Updated Equivalent Axial Load Method for Design of Steel Beam-Columns

MATHEW REYNOLDS and CHIA-MING UANG

ABSTRACT

The equivalent axial load method is a design aid that aims to select candidate wide-flange beam-columns with minimal iteration. The method itself was described in previous editions of the AISC *Steel Construction Manual* but has been removed in recent editions. An expanded version of the tables for sizing of both shallow and deep beam-columns based on the latest edition of the AISC *Manual* is presented. Several examples of uniaxial and biaxially loaded beam-columns are provided to demonstrate the effectiveness of the method. The method is validated using a programmed heuristic that designed beam-columns from randomly generated scenarios.

Keywords: beam-column, equivalent axial load, design.

INTRODUCTION

The equivalent axial load method was first developed by Burgett (1973) to efficiently select a candidate shape for the design of steel W-shape beam-columns. In this method, the required moment is first converted into an equivalent axial load component by using the "*m*" factor before it is added to the required axial load. Together with the column design strength table in the AISC *Steel Construction Manual*, a designer can use this equivalent axial load to select a candidate shape. The goal is that the candidate shape is either the most economical shape or very close to it. A table of *m* values was adopted into Part 3 (Column Design) of the 9th edition of the AISC *Allowable Stress Design* (ASD) *Steel Construction Manual* (1989) and the 1st edition of the AISC *Load and Resistance Factor Design* (LRFD) *Steel Construction Manual* (1986). The LRFD version of the table was found to produce erroneously large equivalent axial loads and was subsequently updated by Uang et al. (1990); these corrected values were incorporated into the 2nd edition of the AISC LRFD *Manual* (1994). Because column tables in these editions list shallow wide-flange shapes up to W14 shapes, *m* values were only derived for W-shapes up to a nominal depth of 14 in.

The 3rd edition of the AISC LRFD *Manual* (2001) saw the development of Part 6 for members subject to combined loading. This edition of the AISC *Manual* elected to adopt a method of using reciprocal coefficients of *b*, *m*, and *n* developed by Aminmansour (2000):

$$b = \frac{1}{\phi P_n} \tag{1}$$

$$m = \frac{8}{9\phi_b M_{nx}} \tag{2}$$

$$n = \frac{8}{9\phi_b M_{ny}} \tag{3}$$

These factors represent the reciprocal of the axial, major-axis bending, and minor-axis bending design strengths, respectively. Note that the *m* in this method does not relate to the *m* from the equivalent axial load method. It was proposed that an initial trial shape be evaluated by using Table 6-1 of the 3rd edition of the AISC LRFD *Manual*, which included median values of the reciprocals for each depth group (from W4 to W40 shapes). Applicable to shallow shapes and shapes deeper than W14, the proposed method to select a trial shape would be to use either the median *m* or *b* and solve for the required *b* or *m*. There was no formal guidance provided to determine the required depth of shape other than to focus on deep shapes when the moment demands dominate.

In the 13th (2005) and 14th (2011) editions of the unified AISC ASD and LRFD *Steel Construction Manual*, the method for selecting a trial shape by Aminmansour was removed and the tables of reciprocals became Table 6-1, where the reciprocals were renamed p, b_x, and b_y. The latest edition, the 15th, of the AISC *Manual* (AISC, 2017) presents a new way of organizing the beam-column design table. In this table, the information from the column design tables (Table 4-1) and beam design tables (Table 3-10) are presented side-by-side in Table 6-2. However, no guidance

Mathew Reynolds, P.Eng., Graduate Student Researcher, Department of Structural Engineering, University of California, San Diego, La Jolla, CA. Email: mpreynol@ucsd.edu (corresponding)

Chia-Ming Uang, PhD, Professor, Department of Structural Engineering, University of California, San Diego, La Jolla, CA. Email: cmu@ucsd.edu.

Paper No. 2018–10

is provided in the *Manual* on the selection of an initial trial beam-column W-shape for design purposes. Designers may resort to other resources—for example, a simplified table of m values for use in the equivalent axial load method in Geschwindner et al. (2017).

