Design of Unstiffened Girders

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THIS PAPER deals with the design of girders without intermediate stiffeners, on the basis of the current 1963 AISC Specification, Part 1 (Working Stress Method). A typical unstiffened girder is shown in section in Fig. 1. The notation used is shown in this figure and Appendix A.

The usual problem in design is to size the girder for a given moment M and shear V. The usual procedure is to try several girder depths, to perform a preliminary design for each depth, to compute the sectional area resulting from each preliminary design, and then to select one of the designs, frequently the one that gives the least area.

Table 1 has been developed with the object of guiding and facilitating the designer's task. It gives in compact form expressions for the sectional area of unstiffened girders and for some other quantities useful in design. The effect on girder weight of changing the overall depth and the grade of steel are clearly apparent in these expressions.

The allowable shear stress is governed by three different formulas depending on the depth-to-thickness ratio of the web. For this reason girders are classed in Table 1 in Shear Groups **A**, **B**, and **C**. Similarly, the allowable bending stress is governed by five different formulas, depending on the bracing of the compressive flange and the compactness of the section. This is reflected in Table 1 in the subdivision of girders into Flexural Groups **1** through **5**.

Thus a girder with a web depth-to-thickness ratio that puts it into Shear Group **B** and with a braced flange that puts it into Flexural Group **2** would be classed in Group **B2**, and the formulas relating to it will be found in the corresponding place in Table 1. The derivation of Table 1 as well as background information to help the user are given in Appendix B.

Member types other than unstiffened girders, such as girders with intermediate stiffeners and trusses, are not included in Table 1; also, any reduction in the sectional area of the girder along its length has not been considered.

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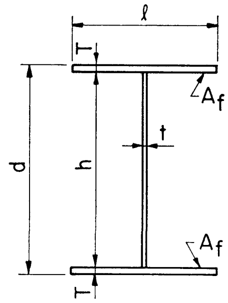

Figure 1

An example of the use of Table 1 is shown in Fig. 2. In this example the steel is A36, the compressive flange is braced, and the design shear is 500 kips. Table 1 (and Fig. 2) indicate that one change in design formulas occurs at a web depth of 47 in., and another at 67½ in. The required web thickness, web area, and total area for design moments of 24,000, 30,000, and 60,000 kip-in. are plotted in Fig. 2.

Charts such as Fig. 2 are not necessary in design, however. The girder size may be established on the basis of Table 1 alone for each given design problem.

APPENDIX A—NOMENCLATURE*

A_f	$= bT$
	= area of one flange (flanges assumed equal)
A_w	$= ht$
	= area of web
A	$= 2A_f + A_w$
	= area of girder
A_0	= least value of A for given M and V
b	= width of flange
C_b	= coefficient defined in AISC Specification Sect. 1.5.1.4.5
d	$= h + 2T$
	= overall vertical dimension of girder
h	= clear height of web between flanges
h_0	= value of h corresponding to A_0
I	= moment of inertia of girder on its strong axis

* *Symbols not listed in this nomenclature are as listed in the AISC Specification nomenclature.*

