200 + \frac{8}{9}\left(\frac{385 \text{ kip-ft}}{573 \text{ kip-ft}}\right) \\ &= 0.796 \end{aligned} \qquad \text{(\textit{Spec}. Eq. H1-1a)}$$

Table A1 shows the interaction equation values from Carter and Geschwindner for the FOM, ELM, simplified ELM, and DM using AISC *Specification* Equation H1-1a. The table adds the IAM value computed above, which precisely matches the DM for this simple system.

APPENDIX B

VERIFICATION: CANTILEVER STABILITY COLUMN WITH LEANING COLUMN (PARAMETRIC STUDY)

To verify that the proposed IAM produces forces consistent with the DM over a broad range of multiple parameters, the results of calculations similar to those in Appendix A are presented here in tabular form. These examples use a frame of the same configuration as that in Appendix A (Figure A1), analyzed using first-order analysis, second-order analysis, and DM (second-order analysis with modified stiffness properties). The stability column is an ASTM A992 W12×120 oriented in the strong axis and restrained from lateral torsional buckling along its length such that the flexural strength is based on the plastic section modulus ($\phi M_n = \phi M_p = 9{,}300$ kip-in.). The column is restrained from minor-axis buckling at the top and bottom. The LRFD method is employed in this example.

Several cases were studied, covering a range of second-order effects and axial-force demands on the stability column and multiple frame heights. The analyses were conducted using formulae for flexural stiffness and do not include shear deformations. The method was confirmed using analysis software and benchmark problems. Using formulae permits a systematic tabular comparison of results from DM and IAM evaluations varying multiple parameters in a spreadsheet.

Following methods developed by LeMessurier (1977), second-order effects incorporate either a force-amplification factor, F_{AF}, or a displacement amplification factor, D_{AF}. Nomenclature and equations for these factors correspond to those used by Griffis and White (2013).

The force amplification factor is presented in Equation A-9 in Griffis and White (2013):

$$F_{AF} = \frac{1}{1 - \dfrac{\sum P}{P_L - \sum C_L P}} \tag{21}$$

where

C_L = factor that accounts for the reduction in column sidesway stiffness due to the presence of axial load on the column

P = applied vertical force, kips

P_L = story lateral force required to induce a first-order unit drift, kips

ΣP = sum of applied vertical forces, kips

In the example frame, the vertical force is:

$$\sum P = P_A + P_B \tag{22}$$

For a cantilever column, the parameter P_L is:

$$P_L = \frac{3EI}{L^2} \tag{23}$$

The factor C_L is zero for columns not providing lateral stiffness. For the cantilever column C_L is:

$$C_L = \left(\frac{12}{\pi^2} - 1\right)\left(\frac{\beta_L}{3 + 9\alpha}\right)^2 \tag{24}$$

where

α = ratio of the smaller to the larger end moment in the moment-frame column, determined from a first-order sidesway analysis

= 0 for a cantilever column

β_L = coefficient corresponding to the contribution of the moment-frame column to the first-order story sidesway stiffness

= 3 for a cantilever column

Thus, for the frame under consideration:

$$F_{AF} = \frac{1}{1 - \dfrac{P_A + P_B}{{}^{3EI}\!/\!{}_{L^2} - \left(\dfrac{12}{\pi^2} - 1\right)P_A}} \tag{25}$$

For the DM, the stiffness term is reduced by $0.8\tau_b$:

$$F_{AF-DM} = \frac{1}{1 - \dfrac{P_A + P_B}{0.8\tau_b\, {}^{3EI}\!/\!{}_{L^2} - \left(\dfrac{12}{\pi^2} - 1\right)P_A}} \tag{26}$$

The displacement amplification factor D_{AF} is presented in Equation A-10 in Griffis and White (2013):

$$D_{AF} = \frac{1}{1 - \dfrac{\sum P + \sum C_L P}{P_L}} \tag{27}$$

For the frame under consideration:

$$D_{AF} = \frac{1}{1 - \dfrac{\dfrac{12}{\pi^2} P_A + P_B}{3EI/L^2}} \tag{28}$$

For the DM the stiffness term is reduced by $0.8\tau_b$:

$$D_{AF-DM} = \frac{1}{1 - \dfrac{12/\pi^2\, P_A + P_B}{0.8\tau_b\, 3EI/L^2}} \tag{29}$$

Moments are determined as follows:

$$M_{FOA} = HL \tag{30}$$

$$M_{SOA} = F_{AF}HL \tag{31}$$

$$M_{DM} = F_{AF-DM}HL \tag{32}$$

Displacements are determined as follows:

$$\Delta_1 = \frac{HL^3}{3EI} \tag{33}$$

$$\Delta_2 = D_{AF}\Delta_1 \tag{34}$$

$$\Delta_{DM} = D_{AF-DM}\Delta_1 \tag{35}$$

Table B1 provides a systematic comparison of DM and IAM demand-to-capacity ratios for varying column height, axial load ratios ($\alpha P_r/P_{ns}$), and second-order effects (B_2). The table shows the analysis and loading information and calculated τ_b factors for the stability column, as well as notional loads where there is no applied lateral load and where the B_2 factor is greater than 1.5. The table also shows the results, including analyzed first-order and second-order displacement (Δ_1 and Δ_2), the second-order displacement corresponding to the DM (Δ_{DM}), and analyzed first-order, second-order, and DM moments (M_{FOA}, M_{SOA}, and M_{DM}).

Table B1 shows factors B_2 and B_3 determined in two ways. First, B_2 is calculated using Equation 12, and the corresponding value of B_3 is determined using Equation 14. Second, a value of B_2' is calculated as the ratio of Δ_2 to Δ_1 from the analysis (Equation 16), and the corresponding value of B_3' is determined (Equation 17). The table presents the F_{AF} and D_{AF} factors (Equations 25–29), and the amplifier products B_2B_3 and $B_2'B_3'$ for comparison.

The table shows calculated demand-to-capacity ratios for the stability column using Chapter H interaction equations from the AISC *Specification* for DM, IAM (using B_3), and IAM′ (using B_3') forces. Frame heights were selected to provide a range of stability column slenderness. Gravity loads were selected to achieve target values of B_2, and to provide both high- and low-axial-stress cases. Lateral loads were selected such that the DM produced demand-to-capacity ratios of 1.00 (as shown in the DCR column).