SCOPE

Using Table 6-2 in the 15th edition of the AISC *Manual*, it is believed that an updated m table for the equivalent axial load method will provide a straightforward and robust methodology for selecting a candidate wide-flange beam-column. This updated table will also encourage the use of shapes deeper than W14 (i.e., beam-type shapes) for beam-columns through the inclusion of m values up to W36 shapes. This is made possible through inclusion of shapes beyond W14 being tabulated in AISC *Manual* Table 6-2. In addition, for the first time an equation has been developed to guide users on selecting an appropriate depth of W-shaped beam-columns subjected to major-axis bending.

DERIVATION

The design of W-shape beam-columns is covered in AISC *Specification* Section H (2016). The design of beam-columns is governed by one of two interaction equations:

$$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0 \text{ when } \frac{P_r}{P_c}\geq 0.2$$

(*Spec*. Eq. H1-1a)

$$\frac{P_r}{2P_c}+\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0 \text{ when } \frac{P_r}{P_c}< 0.2$$

(*Spec*. Eq. H1-1b)

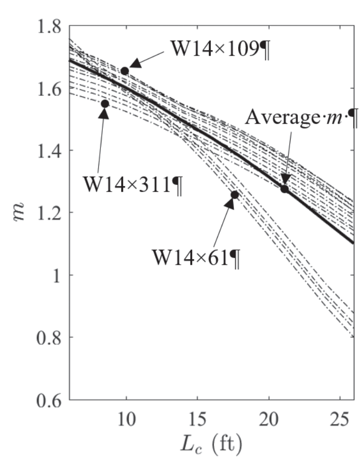

Fig. 1. Values of m *for W14 members.*

The equivalent axial load method rewrites these interaction equations into Equations 4a and 4b.

$$P_{eq}=P_r+mM_{rx}+muM_{ry}\leq P_c \text{ when } \frac{P_r}{P_c}\geq 0.2 \tag{4a}$$

$$P_{eq}=\frac{P_r}{2}+\frac{9}{8}mM_{rx}+\frac{9}{8}muM_{ry}\leq P_c \text{ when } \frac{P_r}{P_c}< 0.2 \tag{4b}$$

The left-hand side of these equations represents the equivalent axial load, P_{eq}. The coefficients m and u are defined in Equations 5 and 6.

$$m=\frac{\frac{8}{9}P_c}{M_{cx}} \tag{5}$$

$$u=\frac{M_{cx}}{M_{cy}} \tag{6}$$

Because the shapes are assumed to be compact, the design flexural strength in the major and minor directions are governed by AISC *Specification* Sections F2 and F6, respectively. Similarly, the design axial strength is governed by AISC *Specification* Section E3 using the least radius of gyration, r_y. Note that ϕ_c for computing P_c was 0.85 in the 1st edition of the AISC LRFD *Specification*, but was changed to and has remained 0.9 since the 13th edition.

The first step in deriving m values is to compute their value for every W-shape over a range of lengths. The range of W-shapes used for the derivation is limited to those which are classified as compact for F_y equal to 50 ksi. The values of m are then averaged across each nominal depth increment to provide an average m for each depth. For example, the m value for the W14 group is averaged across all weights between W14×61 to W14×311 (see Figure 1). Table 1 shows the computed values of m when the moment gradient adjustment factor for lateral-torsional buckling (LTB), C_b, is equal to 1.0. Table 2 shows another set of m values based on the major-axis flexural strength reaching the nominal plastic moment, a condition that is a function of L_b and C_b values. This condition is automatically satisfied in the plastic LTB region—that is, $L_b \leq L_p$. In the inelastic LTB region—that is, $L_b < L_p < L_r$, C_b needs to be sufficiently large ($C_b = 1$ at $L_b = L_p$ and $C_b = 1.43$ at $L_b = L_r$). An even larger value of C_b (>1.43) is required to satisfy this condition in the elastic LTB region ($L_b > L_r$). Table 2 shows in light gray the values that are different from those in Table 1. Values in either table assume a yield stress equal to 50 ksi. In lieu of providing additional tables, it is found that the increase to 65 ksi yield stress results in an approximate reduction factor of 0.95 to these values.