FLEXURE GROUP		AISC Spec: Section	Upper Limit or Condition: b/T	ℓ/b	$\ell d/A_f$	h/t	Other	A_f / A_w	SHEAR GROUP A	SHEAR GROUP B	SHEAR GROUP C
									$h/t \le 63/n$	$63/n \le h/t \le 91/n$	$91/n \le h/t \le 260$
									$h \le 2.10\sqrt{V/n^3}$	$2.10\sqrt{V/n^3} \le h \le 3.02\sqrt{V/n^3}$	$3.02\sqrt{V/n^3} \le h \le 14.5\sqrt{V}$
									$V/14.4\,m$	$h\sqrt{V/n}\,/30.2$	$\sqrt[3]{h^4 V}\,/43.6$
Braced Comp. Flange	1	1.5.1.4.1	$16.9/n$	$12.6/n$	$555/m$	$(70/n)(h/d)$	Continuous Web Weld	$\frac{M}{23.8\,hmu}$	$1/u = 1 - 0.275\,h\,V/M$ $A = (1.82\,M/h + V)/21.6m$ $h_o = 2.10\sqrt{V/n^3}$ $A_o = (0.87\,M/\sqrt{nV} + V/m)/2.16$		
Braced Comp. Flange	2	1.5.1.4.5 1.9.1 1.10.6	$31.6/n$	$11.6\sqrt{u}$ OR	$555/m$	$163/n$		$\frac{M}{21.6\,hmu}$	$1/u = 1 - 0.250\,h\,V/M$ $A = (2\,M/h + V)/21.6\,m$ $h_o = 2.10\sqrt{V/n^3}$ $A_o = (0.96\,M/\sqrt{nV} + V/m)/21.6$	$1/u = 1 - 0.12\,h^2\sqrt{n^3}\,V/M$ $A = M/10.8\,hm + h\sqrt{V/n}\,/45.3$ $h_o = 2.05\sqrt{M}\,/\,\sqrt[4]{n^3 V}$ $A_o = 0.091\sqrt{M}\,\sqrt[4]{V/n^5}$ For $M \le 2.18\sqrt{(V/n)^3}$	$1/u = 1 - 0.082\,m\,\sqrt[3]{h^7}\,V/M$ $A = M/10.8\,hm + \sqrt[3]{h^4}\,V/65.5$ $h_o = 1.91\,\sqrt[7]{(M/m)^3\,V}$ $A_o = 0.085\,\sqrt[7]{(M/m)^4\,V}$ For $M \ge 2.92\sqrt{(V/n)^3}$
Braced Comp. Flange	3	1.5.1.4.5 1.9.1 Formula (11)	$31.6/n$	$11.6\sqrt{u}$ OR	$555/m$	$>163/n$		$\frac{M}{21.6\,hmu} + \frac{h^2 - 163\,A_w/n}{2000}$			$A = M/10.8\,hm + h^2 u/1000 +$ $+ (1 - u/4n)\,\sqrt[3]{h^4 V}\,/65.5;$ u, h_o, A_o by trial & error.
Unbraced Comp. Flange	4	1.9.1 1.10.6 Formula (4)	$31.6/n$	$>11.6\sqrt{u}$	$>555/m$	$163/n$		$M_4/21.6\,hmu$ where $M_4 = \frac{M}{1 - \frac{m\ell^2}{2650\,C_b\,b^2}}$	$1/u, A$: Same as Flexure Group 2, except use M_4 in place of M in all expressions. h_o, A_o: Find by trial and error.		
Unbraced Comp. Flange	5	1.9.1 1.10.6 Formula (5)	$31.6/n$	$>11.6\sqrt{u}$	$>555/m$	$163/n$		$\frac{\sqrt{m\ell d/h}}{110}$	$A = 0.0182\sqrt{M\ell d/h} + 0.85\,A_w$ u, h_o, A_o: Find by trial and error.		

NOTES:
Numerical values of allowable stresses and design parameters are not rounded off in this table as they are in the AISC Specification Appendix.
Units are in sq. in., kips, kip-in. & kips/sq. in.

Table 1

I (tee)	=	moment of inertia of tee-section defined in AISC Specification Sect. 1.5.1.4.5
k	=	b/T
	=	width-to-thickness ratio of flange
l	=	unbraced length
M	=	given design moment
M_4	=	$M/[1 - (mu/2650\ C_b)(l^2/b^2)]$
	=	equivalent moment for AISC Formula (4), see Eq. (7-3) and (7-7)
†m	=	$F_y/36$ ksi
	=	ratio of yield strength of steel used to 36 ksi
†n	=	$\sqrt{m}$
S	=	$I/(\frac{1}{2}d)$
	=	section modulus
t	=	thickness of web
T	=	thickness of flange
u	=	$1 + \frac{1}{6} A_w/A_f$
	=	coefficient of increase in S due to web
V	=	given design shear

APPENDIX B—DERIVATION OF TABLE 1

1. Shear

A. The shear stress in unstiffened girders is governed by AISC Formula (9):

$$F_v = \frac{F_y}{2.89} C_v \tag{1-1}$$

In addition, the shear stress must not exceed:

$$F_v = 0.4F_y \tag{1-2}$$

Two expressions for C_v are given in the AISC Specification (F_y being in psi):

$$C_v = \frac{45{,}000{,}000k}{F_y(h/t)^2} \quad \text{when } C_v \leq 0.8 \tag{1-3}$$

$$C_v = \frac{6{,}000}{h/t} \frac{\sqrt{k}}{F_y} \quad \text{when } C_v \geq 0.8 \tag{1-4}$$