Cases B1–B4 [at the top of the table) are the results from Benchmark Problem Case 2 in AISC *Specification* Figure C-C2.3 (AISC, 2016)], neglecting shear deformations. Results confirm that accurate moments and displacements are obtained using the formulae in the table.

Cases 1–6 cover a range of B_2 from 1.11 to 1.61 with high axial-force in the stability column such that $\alpha P_r/P_{ns} = 0.7$ and $\tau_b = 0.84$. (The very low slenderness permits such high force.) The stability column has $P_A/P_{e1} = 0.10$ (where P_{e1} is the Euler buckling load determined in the plane of bending per AISC *Specification* Equation A-8-5). The system reaches the "limit of stability" (i.e., a demand-to-capacity ratio of 1.0 under vertical and notional loads alone with no applied lateral force) at $B_2 = 1.61$.

Cases 7–17 cover a range of B_2 from 1.10 to 3.00 with no axial force in the stability column, which has low slenderness.

Cases 18–20 cover a range of B_2 from 1.5 to 1.82 with high axial force in the low-slenderness stability column such that $\tau_b = 0.89$. Because of the high stability-column axial force and column height, lower values of B_2 cannot be achieved. The stability column has $P_A/P_{e1} = 0.34$. The system reaches the limit of stability at $B_2 = 1.82$.

Cases 21–28 cover a range of B_2 from 1.40 to 3.00 with moderate axial force in the moderate-slenderness stability column, which has $P_A/P_{e1} = 0.29$.

Cases 29–38 are similar to cases 21–28 and cover a range of B_2 from 1.10 to 3.00. This range of B_2 is achieved by varying the stability-column axial force rather than the leaning-column axial force.

Cases 39–44 are similar to case 34. Both P_A and P_B are varied such that $B_2 = 2.00$ is maintained for a range of P_A/P_{e1} from 0.09 to 0.52.

Cases 45–50 vary both P_A and P_B such that $B_2 = 2.00$ is maintained for a range of slenderness (L/r) from 64 to 383.

Table B1 can be used to assess the correspondence of the IAM to the DM using both the design moments obtained and the design interaction value from AISC *Specification* Section H1.1 (AISC, 2016). Comparison of the moments provides a measure of the accuracy of the IAM. However, in cases where the axial load in the stability column is large, the moment represents a small contribution to the design interaction ratio, and thus differences in moment are of less consequence. Demand-to-capacity ratios differing from 1.00 are highlighted in the table, as are ratios of amplified moments to DM moments differing from 1.00.

Comparison of Moments

The table presents three moment ratios:

- Amplified first-order moment to DM moment ($B_2B_3M_{FOA}/M_{DM}$).
- Amplified second-order moment to DM moment (B_3M_{SOA}/M_{DM}).
- Amplified second-order moment to DM moment ($B_3' M_{SOA}/M_{DM}$) based on analyzed displacements.

Ratios above 1.00 indicate that the IAM is conservative, while ratios below 1.00 indicate the opposite. The results indicate that use of the IAM with amplifiers B_2' and B_3' based on analyzed second-order displacements is always conservative in the range studied. Use of first-order analysis with amplifiers B_2 and B_3 based on first-order displacements is conservative for $B_2 \leq 2.5$. Second-order analysis with the amplifier B_3 based on first-order displacements is conservative for $B_2 \leq 2.0$. For cases with $B_2 \geq 2.0$ and $P_A/P_{e1} \geq 0.29$, use of the amplifier B_3 based on first-order displacements becomes slightly unconservative. As noted by Surovek-Maleck and White (2004), the approximate methods of second-order analysis may become less reliable as column P_A/P_{e1} increases.

Comparison of Interaction Ratios

Comparison of interaction ratios was limited to IAM using second-order analysis; amplified first-order analysis forces were not evaluated. Interaction ratios were evaluated for the DM, the IAM, and the IAM′ (using the amplifiers B_3' based on analyzed second-order displacements). (Horizontal forces were selected to produce DM demand-to-capacity ratios of 1.00.) Cases in which the IAM demand-to-capacity ratio is greater than 1.00 indicate that the IAM is conservative compared to the DM; cases in which the IAM demand-to-capacity ratio is less than 1.00 indicate that the IAM is unconservative. Cases in which the IAM produces an unconservative demand-to-capacity ratio by more than 0.005 are outside the range of proposed applicability. In short, the IAM matches the DM or is slightly conservative for $B_2 \leq 2.0$. Above that range, B_3 (which is a function of B_2) could be unconservative as B_2 deviates slightly from Δ_2/Δ_1. Based on the results, use of the IAM is conservative for $B_2 \leq 2.0$ and use of the IAM′ (which utilizes B_3' from Equation 17) is always conservative in the range studied.