Unless limited by factors like architectural limitations, choosing the nominal shape depth of an economical

Table 1. Values of m and u (F_y = 50 ksi[a] for C_b = 1.0)										
	Values of m									Values of u
L_c (ft)	8	10	12	14	16	18	20	22	24+	
W8	2.7	2.5	2.3	2	1.8	1.6	1.3	1.2	1	2.3
W10	2.2	2.1	2	1.9	1.8	1.6	1.4	1.3	1.2	2.3
W12	1.9	1.8	1.7	1.7	1.6	1.5	1.4	1.3	1.2	2.3
W14	1.6	1.6	1.5	1.5	1.4	1.4	1.3	1.2	1.2	2.3
W16	1.4	1.4	1.3	1.2	1.1	1	0.9	0.9	0.8	4.1
W18	1.3	1.2	1.2	1.1	1	1	0.9	0.8	0.7	4.1
W21	1.1	1.1	1	1	1	0.9	0.8	0.8	0.7	4.3
W24	1	1	0.9	0.9	0.9	0.8	0.8	0.7	0.7	4.4
W27	0.9	0.9	0.8	0.8	0.8	0.8	0.7	0.7	0.7	4.6
W30	0.8	0.8	0.8	0.8	0.7	0.7	0.7	0.7	0.6	4.8
W33	0.8	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.6	5.1
W36	0.7	0.7	0.7	0.6	0.6	0.6	0.6	0.5	0.5	6.1

[a] For F_y = 65 ksi, multiply m value by 0.95.

Table 2. Values of m and u for High Moment Gradient (F_y = 50 ksi, $M_{cx} = M_p$)[a]										
	Values of m									Values of u
L_c (ft)	8	10	12	14	16	18	20	22	24+	
W8	2.6	2.3	2.1	1.8	1.5	1.3	1	0.9	0.7	2.3
W10	2.2	2.1	1.9	1.7	1.6	1.4	1.2	1	0.9	2.3
W12	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1	2.3
W14	1.6	1.6	1.5	1.5	1.4	1.3	1.2	1.1	1	2.3
W16	1.4	1.3	1.2	1	0.9	0.8	0.7	0.6	0.5	4.1
W18	1.3	1.2	1.1	1	0.9	0.8	0.7	0.6	0.6	4.1
W21	1.1	1.1	1	0.9	0.9	0.8	0.7	0.6	0.6	4.3
W24	1	1	0.9	0.9	0.8	0.7	0.7	0.6	0.6	4.4
W27	0.9	0.9	0.8	0.8	0.8	0.7	0.7	0.6	0.6	4.6
W30	0.8	0.8	0.8	0.7	0.7	0.7	0.6	0.6	0.5	4.8
W33	0.8	0.7	0.7	0.7	0.6	0.6	0.6	0.5	0.5	5.1
W36	0.7	0.7	0.7	0.6	0.6	0.5	0.5	0.5	0.4	6.1

[a] For convenience, values different from Table 1 are shown in gray.

beam-column is not straightforward. Previous editions of the m table have included first approximation values that would assist in choosing a nominal depth range. Based on the nominal shape depth of the first trial shape, the user then can update the equivalent axial load based on the subsequent approximation m values for an improved shape. This approach may result in conservative designs as the efficiency of moving to a deeper shape when under the influence of relatively high moment may not be realized unless the designer checks many different shapes. This effect can readily be observed by comparing the gradient of the equivalent axial load for a given set of required axial load and moment as one selects incrementally deeper shapes. For example, consider a 16-ft-long beam-column subjected to an axial load of 500 kips and a major-axis bending moment of 700 kip-ft. Using Table 1, P_{eq} equals 1,480 kips when considering a W14 shape compared to 1,060 kips when considering a W27 shape. A decrease of 30% of equivalent axial load is realized by moving to a deeper W-shape.

Instead of suggesting a first approximation for the value of m, an approximate depth of shape can be computed from Equation 7, which has been derived based on curve-fitting randomly generated design scenarios compared with the most efficient W-shape as determined from an exhaustive analysis. The exhaustive analysis determines the most efficient wide-flange shape after performing the appropriate axial and bending interaction check using AISC *Specification* Equation H1-1a or H1-1b on every shape in the AISC W-shape database.

$$D = 1.2\sqrt[4]{P_u L_c} + 1.1\sqrt[4]{M_u L_c} + \frac{\sqrt{P_u M_u}}{3L_c^2} \quad (7)$$

where the units of M_u and P_u are in kip-ft and kips, L_c in ft, and D in inches.