The Specification also gives, for unstiffened girders:

$$k = 5.34 \tag{1-5}$$

In the notation of this paper:

$$F_y = 36m/\text{ksi} = 36{,}000m \text{ psi} \tag{1-6}$$

$$n = \sqrt{m} \tag{1-7}$$

Introduce the value of C_v from Eqs. (1-1) and (1-2), and k, F_y, n from Eqs. (1-5) through (1-7) into (1-4):

$$h/t = 63/n \tag{1-8}$$

† *Values of m and n for some common grades of steel are:*

F_y (ksi)	33	36	42	46	50
m	0.917	1.000	1.167	1.278	1.389
n	0.957	1.000	1.080	1.130	1.179

Girders with webs having h/t not greater than $63/n$ are classed in Shear Group **A**. For this Group the shear stress is given by Eq. (1-2), and the shear capacity by:

$$V = 14.4mht \tag{1-9}$$

From Eqs. (1-8) and (1-9):

$$h = 2.10\sqrt{V/n^3} \tag{1-10}$$

This is the upper limit of h for Shear Group **A**.

The web area is given by:

$$A_w = V/F_v \tag{1-11}$$

Therefore, for Shear Group **A**:

$$A_w = V/14.4m \tag{1-12}$$

B. Put $C_v = 0.8$ in either Eq. (1-3) or Eq. (1-4):

$$h/t = 91/n \tag{1-13}$$

This value of h/t is the boundary between Shear Groups **B** and **C**.

In Shear Group **B**, C_v is given by Eq. (1-4) and the shear stress by Eq. (1-1), which yields:

$$F_v = \frac{910n}{h/t} \tag{1-14}$$

$$V = 910nt^2 \tag{1-15}$$

Thus, for girders in Shear Group **B** the shear capacity depends only on the web thickness and not on the depth.

From Eqs. (1-14) and (1-15):

$$h = 3.02\sqrt{V/n^3} \tag{1-16}$$

This is the upper limit of h for Shear Group **B**, and the lower limit for Shear Group **C**.

From Eq. (1-11) together with Eqs. (1-14) and (1-15):

$$A_w = \frac{h\sqrt{V/n}}{30.2} \tag{1-17}$$

C. The upper limit of h/t for Shear Group **C** (and for unstiffened girders generally) is laid down in AISC Specification Sect. 1.10.5.3:

$$h/t = 260 \tag{1-18}$$

For this Group, from Eqs. (1-1) and (1-4):

$$F_v = \frac{83{,}000}{(h/t)^2} \tag{1-19}$$

$$V = 83{,}000t^3/h \tag{1-20}$$

Thus, for Shear Group **C** the shear capacity is governed by considerations of elastic instability and is therefore independent of F_y. From Eqs. (1-18) and (1-20):