Table B1. Verification Analysis Summary																	
	L (in.)	L/r	H (kips)	N (kips)	P_A (kips)	τ_b	P_A/P_{e1}	P_B (kips)	P_{story} (kips)	R_M	B_2	B_3	B_2B_3	F_{AF}	$F_{AF\text{-}DM}$	D_{AF}	$D_{AF\text{-}DM}$
Notes	**1**		**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Case																	
B1	336	176	1.0	0.0	0	1.00	0.00	0	0	1.00	1.00			1.00		1.00	
B2	336	176	1.0	0.0	100	1.00	0.33	0	100	0.85	1.46			1.40		1.48	
B3	336	176	1.0	0.0	150	1.00	0.49	0	150	0.85	1.90			1.79		1.96	
B4	336	176	1.0	0.0	200	1.00	0.65	0	200	0.85	2.71			2.54		2.87	
1	80	26	21.0	0.0	1232	0.84	0.10	0	1,232	0.85	1.11	1.06	1.17	1.09	1.15	1.11	1.18
2	80	26	18.1	0.0	1232	0.84	0.10	991	2,223	0.92	1.20	1.11	1.33	1.18	1.31	1.21	1.34
3	80	26	16.0	0.0	1232	0.84	0.10	1,928	3,160	0.94	1.30	1.17	1.52	1.28	1.50	1.31	1.54
4	80	26	14.0	0.0	1232	0.84	0.10	2,730	3,962	0.95	1.40	1.24	1.74	1.38	1.71	1.41	1.76
5	80	26	12.5	0.0	1232	0.84	0.10	3,424	4,656	0.96	1.50	1.32	1.98	1.48	1.96	1.51	2.01
6	80	26	0.0	10.6	1232	0.84	0.10	4,061	5,293	0.97	1.61	1.42	2.28	1.59	2.26	1.62	2.32
7	150	48	55.0	0.0	0	1.00	0.00	376	376	1.00	1.10	1.03	1.13	1.10	1.13	1.10	1.13
8	150	48	49.1	0.0	0	1.00	0.00	690	690	1.00	1.20	1.05	1.26	1.20	1.26	1.20	1.26
9	150	48	44.1	0.0	0	1.00	0.00	955	955	1.00	1.30	1.08	1.41	1.30	1.41	1.30	1.41
10	150	48	39.9	0.0	0	1.00	0.00	1,182	1,182	1.00	1.40	1.11	1.56	1.40	1.56	1.40	1.56
11	150	48	36.2	0.0	0	1.00	0.00	1,379	1,379	1.00	1.50	1.14	1.71	1.50	1.71	1.50	1.71
12	150	48	25.2	3.5	0	1.00	0.00	1,773	1,773	1.00	1.75	1.23	2.15	1.75	2.15	1.75	2.15
13	150	48	19.1	4.1	0	1.00	0.00	2,069	2,069	1.00	2.00	1.33	2.67	2.00	2.67	2.00	2.67
14	150	48	14.4	4.6	0	1.00	0.00	2,299	2,299	1.00	2.25	1.45	3.27	2.25	3.27	2.25	3.27
15	150	48	10.5	5.0	0	1.00	0.00	2,482	2,482	1.00	2.50	1.60	4.00	2.50	4.00	2.50	4.00
16	150	48	7.4	5.3	0	1.00	0.00	2,633	2,633	1.00	2.75	1.78	4.89	2.75	4.89	2.75	4.89
17	150	48	4.8	5.5	0	1.00	0.00	2,758	2,758	1.00	3.00	2.00	6.00	3.00	6.00	3.00	6.00
18	150	48	5.0	0.0	1172	0.89	0.34	0	1,172	0.85	1.50	1.25	1.88	1.43	1.77	1.53	1.94
19	150	48	0.6	3.2	1172	0.89	0.34	403	1,575	0.89	1.75	1.44	2.51	1.68	2.41	1.79	2.64
20	150	48	0.0	3.3	1172	0.89	0.34	500	1,672	0.89	1.82	1.50	2.73	1.76	2.64	1.87	2.89
21	300	96	16.3	0.0	250	1.00	0.29	1.4	251	0.85	1.40	1.11	1.56	1.34	1.48	1.42	1.58
22	300	96	14.7	0.0	250	1.00	0.29	52	302	0.88	1.50	1.14	1.71	1.45	1.64	1.52	1.75
23	300	96	10.8	0.8	250	1.00	0.29	152	402	0.91	1.75	1.23	2.15	1.70	2.08	1.79	2.23
24	300	96	8.3	1.0	250	1.00	0.29	226	476	0.92	2.00	1.33	2.67	1.95	2.60	2.05	2.79
25	300	96	6.4	1.1	250	1.00	0.29	284	534	0.93	2.25	1.45	3.27	2.20	3.23	2.32	3.46
26	300	96	4.9	1.2	250	1.00	0.29	330	580	0.94	2.50	1.60	4.00	2.45	4.01	2.59	4.29
27	300	96	3.6	1.2	250	1.00	0.29	368	618	0.94	2.75	1.78	4.89	2.71	4.98	2.86	5.33
28	300	96	2.6	1.3	250	1.00	0.29	400	650	0.94	3.00	2.00	5.99	2.96	6.24	3.13	6.67
29	300	96	26.6	0.0	80	1.00	0.09	0	80	0.85	1.10	1.03	1.13	1.09	1.11	1.10	1.13
30	300	96	23.0	0.0	147	1.00	0.17	0	147	0.85	1.20	1.05	1.26	1.17	1.23	1.21	1.27
31	300	96	19.4	0.0	203	1.00	0.24	0	203	0.85	1.30	1.08	1.41	1.26	1.35	1.31	1.42
32	300	96	13.7	0.0	293	1.00	0.34	0	293	0.85	1.50	1.14	1.71	1.43	1.62	1.53	1.76
33	300	96	8.5	0.8	377	1.00	0.44	0	377	0.85	1.75	1.23	2.15	1.65	2.02	1.80	2.24
34	300	96	5.5	0.9	440	1.00	0.52	0	440	0.85	2.00	1.33	2.67	1.88	2.50	2.07	2.82
35	300	96	3.5	1.0	488	1.00	0.57	0	488	0.85	2.25	1.45	3.27	2.11	3.09	2.35	3.54
36	300	96	2.1	1.1	528	1.00	0.62	0	528	0.85	2.50	1.60	4.00	2.34	3.83	2.63	4.45
37	300	96	1.1	1.1	559	1.00	0.66	0	559	0.85	2.75	1.78	4.89	2.58	4.80	2.92	5.62
38	300	96	0.4	1.2	586	1.00	0.69	0	586	0.85	3.00	2.00	6.00	2.82	6.10	3.22	7.21
39	300	96	10.1	1.0	80	1.00	0.09	426	506	0.98	2.00	1.34	2.68	1.99	2.66	2.02	2.72
40	300	96	9.7	1.0	147	1.00	0.17	347	494	0.96	2.00	1.33	2.66	1.97	2.63	2.03	2.74
41	300	96	9.0	1.0	203	1.00	0.24	282	485	0.94	2.00	1.33	2.67	1.96	2.62	2.05	2.77
42	300	96	7.7	0.9	293	1.00	0.34	176	469	0.91	2.00	1.33	2.67	1.93	2.59	2.06	2.80
43	300	96	6.4	0.9	377	1.00	0.44	77	454	0.88	2.00	1.34	2.68	1.91	2.55	2.07	2.83
44	300	96	5.5	0.9	440	1.00	0.52	0	440	0.85	2.00	1.33	2.67	1.88	2.50	2.07	2.82
45	200	64	1.2	2.0	989	0.98	0.52	0	989	0.85	2.00	1.37	2.74	1.88	2.57	2.07	2.91
46	400	128	4.7	0.5	247	1.00	0.52	0	247	0.85	2.00	1.33	2.67	1.88	2.50	2.07	2.82
47	600	192	3.2	0.2	110	1.00	0.52	0	110	0.85	2.00	1.33	2.67	1.88	2.50	2.07	2.82
48	800	256	2.5	0.1	62	1.00	0.52	0	62	0.85	2.00	1.33	2.67	1.88	2.50	2.07	2.82
49	1000	319	2.0	0.1	40	1.00	0.52	0	40	0.85	2.00	1.33	2.67	1.88	2.50	2.07	2.82
50	1200	383	1.7	0.1	27	1.00	0.52	0	27	0.85	2.00	1.33	2.67	1.88	2.50	2.07	2.82
25'	300	96	6.4	1.1	250	1.00	0.29	284	534	0.92	2.29	1.48	3.38	2.20	3.23	2.32	3.46
26'	300	96	4.9	1.2	250	1.00	0.29	330	580	0.92	2.55	1.63	4.16	2.45	4.01	2.59	4.29
27'	300	96	3.6	1.2	250	1.00	0.29	368	618	0.93	2.81	1.82	5.12	2.71	4.98	2.86	5.33
28'	300	96	2.6	1.3	250	1.00	0.29	400	650	0.93	3.07	2.07	6.35	2.96	6.24	3.13	6.67
35'	300	96	3.5	1.0	488	1.00	0.57	0	488	0.82	2.35	1.51	3.54	2.11	3.09	2.35	3.54
36'	300	96	2.1	1.1	528	1.00	0.62	0	528	0.82	2.63	1.69	4.45	2.34	3.83	2.63	4.45
37'	300	96	1.1	1.1	559	1.00	0.66	0	559	0.82	2.92	1.92	5.62	2.58	4.80	2.92	5.62