The first term, $1.2\sqrt[4]{P_u L_c}$, represents the most efficient depth of column for a given axial load and length. The second term, $1.1\sqrt[4]{M_u L_c}$, represents the additional depth required for a given major-axis flexural demand. The last term is a correction factor that dominates for shorter columns with lengths less than 14 ft. Considering the preceding example, the estimated depth as computed using Equation 7 is 23.4 in. Therefore, the closest depth available of 24 in. can be chosen for a trial W-shape. The most efficient W-shape, determined by exhaustive analysis, is a W24×131. This step (using Equation 7) is only required if the designer has no other considerations on the depth of the shape.

Limitations

The key limitation to the values of m presented in Tables 1 and 2 is the assumption that the unbraced length for bending, L_b, is equal to the effective length of the column for minor-axis buckling. Two common situations where this limitation is violated are:

1. The beam-column has some end fixity or minor-axis bracing.
2. The major-axis flexural buckling governs such that $L_{cy\ eq} > L_{cy}$, where $L_{cy\ eq}$ represents the equivalent length in the minor-axis as determined using AISC *Manual* Equation 4–1 (2017), $L_{cy\ eq} = L_{cx}/(r_x/r_y)$.

In both situations, it is recommended to conservatively use the unbraced length for bending as the length to use with the m table to start the design process. More iterations using AISC *Specification* Equation H1-1a or H1-1b may be required to accurately account for these effects.

Second-Order and Biaxial Bending Considerations

Beam-columns may be subject to second-order amplification of the applied moments. Using B_1 and B_2 from the approximate second-order analysis in AISC *Specification* Appendix 8 results in an iterative procedure because the amplification of flexural demands depends on the flexural stiffness. Some designers suggest that a value of B_1 equal to 1.1 be chosen for trial selection for sidesway inhibited beam-columns. This value is found to be adequate for conservative estimation for beam-columns of moderate length. As a design aid, a similar procedure that was used to develop m values was implemented to develop approximate B_1 values for the amplification of major-axis flexural demands. To derive the B_1 value, Equation 8 was used by assuming a factored axial load of $0.25A_gF_y$:

$$B_1 = \frac{C_m}{1 - \dfrac{P_r}{P_{el}}} \quad (8)$$

where C_m is the equivalent uniform moment factor, and P_{el} is the elastic critical buckling strength in the plane of bending. The values of B_1 are then averaged across each nominal W-shape depth group (see Figure 2). Figure 3 shows the variation of B_1 for major-axis bending for each nominal depth group. For second-order amplification of sway effects or minor-axis bending, arbitrary values of B_1 and B_2 should be chosen based on experience.

For a beam-column with biaxial bending, a conversion factor, u, between minor- and major-axis design flexural strengths is required (see Equations 4 and 6). Previous editions of the AISC *Manual* included a single value of u in the column design table based on the ratio of plastic moment capacities. A similar procedure was used for the development of the B_1 design aid was implemented to produce values of u for each depth group for a range of lengths (see Figure 4). Note that the influence of length arises from the length dependence of major-axis LTB strength. The last column in Tables 1 and 2 shows the recommended u value for each nominal depth group; other than providing a complex

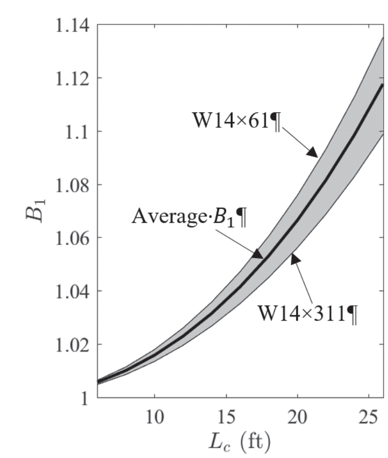

Fig. 2. B_1 *for W14 members (major axis).*

equation to reflect the dependence of member length, the recommended u value is based on the major-axis plastic moment for M_{cx}.