$$h = 14.5\sqrt{V} \tag{1-21}$$

This is the upper limit of h for Shear Group **C** (and

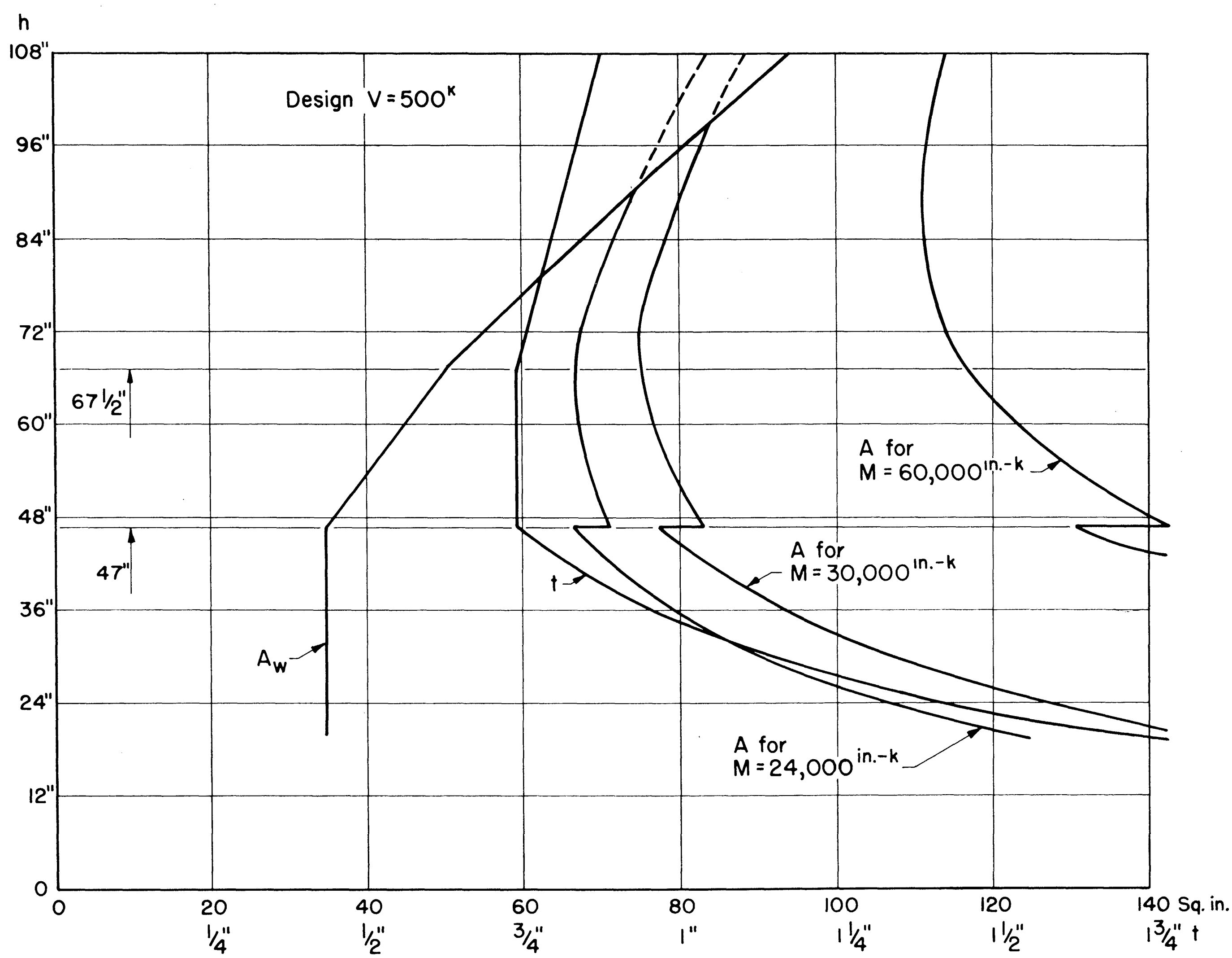

Figure 2

for unstiffened girders generally). If a larger h is used, t must be calculated from Eq. (1-18), and an unnecessarily large shear capacity results from Eq. (1-20). Such girders are in general uneconomical and are not further considered here.

From Eq. (1-11) together with Eqs. (1-19) and (1-20):

$$A_w = \frac{\sqrt[3]{h^4 V}}{43.6} \tag{1-22}$$

2. Contribution of the Web to the Flexural Capacity

Referring to Fig. 1, put:

$$S = A_f h u \tag{2-1}$$

The strong moment of inertia is given by:

$$I = \tfrac{1}{2}A_f(h + T)^2 + \tfrac{1}{6}A_f T^2 + \tfrac{1}{12}A_w h^2 \tag{2-2}$$

Therefore the section modulus is given by:

$$S = \frac{I}{\tfrac{1}{2}h + T} = (A_f + \tfrac{1}{6}A_w)h - \frac{(A_w h - 4A_f T)}{3(h + 2T)}\,T \tag{2-3}$$