Table B1. Verification Analysis Summary *(continued)*

Δ_1 (in.)	Δ_2 (in.)	Δ_{DM} (in.)	B'_2	B'_3	$B'_2B'_3$	M_{FOA} (kip-in.)	M_{SOA} (kip-in.)	M_{DM} (kip-in.)	DCR DM	DCR IAM (B_2B_3)	DCR IAM′ (B_3SOA)	$B_2B_3M_{FOA}/M_{DM}$	B_3M_{SOA}/M_{DM}	B'_3M_{SOA}/M_{DM}	
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Notes Case
0.90	0.90					336	336								B1
0.90	1.34					336	470								B2
0.90	1.76					336	600								B3
0.90	2.59					336	854								B4
0.12	0.13	0.14	1.11	1.06	1.18	1680	1838	1930	1.00	1.00	1.00	1.02	1.01	1.01	1
0.10	0.12	0.13	1.21	1.11	1.34	1447	1713	1888	1.00	1.00	1.00	1.02	1.01	1.01	2
0.09	0.12	0.14	1.31	1.18	1.54	1281	1645	1919	1.00	1.00	1.00	1.02	1.00	1.01	3
0.08	0.11	0.14	1.41	1.25	1.76	1121	1551	1921	1.00	1.00	1.00	1.01	1.00	1.01	4
0.07	0.10	0.14	1.51	1.33	2.01	997	1479	1953	1.00	1.00	1.00	1.01	1.00	1.01	5
0.06	0.09	0.14	1.62	1.43	2.32	847	1346	1910	1.00	1.00	1.00	1.01	1.00	1.01	6
1.99	2.19	2.25	1.10	1.03	1.13	8243	9067	9299	1.00	1.00	1.00	1.00	1.00	1.00	7
1.78	2.14	2.25	1.20	1.05	1.26	7362	8834	9299	1.00	1.00	1.00	1.00	1.00	1.00	8
1.60	2.08	2.25	1.30	1.08	1.41	6618	8603	9301	1.00	1.00	1.00	1.00	1.00	1.00	9
1.45	2.02	2.25	1.40	1.11	1.56	5979	8371	9301	1.00	1.00	1.00	1.00	1.00	1.00	10
1.31	1.97	2.25	1.50	1.14	1.71	5426	8138	9301	1.00	1.00	1.00	1.00	1.00	1.00	11
1.04	1.83	2.25	1.75	1.23	2.15	4318	7556	9300	1.00	1.00	1.00	1.00	1.00	1.00	12
0.84	1.69	2.25	2.00	1.33	2.67	3487	6974	9299	1.00	1.00	1.00	1.00	1.00	1.00	13
0.69	1.55	2.25	2.25	1.45	3.27	2842	6395	9301	1.00	1.00	1.00	1.00	1.00	1.00	14
0.56	1.41	2.25	2.50	1.60	4.00	2326	5814	9303	1.00	1.00	1.00	1.00	1.00	1.00	15
0.46	1.26	2.25	2.75	1.78	4.89	1903	5233	9303	1.00	1.00	1.00	1.00	1.00	1.00	16
0.37	1.12	2.25	3.00	2.00	6.00	1550	4651	9303	1.00	1.00	1.00	1.00	1.00	1.00	17
0.18	0.28	0.35	1.53	1.27	1.94	750	1074	1328	1.00	1.00	1.01	1.06	1.01	1.03	18
0.14	0.24	0.36	1.79	1.47	2.64	563	947	1356	1.00	1.01	1.01	1.04	1.00	1.03	19
0.12	0.23	0.35	1.87	1.54	2.89	502	881	1323	1.00	1.00	1.01	1.04	1.00	1.03	20
4.72	6.70	7.48	1.42	1.12	1.58	4881	6564	7231	1.00	1.01	1.01	1.05	1.01	1.01	21
4.26	6.50	7.48	1.52	1.15	1.75	4408	6370	7231	1.00	1.00	1.01	1.05	1.01	1.01	22
3.36	6.00	7.48	1.79	1.25	2.23	3472	5886	7231	1.00	1.00	1.01	1.03	1.00	1.01	23
2.68	5.51	7.48	2.05	1.36	2.79	2777	5402	7231	1.00	1.00	1.01	1.02	1.00	1.01	24
2.16	5.01	7.48	2.32	1.49	3.46	2237	4915	7231	1.00	0.99	1.01	1.01	0.99	1.01	25
1.75	4.51	7.48	2.59	1.66	4.29	1805	4425	7231	1.00	0.99	1.01	1.00	0.98	1.01	26
1.40	4.01	7.48	2.86	1.87	5.33	1452	3931	7231	1.00	0.98	1.01	0.98	0.97	1.01	27
1.12	3.50	7.48	3.13	2.13	6.67	1159	3435	7231	1.00	0.96	1.01	0.96	0.95	1.01	28
7.70	8.50	8.73	1.10	1.03	1.13	7969	8648	8841	1.00	1.00	1.00	1.02	1.00	1.00	29
6.67	8.06	8.50	1.21	1.05	1.27	6901	8082	8458	1.00	1.01	1.01	1.03	1.01	1.01	30
5.62	7.38	8.00	1.31	1.08	1.42	5810	7307	7840	1.00	1.01	1.01	1.04	1.01	1.01	31
3.98	6.07	6.99	1.53	1.15	1.76	4115	5893	6674	1.00	1.01	1.01	1.06	1.01	1.02	32
2.68	4.80	6.00	1.80	1.25	2.24	2768	4578	5592	1.00	1.00	1.01	1.07	1.01	1.02	33
1.85	3.82	5.22	2.07	1.36	2.82	1912	3593	4780	1.00	1.00	1.01	1.07	1.00	1.03	34
1.30	3.05	4.60	2.35	1.51	3.54	1342	2831	4149	1.00	1.00	1.01	1.06	0.99	1.03	35
0.92	2.42	4.09	2.63	1.69	4.45	950	2226	3644	1.00	0.99	1.01	1.04	0.98	1.03	36
0.65	1.90	3.66	2.92	1.92	5.62	673	1736	3231	1.00	0.99	1.01	1.02	0.96	1.03	37
0.46	1.47	3.30	3.22	2.24	7.21	473	1335	2887	1.00	0.98	1.01	0.98	0.92	1.04	38
3.21	6.50	8.73	2.02	1.34	2.72	3320	6607	8841	1.00	1.00	1.00	1.00	1.00	1.00	39
3.11	6.31	8.50	2.03	1.35	2.74	3213	6327	8458	1.00	1.00	1.01	1.01	1.00	1.01	40
2.89	5.91	8.00	2.05	1.35	2.77	2989	5855	7840	1.00	1.00	1.01	1.02	1.00	1.01	41
2.49	5.13	6.99	2.06	1.36	2.80	2578	4986	6674	1.00	1.00	1.01	1.03	1.00	1.02	42
2.12	4.39	6.00	2.07	1.37	2.83	2191	4183	5592	1.00	1.00	1.01	1.05	1.00	1.02	43
1.85	3.82	5.22	2.07	1.36	2.82	1912	3593	4780	1.00	1.00	1.01	1.07	1.00	1.03	44
0.27	0.56	0.79	2.07	1.40	2.91	632	1187	1622	1.00	1.00	1.00	1.07	1.00	1.03	45
3.54	7.32	9.99	2.07	1.36	2.82	2058	3868	5145	1.00	1.00	1.01	1.07	1.00	1.03	46
7.98	16.52	22.54	2.07	1.36	2.82	2064	3879	5160	1.00	1.00	1.01	1.07	1.00	1.03	47
14.21	29.40	40.13	2.07	1.36	2.82	2067	3885	5168	1.00	1.00	1.01	1.07	1.00	1.03	48
22.23	45.99	62.77	2.07	1.36	2.82	2069	3889	5174	1.00	1.00	1.01	1.07	1.00	1.03	49
32.02	66.26	90.43	2.07	1.36	2.82	2070	3890	5176	1.00	1.00	1.02	1.07	1.00	1.03	50
2.16	5.02	7.48	2.32	1.49	3.46	2238	4917	7233	1.00	1.00	1.01	1.05	1.00	1.01	25'
1.75	4.52	7.48	2.59	1.66	4.29	1806	4427	7235	1.00	1.00	1.01	1.04	1.00	1.01	26'
1.41	4.01	7.49	2.86	1.87	5.33	1454	3935	7240	1.00	1.00	1.01	1.03	0.99	1.01	27'
1.12	3.49	7.45	3.13	2.13	6.67	1155	3422	7204	1.00	0.99	1.01	1.02	0.98	1.01	28'
1.30	3.05	4.60	2.35	1.51	3.54	1342	2831	4149	1.00	1.01	1.01	1.15	1.03	1.03	35'
0.92	2.42	4.09	2.63	1.69	4.45	950	2226	3644	1.00	1.01	1.01	1.16	1.03	1.03	36'
0.65	1.90	3.66	2.92	1.92	5.62	673	1736	3231	1.00	1.01	1.01	1.17	1.03	1.03	37'