RECOMMENDED DESIGN PROCEDURE

1. For a given L_c, select an appropriate value of m from Table 1 (for C_b close to 1.0) or Table 2 (for C_b between 1.5 and 2.0). If the nominal depth of the shape is not predetermined, use Equation 7 to estimate a shape depth. Select or determine B_1 (Figure 2) and u (Tables 1 and 2 or Figure 4) if there is biaxial bending.
2. Calculate the equivalent axial load, $P_{eq} = P_r + mM_{rx} + muM_{ry}$. Use the first-order flexural demands and estimated values of B_1 and B_2 to calculate M_{rx} and M_{ry}.
3. If $P_r/P_{eq} < 0.2$, modify the equivalent axial load as $P_{eq} = \frac{P_r}{2} + \frac{9}{8}mM_{rx} + \frac{9}{8}muM_{ry}$.
4. Select a candidate shape from AISC *Manual* Table 6-2. Alternatively, AISC *Manual* Table 4-1 may be used for W-shapes with a nominal depth no greater than 14 in. If an efficient candidate shape cannot be found within the selected nominal depth group, move to step 5. Otherwise, proceed to step 6.
5. Consider shapes of the lesser or greater nominal depth group, and select new m, B_1, and u values. Repeat steps 2 through 4.
6. Verify the adequacy of the W-shape using the AISC *Specification* interaction Equations H1-1a or H1-b. This is easily performed using AISC *Manual* Table 6-2, which lists the design axial and flexural strength of each shape. If the shape does not satisfy the interaction formula or more efficiency is sought, additional iterations may be performed.

Several representative examples are included in the Appendix to illustrate the efficiency of the design procedure.

VALIDATION

To test the values contained in Tables 1 and 2, a design heuristic was programmed to compare the W-shape determined from the equivalent axial load method to the most efficient W-shape as determined from exhaustive analysis. The heuristic uses the depth estimate from Equation 7 to select the closest applicable nominal depth for initial shape selection. If the heuristic fails to determine a W-shape with a demand-capacity ratio (DCR) between 0.85 and 1.0 (see Equation 9) within the first nominal depth group selected, then the heuristic is permitted one additional adjacent nominal depth group for candidate selection. This DCR range was chosen to represent an adequate level of efficiency a designer might use in selecting a beam-column shape. The heuristic uses AISC *Manual* Table 6-2 to determine P_c. A tabulated version of Figure 3 is provided to the heuristic for an initial estimate of B_1. The error in the equivalent axial load method is then determined by comparing the weight of the shape as determined from the design heuristic and the most efficient shape determined by exhaustive analysis (see Equation 10).

$$DCR = \frac{P_{eq}}{P_c} \tag{9}$$

$$Error = \frac{W_{eq} - W_{act}}{W_{act}} \tag{10}$$

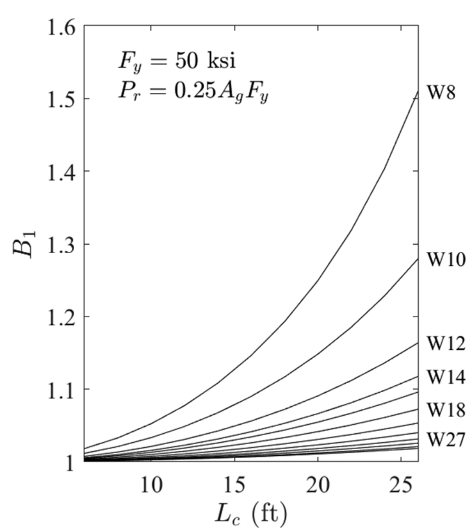

Fig. 3. Value of B_1 *(major-axis bending).*

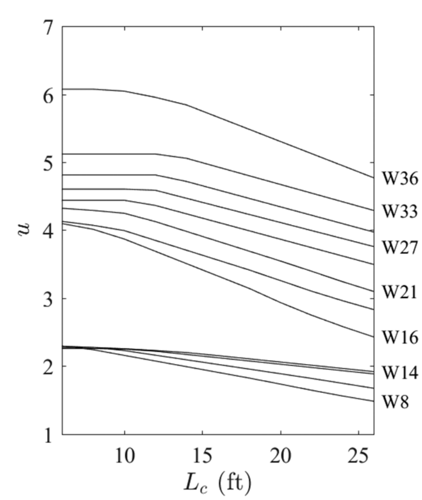

Fig. 4. Value of u.

where W_{eq} is the unit weight of the shape determined using the equivalent axial load method, and W_{act} is the unit weight of the most efficient shape for the given force demands.

Figure 5 shows the probability density function (PDF) of the error obtained after performing 10,000 randomly generated uniaxial design scenarios. The mean error was equal to +0.5%. Integrating the PDF results in a 90% probability of selecting a shape within 5% of the most efficient shape in terms of unit weight. Additionally, it was found that 77% of the time the shape was selected from the first nominal depth based on Equation 7. Otherwise, it was found to be within one adjacent group.