For most girders T/h is relatively small, and the second term in Eq. (2-3) is relatively unimportant. For such girders, S may be approximated by:

$$S = (A_f + \tfrac{1}{6}A_w)h \tag{2-4}$$

This well-known approximation, rather than Eq. (2-3), is used in this paper. On this basis, from Eqs. (2-1) and (2-4):

$$u = 1 + \tfrac{1}{6}A_w/A_f \tag{2-5}$$

Denote the bending stress by F_b and the shear stress by F_v. Then, for a given moment M and shear V:

$$A_f = M/(F_b h u) \tag{2-6}$$
$$A_w = V/F_v \tag{2-7}$$

From Eqs. (2-5), (2-6), and (2-7):

$$u = \frac{M/(F_b h)}{M/(F_b h) - \frac{1}{6} V/F_v} \tag{2-8}$$

The reciprocal of u is often easier to compute:

$$\frac{1}{u} = 1 - \frac{1}{6} \frac{hVF_b}{MF_v} \tag{2-9}$$

3. Groups A1 and A2

Girders with braced compressive flange are classed in Flexural Groups **1**, **2**, and **3**.

A. Flexural Group **1** consists of the compact girders of AISC Specification Sect. 1.5.1.4.1. The limitations on b/T, l/b, ld/A_f, and h/t contained in Table 1 are taken from that Section.

The bending stress for this Group is:

$$F_b = 0.66F_y = 23.6m \tag{3-1}$$

Usually, because of the limitation on h/t, the girders of this Flexural Group belong to Shear Group **A**, i.e., they may be classed in Group **A1.** Therefore, from Eqs. (3-1), (1-2), and (2-9):

$$1/u = 1 - 0.275hV/M \tag{3-2}$$

From Eqs. (2-6), (3-1), and (3-2):

$$A_f = \frac{M}{23.8hmu} = \frac{M/h - 0.275V}{23.6m} \tag{3-3}$$

Add $2A_f$ from Eq. (3-3) to A_w from Eq. (1-12):

$$A = (1.82M/h + V)/21.6m \tag{3-4}$$

Obviously, to reduce A, h should be increased as much as possible. Therefore, the most economical h/t is at the upper end of the range of Shear Group **A**, $63/n$, or of Flexural Group **1,** $(70/n)(h/d)$; assuming the former governs, Eq. (1-10) may be used:

$$h_0 = 2.10\sqrt{V/n^3} \tag{3-5}$$

From Eqs. (3-4) and (3-5):

$$A_0 = (0.87M/\sqrt{nV} + V/m)/21.6 \tag{3-6}$$

This holds only as long as the depth h does not exceed the limits for Group **A1.** It is quite probable that with a greater h a smaller sectional area is attainable, but then the girder would be classed outside this Group, and different formulas would apply.

B. Flexural Group **2** comprises noncompact but braced girders, with h/t limited by AISC Specification Sect. 1.10.6. The bending stress for this Group is:

$$F_b = 0.6F_y = 21.6m \tag{3-7}$$

To qualify for this stress, either of AISC Formulas (4) or (5) must show that no stress reduction is required. For AISC Formula (4), the condition for no stress reduction is:

$$l/r = 40 \tag{3-8}$$

The term r is defined in AISC Specification Sect. 1.5.1.4.5:

$$I(\text{tee}) = (A_f + \tfrac{1}{6} A_w)r^2 = A_f b^2/12 \tag{3-9}$$

or, from Eq. (2-5):

$$ur^2 = b^2/12 \tag{3-10}$$

Eq. (3-8) now becomes:

$$l/b = 11.6/\sqrt{u} \tag{3-11}$$

AISC Formula (5), expressed in ksi, is:

$$F_b = \frac{12{,}000}{ld/A_f} \tag{3-12}$$

The condition for no stress reduction is, from Eqs. (3-7) and (3-12):

$$ld/A_f = 555/m \tag{3-13}$$

Limitation Eqs. (3-11) and (3-13) are alternative as already stated. If one of them is not exceeded, the girder qualifies for Flexural Group **2** as far as bracing of the flange is concerned. The limitation on h/t is, by AISC Specification Sect. 1.10.6 and Eq. (3-7):

$$h/t = 24{,}000/\sqrt{F_b(\text{psi})} = 163/n \tag{3-14}$$

The derivation of the formulas for **A2** girders is analogous to that for **A1** girders, Eq. (3-2) through (3-6), except that Eq. (3-7) replaces Eq. (3-1).