Notes for Table B1:

1. Cantilever height. Darker values are larger.
2. Horizontal load selected to produce demand-to-capacity ratio of 1.0 for the IAM. Darker values are larger within each group.
3. Notional load N determined using Equation 18.
4. Axial force in the stability column. Darker values are larger within each group.
5. The τ_b parameter for the stability column determined using Equation 3. Values less than 1.0 are highlighted.
6. The ratio for the stability column, with the Euler buckling load P_{e1} determined in the plane of bending per AISC *Specification* Equation A-8-5. Darker values are larger.
7. Axial force in the leaning column. Darker values are larger within each group.
8. The sum of the axial forces on the stability column and on the leaning column ($P_{story} = P_A + P_B$).
9. $R_M = 1 - 0.15P_A/P_{story}$, AISC *Specification* Equation A-8-8. Darker values are smaller.
10. The B_2 factor determined using Equation 12. Darker values are larger.
11. The B_3 factor determined using Equation 13. Darker values are larger.
12. Product of B_2 and B_3. Darker values of B_2 are larger.
13. Force amplification factor using Equation 25.
14. Force amplification factor for DM using Equation 26.
15. Displacement amplification factor using Equation 28.
16. Displacement amplification factor for DM using Equation 29.
17. First-order displacement, Δ_1, determined using Equation 33.
18. Second-order displacement, Δ_2, determined using Equation 34.
19. Second-order displacement using DM, Δ_{DM}, determined using Equation 35.
20. Ratio of second-order to first-order displacements (Equation 16).
21. The B'_3 factor determined using Equation 17.
22. Product of B'_2 and B'_3.
23. First-order overturning moment M_{FOA} determined using Equation 30.
24. Second-order overturning moment M_{SOA} determined using Equation 31.
25. Second-order overturning moment for DM M_{DM} determined using Equation 32.
26. Stability-column interaction check using AISC *Specification* Equations H1-1a and H1-1b, using PA and M_{DM}. Values are 1.00 because the lateral force H is selected to produce this value.
27. Stability-column interaction check using AISC *Specification* Equations H1-1a and H1-1b, using P_A and B_3M_{SOA}.
28. Stability-column interaction check using AISC *Specification* Equations H1-1a and H1-1b, using P_A and B'_3M_{SOA}.
29. Ratio of amplified first-order moment to DM moment ($B_2B_3M_{FOA}/M_{DM}$).
30. Ratio of amplified second-order moment to DM moment (B_3M_{SOA}/M_{DM}).
31. Ratio of amplified second-order moment to DM moment (B'_3M_{SOA}/M_{DM}), Equation 17.