CONCLUSION

The equivalent axial load method consists of converting the minor- and major-axis bending moments into an equivalent axial load for adding to the applied axial load that a designer can use to compare against tabulated design axial strengths. This methodology simplifies the candidate selection of beam-columns by reducing the number of iterations a designer needs to perform while using the tabulated design values found in the AISC *Steel Construction Manual*. To use the method, a designer uses values of m found in Tables 1 or 2, which have been tabulated beyond shapes of 14-in. nominal depths. This updated and expanded m table in combination with AISC *Manual* Table 6-2 provides a powerful methodology in selecting candidate W-shapes. Once a candidate shape is selected, the governing interaction equation can be verified using AISC *Manual* Table 6-2. Potential inefficiencies are avoided by using Equation 7 to select the appropriate trial depth, which encourages the use of deeper and more efficient W-shapes for bending dominated beam-columns. Simplified u factors are provided in Tables 1 and 2 that can be used for biaxial bending design problems.

APPENDIX

Example 1. Select an economical W12 shape for a member with minor-axis effective length and unbraced beam length equal to 12 ft. The factored axial load and moments calculated from a first-order structural analysis are given as P_u = 200 kips, M_{u1} = 100 kip-ft, and M_{u2} = −200 kip-ft, where M_{u1} and M_{u2} are the major-axis moments at member ends. The negative sign of M_{u2} represents reverse-curvature bending. Assume minor-axis bending is negligible.

Solution:

1. Select m = 17 from Table 2. Because $C_m = 0.6 - 0.4(M_1/M_2) = 0.4$, we assume $B_1 = 1.0$ (Equation 8).

2. Calculate the equivalent axial load:

$$\begin{aligned} P_{eq} &= P_r + mM_{rx} + muM_{ry} \le P_c \qquad (4a) \\ &= 200 \text{ kips} + 1.7(200 \text{ kip-ft}) \\ &= 500 \text{ kips} \end{aligned}$$

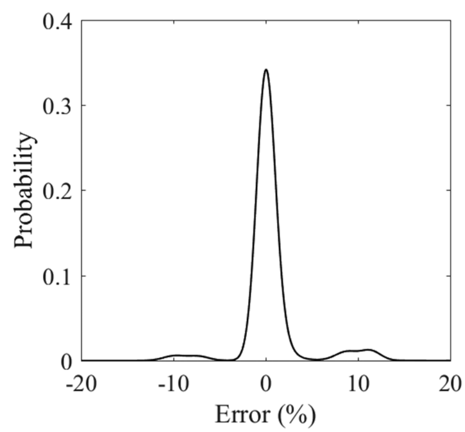

Fig. 5. Probability distribution of percentage error of the equivalent axial load method.

3. Because P_r/P_{eq} = 200 kips/500 kips = 0.4000 ≥ 0.2, the assumption of using Equation 4a was correct and no changes are required.

4. Using AISC *Manual* Table 6-2, select a W12×53 as the most economical shape with $\phi_c P_n$ = 549 kips.

5. No revision is required because an appropriate shape was found.

6. Check the appropriate interaction using AISC *Specification* Equation H1-1a and AISC *Manual* Table 6-2. Note that the moment gradient gives C_b = 2.17, which results in $\phi_b M_{nx}$ = 292 kip-ft (for L_b = 0 ft) rather than the value listed for L_b = 12 ft.

$$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0 \text{ when } \frac{P_r}{P_c}\geq 0.2 \qquad (\textit{Spec}.\text{ Eq. H1-1a})$$

$$\frac{200\text{ kips}}{549\text{ kips}}+\frac{8}{9}\left(\frac{200\text{ kip-ft}}{292\text{ kip-ft}}\right)=0.97<1.0 \qquad \textbf{o.k.}$$

Example 2. Select an economical W-shape for an 18-ft-long member in a braced frame assuming pinned end connections. The factored axial loads and moment calculated from a first-order structural analysis are given as P_u = 600 kips and M_u = 350 kip-ft due to a transverse loading. Assume minor-axis bending is negligible and C_b = 1.0.

Solution:

1. Use the first two terms of Equation 7 to estimate the depth (the third term is small for L_c > 12 ft).

$$1.2\sqrt[4]{P_u L_c}+1.1\sqrt[4]{M_u L_c}=22.0\text{ in.} \qquad \text{(from Eq. 7)}$$

Select the W21 group as the first estimate, and select m = 0.9 from Table 1. Because C_m = 1.0, the value of B_1 may be determined from Figure 3 as 1.03.