4. Group B2

For girders belonging to shear Group **B** and Flexural Group **2,** the shearing and bending stress, respectively, are given by Eqs. (1-14) and (3-7). From these expressions, together with Eqs. (1-15) and (2-9):

$$1/u = 1 - 0.12h^2\sqrt{n^3V/M} \tag{4-1}$$

Therefore the flange area is:

$$A_f = \frac{M}{21.6hmu} = \frac{M}{21.6hm} - \frac{h\sqrt{V/n}}{181} \tag{4-2}$$

Add $2A_f$ from Eq. (4-2) to A_w from Eq. (1-17):

$$A = \frac{M}{10.8hm} + \frac{h\sqrt{V/n}}{45.3} \tag{4-3}$$

To find the least area A_0 and the corresponding depth h_0, equate $\partial A/\partial h$ to zero. This gives:

$$h_0 = \frac{2.05\sqrt{M}}{\sqrt[4]{n^3V}} \tag{4-4}$$

$$A_0 = 0.091 \sqrt[4]{M^2V/n^5} \tag{4-5}$$

The greatest girder depth for Shear Group **B** is given by Eq. (1-16). Equate Eq. (1-16) to Eq. (4-4):

$$M = 2.18 \sqrt{(V/n)^3} \text{ or } M^2n^3/V^3 = 4.75 \qquad (4\text{-}6)$$

For M larger than this value, Eq. (1-6) rather than Eq. (4-4) gives h_0, and h_0 thus obtained should be inserted in Eq. (4-3) to compute A_0. Of course, the possibility that the least value of A corresponds to h outside the limits of Shear Group **B** may have to be investigated.

5. Group C2

Girders complying with the requirements of Flexural Group **2** (braced flanges, etc.) and having h/t between $91/n$ and $163/n$ belong to this Group. The bending stress is still given by Eq. (3-7), but the shear stress is given by Eq. (1-19).

On this basis, together with Eq. (1-20), Eq. (2-9) gives:

$$1/u = 1 - 0.082(m/M) \sqrt[3]{7hV} \qquad (5\text{-}1)$$

The flange area is:

$$A_f = \frac{M}{21.6hmu} = \frac{M}{21.6hm} - \frac{\sqrt[3]{h^4V}}{262} \qquad (5\text{-}2)$$

Add $2A_f$ to A_w from Eq. (1-22):

$$A = \frac{M}{10.8hm} + \frac{\sqrt[3]{h^4V}}{65.5} \qquad (5\text{-}3)$$

Equate $\partial A/\partial h$ to zero; this yields:

$$h_0 = 1.91 \sqrt[7]{M^3/m^3V} \qquad (5\text{-}4)$$

$$A_0 = 0.085 \sqrt[7]{M^4V/m^4} \qquad (5\text{-}5)$$

Equating h_0 from Eq. (5-4) to the least girder depth for Group **C** as given by Eq. (1-16) leads to:

$$M = 2.92\sqrt{V^3/n^3} \quad \text{or} \quad M^2n^3/V^3 = 8.50 \qquad (5\text{-}6)$$

Eqs. (4-6) and (5-6) may be summarized as follows:

If $M^2n^3/V^3 \leq 4.75$, h_0 is given by Eq. (4-4), A_0 by Eq. (4-5) (or, if the girder qualifies for Flexural Group **1**, by Eqs. (3-5) and (3-6)).

If $M^2n^3/V^3 \geq 8.50$, h_0 is given by Eq. (5-4), A_0 by Eq. (5-5).

If $4.75 \leq M^2n^3/V^3 \leq 8.50$, h_0 is given by Eq. (1-16) and A_0 by either Eq. (4-3) or Eq. (5-3), which give the same result.

In the infrequent case of very low V, Eq. (5-4) may give a girder depth greater than Eq. (1–21). In such cases Eq. (1-21) of course overrides.