Figure B1 shows demand-to-capacity ratios from selected cases: 21–28, in which the axial force in column B is varied such that there is a range of B_2; 39–44, in which the axial forces in column A and column B are varied such that there is range of axial load ratios P_A/P_{e1} and $B_2 = 2.0$ is maintained; and 29–38, in which the axial force in column A is varied such that there is a range of both P_A/P_{e1} and B_2. The graphs show that for values of $B_2 \le 2.0$, the IAM can be used to approximate the DM. Above $B_2 = 2.0$, there is the potential for underestimation of demands if the IAM is used with an approximate second-order analysis. Use of the IAM with B'_3 determined using the results of an explicit second-order analysis from Equation 17 does not show this unconservatism for the cases examined.

Based on these results, the IAM provides results consistent with the DM for the range proposed ($B_2 \le 2.0$).

Reduction of Error by Use of More Exact Formulae

At the bottom of Table B1, cases 25–28 and 35–37 (for which the IAM produced unconservative results for $B_2 > 2.0$) are reanalyzed. (These are presented as cases 25'–28' and 35'–37'.) In these revised cases, a more theoretically correct equation is used for the parameter R_M based on work by LeMessurier (1977):

$$R_M = 1 - \left[1 - \frac{\left(\pi/2\right)^2}{3}\right]\frac{P_A}{P_{Story}} = \frac{\pi^2/12\,P_A + P_B}{P_A + P_B} \tag{36}$$

The results show the unconservative error eliminated for all but one case (28'), in which the unconservative error is reduced to 1% (compared to 3% for case 28). The majority of the error in these cases can therefore be attributed to the simplified equation for R_M in AISC *Specification* Appendix 8, rather than to an inherent deficiency of the IAM.

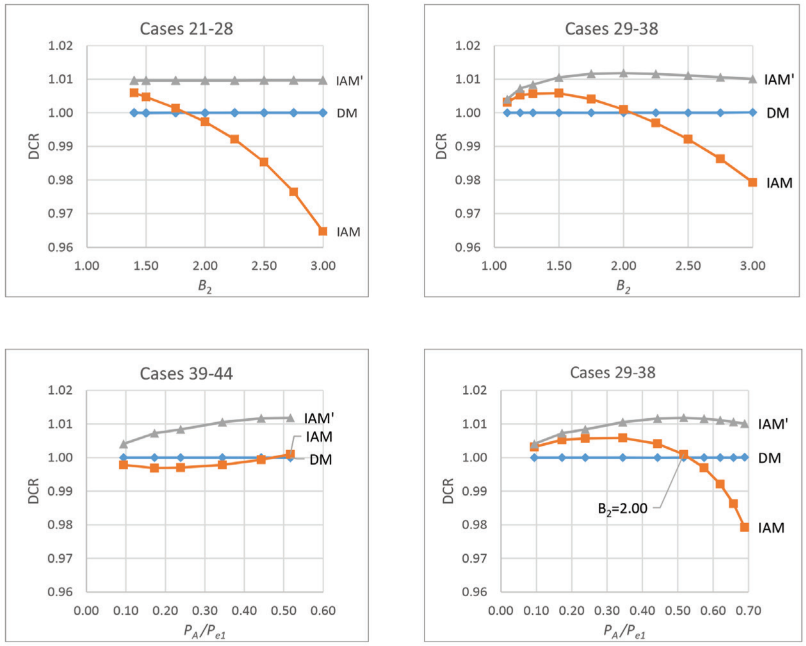

Fig. B1. Selected results from parametric study.

APPENDIX C
EXAMPLE APPLICATION

The following example illustrates the application of the IAM for stability design of a symmetrical eight-story building with moment frames in one direction. Figure C1 shows the plan. The configuration and loads were selected to result in relatively high second-order effects for a building of this height. For simplicity, only two load combinations are considered: one with service-level lateral loads and one with strength-level lateral loads. Two-dimensional analyses of one frame (on line D) are performed. The LRFD method is employed in this example.

The relevant design data are presented in Tables C1 (general design information), C2 (drift evaluation information), and C3 (strength evaluation information). Gravity loads and lateral loads are both given for the entire building, with 50% of each being tributary to the frame being designed. (The tributary gravity load includes that resisted by leaning columns.) Loads directly on the moment-frame columns include a large cladding load. For simplicity, vertical and lateral load values are unrealistically uniform. Factors B_2 and B_3 are calculated in advance of any design, assuming the frames have exactly the stiffness required to meet the drift limit for the appropriate lateral loads, including consideration of second-order effects.

Table C2 contains the information used for the serviceability evaluation. Wind loads are based on a uniform pressure of 14.33 psf, and a tall parapet is assumed, resulting in identical wind forces at each story. (Seismic loads are not considered.) The serviceability drift limit is $L/400$; this is a limit on the second-order drift. The required stiffness is the first-order stiffness that will result in the desired second-order drift (the drift limit) in the presence of the vertical loads. (See note 3. The derivation of this required stiffness which is too large to include in this example.) This required stiffness is used to compute the first-order drift Δ_1, which is then used to compute B_2. Although second-order effects are addressed in these calculations by means of this determination of required stiffness, no design has yet been performed, and only the predesign information is used.

Similarly, Table C3 is constructed using predesign information. Wind loads are based on a uniform pressure of 21.5 lb/ft^2. The first-order lateral stiffness of the system is assumed to be that determined in Table C2 to meet the drift limit in the presence of the serviceability vertical loads. First-order drifts are computed using that stiffness and strength-level lateral loads. (These values are upper bounds based on the assumption of minimum permissible stiffness for serviceability requirements.) The strength-level amplifiers B_2 are then computed based on the vertical loads considered in the strength evaluation. These B_2 values are then used to determine B_3 values, assuming $\alpha P_r/P_{ns} \leq 0.5$.

A frame is designed and analyzed using analysis software to meet the drift limits in Table C2 with the corresponding vertical and lateral loads. Members are selected (and revised) to result in drifts at or just below the drift limit. Second-order effects are considered, with members appropriately meshed to capture P-δ effects. Results are shown in Table C4. Under "Service," the ratio of the second-order drift under service-level lateral loads is compared to the drift limit; these ratios are reasonably close to 1.0 for most of the stories, indicating a design optimized for drift control. The design is shown in Figure C2. Column sizes shown are repeated for all moment-frame columns.