2. Calculate the equivalent axial load:

$$\begin{aligned} P_{eq} &= P_r + mM_{rx} + muM_{ry} \leq P_c \qquad (4a)\\ &= 600\text{ kips} + 0.9\left[350\text{ kip-ft}(1.03)\right]\\ &= 924\text{ kips}\end{aligned}$$

3. Because P_r/P_{eq} = 600 kips/924 kips = 0.650 ≥ 0.2, the assumption of using Equation 4a was correct and no changes are required.

4. Using AISC *Manual* Table 6-2, select a W21×111 as the most economical shape with $\phi_c P_n$ = 978 kips.

6. Check the appropriate interaction using AISC *Specification* Equation H1-1a and AISC *Manual* Table 6-2. The actual B_1 value is determined to be 1.04.

$$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0 \text{ when } \frac{P_r}{P_c}\geq 0.2 \qquad (\textit{Spec}.\text{ Eq. H1-1a})$$

$$\frac{600\text{ kips}}{978\text{ kips}}+\frac{8}{9}\left[\frac{350\text{ kip-ft (1.04)}}{901\text{ kip-ft}}\right]=0.97<1.0 \qquad \textbf{o.k.}$$

Note: If the designer had chosen a W24 section in Step 1, the resulting P_{eq} would have been 888 kips, which would have permitted the selection of the slender W24×104 shape because AISC *Manual* Table 6-2 already includes the reduction for effective widths of slender elements. Checking the interaction formula of this shape results in a DCR of 1.00; either shape is a valid selection.

Example 3. Select an economical shape for a member with the major- and minor-axis effective lengths and unbraced length equal to 18 ft. The factored axial load and moments calculated from a first-order structural analysis are given as P_u = 1,150 kips, M_{u1} = 760 kip-ft, and M_{u2} = −760 kip-ft, where M_{u1} and M_{u2} are the major-axis moments at member ends. Assume minor-axis bending is negligible.

Solution:

1. Use the first two terms of Equation 7 to estimate the depth (third term is small for $L_c > 12$ ft).

   $$1.2\sqrt[4]{P_u L_c} + 1.1\sqrt[4]{M_u L_c} = 26.3 \text{ in.} \qquad \text{(from Eq. 7)}$$

   The value of C_b is expected to be large because the member is bent in double curvature. Therefore, select m as 0.7 from Table 2, assuming a nominal depth of 27 in. Because $C_m = 0.6 - 0.4(M_1/M_2) = 0.2$, determine $B_1 = 1.0$ using Equation 8.

2. Calculate the equivalent axial load:

   $$\begin{aligned} P_{eq} &= P_r + mM_{rx} + muM_{ry} \le P_c \qquad (4a) \\ &= 1{,}150 \text{ kips} + 0.7(760 \text{ kip-ft}) \\ &= 1{,}680 \text{ kips} \end{aligned}$$

3. Because $P_r/P_{eq} \ge 0.2$, the assumption of using Equation 4a was correct and no changes are required.

4. Using AISC *Manual* Table 6-2, select a W27×178 as the most economical shape with $\phi_c P_n = 1{,}710$ kips.

5. No revision is required because an appropriate shape was found.

6. Check the appropriate interaction using AISC *Specification* Equation H1–1a and AISC *Manual* Table 6-2. Note that the moment gradient gives $C_b = 2.3$, which results in $\phi_b M_{nx} = 2{,}140$ kip-ft (for $L_b = 0$ ft) rather than the value listed for $L_b = 18$ ft.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \text{ when } \frac{P_r}{P_c} \ge 0.2 \qquad (\textit{Spec}. \text{ Eq. H1-1a})$$

$$\frac{1{,}150 \text{ kips}}{1{,}710 \text{ kips}} + \frac{8}{9}\left(\frac{760 \text{ kip-ft}}{2{,}140 \text{ kip-ft}}\right) = 0.99 < 1.0 \quad \textbf{o.k.}$$

Example 4. Select an economical W12 shape for a 20-ft-long member in a braced frame. The effective length factor, K, is assumed to be 1.0 in both directions. The factored axial load and moments calculated from a first-order structural analysis are given as $P_u = 400$ kips, $M_{u1} = 200$ kip-ft, and $M_{u2} = 200$ kip-ft, where M_{u1} and M_{u2} are the major-axis moments at member ends. Assume a minor-axis bending moment of $M_{uy} = 50$ kip-ft due to the application of a transverse load.