6. Group C3

This is similar to Group **C2** except that the h/t ratio is greater than $163/n$, and the allowable bending stress must be reduced by AISC Formula (11):

$$F_b = 21.6m[1 - (h^2 - 163A_w/n)/2000A_f] \qquad (6\text{-}1)$$

From Eqs. (2-6) and (6-1):

$$A_f = \frac{M}{21.6hmu} + \frac{h^2 - 163A_w/n}{2{,}000} \qquad (6\text{-}2)$$

Eq. (2-5) may be written $A = 2A_fu + \tfrac{2}{3}A_w$. On this basis, from Eqs. (6-2) and (1-22):

$$A = \frac{M}{10.8hm} + \frac{uh^2}{1{,}000} + \frac{(1 - u/4n) \sqrt[3]{h^4V}}{65.5} \qquad (6\text{-}3)$$

7. Flexural Groups 4 and 5—Unbraced Top Flanges.

Girders for which the distance l between braced points on the compressive flange exceeds both limits calculated from Eqs. (3-11) and (3-13) must be designed by either AISC Formula (4) or (5). Any such girder belongs to both Flexural Groups **4** and **5**. The equations of the Group that results in a smaller A govern.

Flexural Group **4** comprises girders governed by AISC Formula (4), from which:

$$F_b = 21.6m \left(1 - \frac{l^2}{2C_br^2} \cdot \frac{36m}{2\pi^2E}\right) \qquad (7\text{-}1)$$

Combine with Eq. (3-10), and insert 29,000 ksi for E:

$$F_b = 21.6m \left(1 - \frac{mu}{2650C_b} \cdot \frac{l^2}{b^2}\right) \qquad (7\text{-}2)$$

Define the equivalent moment for AISC Formula (4):

$$M_4 = M/\left(1 - \frac{mu}{2650C_b} \cdot \frac{l^2}{b^2}\right) \qquad (7\text{-}3)$$

Then:

$$F_b = 21.6mM/M_4 \qquad (7\text{-}4)$$

and

$$A_f = M_4/(21.6hmu) \qquad (7\text{-}5)$$

It is seen that the expressions for A and u in Groups **A4, B4, C4** are similar to Groups **A2, B2, C2,** except that M_4 replaces M. The expressions for $1/u$ now contain M_4, which is not independent of u; these expressions are, strictly speaking, quadratics in u. If, however, a reasonably close trial value of u is used for estimating M_4, the expressions for $1/u$ will give a result sufficiently accurate for practical purposes on the first trial. An alternative formula for M_4 is obtained by writing:

$$b^2 = kA_f = A_fb/T \qquad (7\text{-}6)$$

where $k = b/T$ is limited to $31.6/n$.

From Eqs. (7-3) through (7-6):

$$M_4 = M + \frac{hl^2m^2u^2}{123C_bb/T} \qquad (7\text{-}7)$$

Flexural Group **5** consists of girders governed by AISC Formula (5). For these girders, from Eqs. (2-6) and (3-12):

$$A_f = \frac{\sqrt{Mld/hu}}{110} \tag{7-8}$$

Eq. (2-9) together with Eqs. (3-12) and (7-8) gives:

$$\frac{1}{u} = 1 - \frac{A_w}{18.2}\sqrt{\frac{h}{Mld}}\sqrt{\frac{1}{u}} \tag{7-9}$$

from which:

$$\sqrt{\frac{1}{u}} = \sqrt{1 + \frac{(9.1A_w)^2 h}{Mld}} - 9.1A_w\sqrt{\frac{h}{Mld}} \tag{7-10}$$

or, approximately:

$$\sqrt{\frac{1}{u}} = 1 - 9.1A_w\sqrt{\frac{h}{Mld}} \tag{7-11}$$

Eq. (7-8) with Eq. (7-11) gives:

$$A_f = \frac{\sqrt{Mld/h}}{110} - \frac{A_w}{12} \tag{7-12}$$

Add $2A_f$ to A_w:

$$A = 0.0182\sqrt{Mld/h} + \tfrac{5}{6}A_w \tag{7-13}$$

This expression underestimates A, in view of the approximation involved in Eq. (7-11). To balance this, round off $\frac{5}{6}$ to 0.85:

$$A = 0.0182\sqrt{Mld/h} + 0.85A_w \tag{7-14}$$

It is clear from Eq. 7-14 that, for the least value of A, A_w should be as small as possible, i.e., if stiffness requirements and geometry allow, the h selected should place the girder in Shear Group **A.**