The frame is then analyzed using the loads from Table C3 for strength and stability; lateral loads are amplified with the maximum value of B_3. Results of the strength and stability evaluation are shown in Table C4. These include the beam and column demand-to-capacity ratios at each story determined using analysis software. As the demand-to-capacity ratios are below 1.0, no redesign is required; member selection to meet drift limits governs the design, and the strength and stability evaluation confirms the acceptability of the design.

The frame is reanalyzed using the DM, and beam and column demand-to-capacity ratios are computed. Additionally, to better understand the simplifications and conservatism of the process employed in this example, B_2' (determined from the analysis using Equation 16) and B_3' (using Equation 17) are presented, although these values are not used in the evaluation.

Table C4 presents the results of these analyses and design evaluations. The results show a reasonably close correlation between the IAM and the DM design checks. Typical of many moment frame buildings, the member sizes are selected to meet the drift limits, and the minor differences in the strength-design demand-to-capacity ratios do not affect the economy of the design. The differences in demand-to-capacity ratios are due to two conservative measures. First, the use of the allowable drift in determining the B_3 amplifier overestimates the value (although it simplifies the design process); this could be avoided by using the analysis-based factor B_3' form, Equation 17. Second, the use of the largest value of B_3 for all stories overestimates the effect at upper stories. Level-by-level application of B_3 (or B_3') could address this, as discussed under "Application."

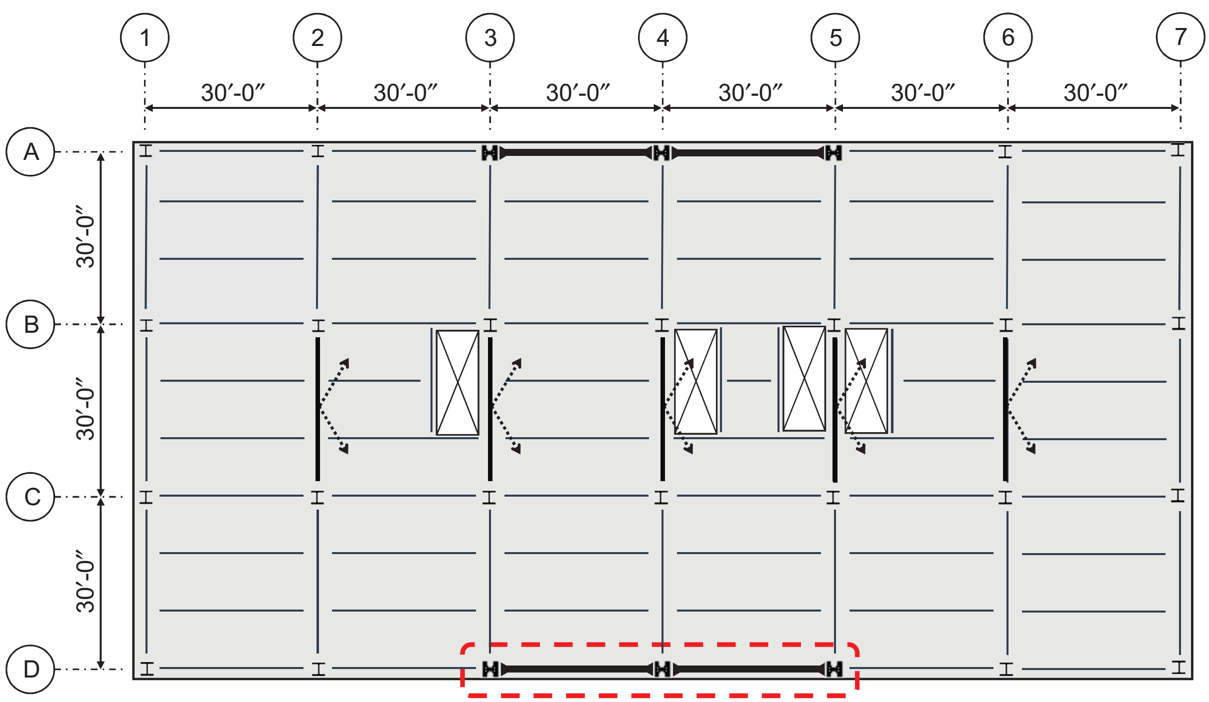

Fig. C1. Typical plan of example building.

Table C1. Pre-Analysis Design Information (General)					
Level	**L (in.)**	**Dead Load (kips)**	**Live Load (kips)**	**P_{mf}/P_{story}**	**R_M**
8	180	2000	0	0.275	0.959
7	180	2000	1600	0.275	0.959
6	180	2000	1600	0.275	0.959
5	180	2000	1600	0.275	0.959
4	180	2000	1600	0.275	0.959
3	180	2000	1600	0.275	0.959
2	180	2000	1600	0.275	0.959
1	180	2000	1600	0.275	0.959
Notes		1	1	1	2

Notes:
1. Given information.
2. $R_M = 1 - 0.15 P_{mf}/P_{story}$ using AISC *Specification* Equation A-8-8.

Table C2. Pre-Analysis Design Information (Drift Evaluation)						
Level	**$H_{service}$ (kips)**	**$\Delta_{allowable}$ (in.)**	**P_{story} (kips)**	**$K_{required}$ (kip/in.)**	**Δ_1 (in.)**	**B_2**
8	20.0	0.450	2,000	56	0.357	1.26
7	40.0	0.450	4,400	114	0.350	1.29
6	60.0	0.450	6,800	173	0.347	1.30
5	80.0	0.450	9,200	231	0.346	1.30
4	100.0	0.450	11,600	289	0.345	1.30
3	120.0	0.450	14,000	348	0.345	1.30
2	140.0	0.450	16,400	406	0.345	1.31
1	160.0	0.450	18,800	464	0.344	1.31
Notes	1	1	2	3	4	5

Notes.
1. Given information.
2. P_{story} is based on 1.0 times the dead load plus 0.25 times the live load for the service drift check.
3. The required stiffness is based on the lateral load and the drift limit, considering the second-order effect: $K_{required} = H/\Delta_{allowable} + P_{story}/R_M L$.
4. $\Delta_1 = H/K_{required}$
5. The B_2 factor using Equation 12 based on P_{story} for the drift load combination (and on Δ_1 and $H_{service}$).