Solution:

1. Select $m = 1.4$ from Table 1. Because $C_m = 10$, the value of B_1 for major-axis bending may be determined from Figure 3 as 1.09. The value of u is determined to be 2.0 from Figure 4. The value of B_1 for minor-axis bending is assumed to be 1.3.

2. Calculate the equivalent axial load:

   $$\begin{aligned} P_{eq} &= P_r + mM_{rx} + muM_{ry} \le P_c \qquad (4a) \\ &= 400 \text{ kips} + 1.4\left[200 \text{ kip-ft}(1.09) + 50 \text{ kip-ft}(2.0)(1.3)\right] \\ &= 887 \text{ kips} \end{aligned}$$

3. Because $P_r/P_{eq} \ge 0.2$, the assumption of using Equation 4a was correct and no changes are required.

4. Using AISC *Manual* Table 6-2, select a W12×106 as the most economical shape with $\phi_c P_n = 908$ kips.

6. Check the appropriate interaction using AISC *Manual* Table 6-2. The actual major- and minor-axis values of B_1 are 1.09 and 1.37, respectively. C_b is equal to 1.0.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \text{ when } \frac{P_r}{P_c} \ge 0.2 \qquad (\textit{Spec}. \text{ Eq. H1-1a})$$

$$\frac{400 \text{ kips}}{908 \text{ kips}} + \frac{8}{9}\left(\frac{200 \text{ kip-ft}(1.09)}{562 \text{ kip-ft}} + \frac{50 \text{ kip-ft}(1.37)}{282 \text{ kip-ft}}\right) = 1.0 \le 1.0 \quad \textbf{o.k.}$$

Note: If the designer had instead used $u = 23$ from Table 1, the equivalent axial load, P_{eq}, would be equal to 915 kips. Because this is marginally greater than $\phi_c P_n$ of W12×106 at 20 ft, the designer may elect to initially choose a W12×120 instead, but expect a DCR of approximately 0.89 (determined as 915 kips/1,030 kips). The actual DCR for the W12×120 is found to be 0.87 after checking the interaction formula using AISC *Specification* Eq. H1–1a.

REFERENCES

AISC (1986), *Load and Resistance Factor Design Manual of Steel Construction*, 1st Ed., American Institute of Steel Construction, Chicago, Ill.

AISC (1989), *Allowable Stress Design Manual of Steel Construction*, 9th Ed., American Institute of Steel Construction, Chicago, Ill.

AISC (1994), *Load and Resistance Factor Design Manual of Steel Construction*, 2nd Ed., American Institute of Steel Construction, Chicago, Ill.

AISC (2001), *Load and Resistance Factor Design Manual of Steel Construction*, 3rd Ed., American Institute of Steel Construction, Chicago, Ill.

AISC (2005), *Steel Construction Manual*, 13th Ed., American Institute of Steel Construction, Chicago, Ill.

AISC (2011), *Steel Construction Manual*, 14th Ed., American Institute of Steel Construction, Chicago, Ill.

AISC (2016), *Specification for Structural Steel Buildings*, ANSI/AISC 360–16, American Institute of Steel Construction, Chicago, Ill.

AISC (2017), *Steel Construction Manual*, 15th Ed., American Institute of Steel Construction, Chicago, Ill.

Aminmansour, A. (2000), "A New Approach for Design of Steel Beam-Columns," *Engineering Journal*, AISC, Vol. 37, No. 2, pp. 41–72.

Burgett, L.B. (1973), "Selection of a Trial Column Section," *Engineering Journal*, AISC, Vol. 10, No. 2, pp. 56–60.

Geschwindner, L.F., Liu, J., and Carter, C.J. (2017), *Unified Design of Steel Structures*, 3rd Ed., North Charleston, S.C.

Uang, C-M., Wattar, S.W., and Leet, K.M. (1990), "Proposed Revision of the Equivalent Axial Load Method for LRFD Steel and Composite Beam-Column Design," *Engineering Journal*, AISC, Vol. 27, No.4, pp.150–157.