Table C3. Pre-Analysis Design Information (Strength Evaluation)									
Level	H (kips)	F (kips)	$K = K_{required}$ (kip/in.)	Δ_1 (in.)	P_{story} (kips)	B_2	θ	B_3	B_3F (kips)
8	30	30.0	56	0.535	2,400	1.33	0.24	1.09	33.78
7	60	30.0	114	0.525	5,600	1.40	0.27	1.11	33.78
6	90	30.0	173	0.521	8,800	1.42	0.28	1.12	33.78
5	120	30.0	231	0.519	12,000	1.43	0.29	1.12	33.78
4	150	30.0	289	0.518	15,200	1.44	0.29	1.12	33.78
3	180	30.0	348	0.518	18,400	1.44	0.29	1.12	33.78
2	210	30.0	406	0.517	21,600	1.45	0.30	1.13	33.78
1	240	30.0	464	0.517	24,800	1.45	0.30	**1.13**	33.78
Notes	1	1	2	3	4	5	6	7	8

Notes.
1. Given information.
2. It is conservatively assumed that the stiffness is exactly equal to that required to meet the drift limit.
3. The first-order displacement corresponding to the assumed stiffness and the applied loads.
4. P_{story} is based on 1.2 times the dead load plus 0.5 times the live load for the strength check.
5. The B_2 factor determined using Equation 12 based on P_{story} for the strength load combination.
6. The stability coefficient θ calculated per Equation 1.
7. The B_3 factor determined using Equation 15. The maximum value is highlighted.
8. The amplified forces B_3F to be used in a second-order analysis. The maximum value of B_3 is used at all levels so that overturning effects at lower levels are not underestimated.

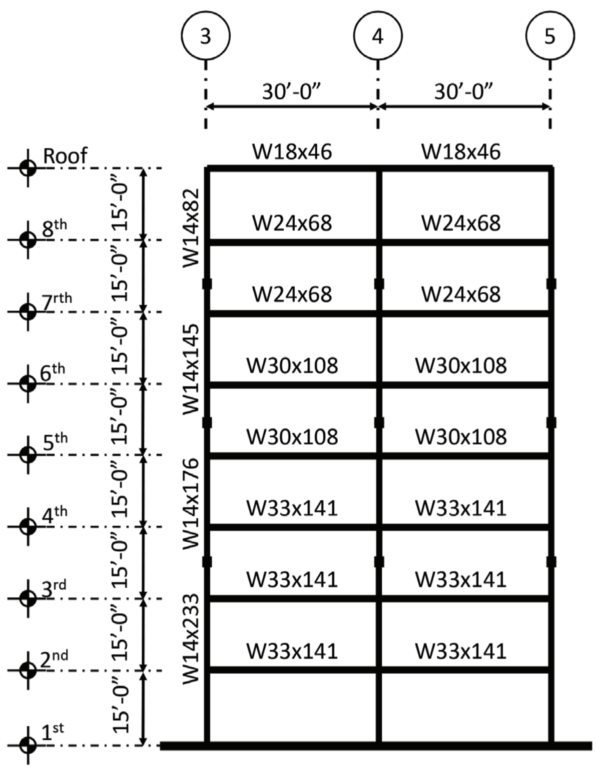

Fig. C2. Moment frame elevation.

Table C4. Analysis Results and Design Checks													
	Service	**Strength Evaluation**											
								IAM	**DM**	**Ratio**	**IAM**	**DM**	**Ratio**
	Δ_2/Δ_{all}	$\alpha P_r/P_{ns}$	τ_b	Δ_1 (in.)	Δ_2 (in.)	B'_2	B'_3	**Beam (DCR)**	**Beam (DCR)**	**IAM/ DM**	**Column (DCR)**	**Column (DCR)**	**IAM/ DM**
Level													
8	0.61	0.07	1.00	0.39	0.49	1.25	1.07	0.13	0.12	1.06	0.14	0.14	1.04
7	0.96	0.16	1.00	0.59	0.79	1.35	1.10	0.21	0.20	1.04	0.45	0.45	1.01
6	0.89	0.15	1.00	0.54	0.74	1.36	1.10	0.31	0.30	1.02	0.25	0.25	1.02
5	0.95	0.21	1.00	0.57	0.79	1.38	1.11	0.27	0.27	1.02	0.45	0.45	1.01
4	0.93	0.22	1.00	0.56	0.78	1.39	1.11	0.32	0.31	1.02	0.47	0.47	1.01
3	1.00	0.28	1.00	0.60	0.84	1.41	1.11	0.29	0.29	1.01	0.58	0.58	1.01
2	0.93	0.25	1.00	0.56	0.78	1.38	1.11	0.33	0.32	1.01	0.50	0.50	1.01
1	0.72	0.29	1.00	0.45	0.59	1.32	1.09	0.33	0.32	1.03	0.65	0.64	1.02
Notes	1	2	3	4	5	6	7	8	9	10	11	12	13

Notes.
1. Second-order serviceability drift divided by drift limit.
2. Gridline 5 column $\alpha P_r/P_{ns}$ ratio.
3. Gridline 5 column τ_b parameter.
4. Δ_1 from a first-order analysis using strength-level loads.
5. Δ_2 from a second-order analysis using strength-level loads.
6. The B'_2 factor determined using Equation 16 based on Δ_1 and Δ_2 from the analysis.
7. The B'_3 factor determined using Equation 17 based on B'_2.
8. Beam demand-to-capacity ratio from IAM second-order analysis amplified by B_3 using AISC *Specification* Equations H1-1a and H1-1b axial-flexure interaction equations.
9. Beam demand-to-capacity ratio from DM second-order analysis with reduced stiffness using AISC *Specification* Equations H1-1a and H1-1b axial-flexure interaction equations.
10. Ratio of IAM to DM beam demand-to-capacity ratio.
11. Gridline 5 column demand-to-capacity ratio from IAM second-order analysis amplified by B_3 using AISC *Specification* Equations H1-1a and H1-1b axial-flexure interaction equations.
12. Gridline 5 column demand-to-capacity ratio from DM second-order analysis with reduced stiffness using AISC *Specification* Equations H1-1a and H1-1b axial-flexure interaction equations. *H* is used for the lateral load.
13. Ratio of IAM to DM gridline 5 column demand-to-capacity ratio.