Web Crippling Strength of Longitudinally Stiffened Steel Plate Girder Webs Subjected to Concentrated Loading

NELSON LOAIZA, CARLOS GRACIANO and ROLANDO CHACÓN

ABSTRACT

Currently, the AISC *Specification* provides guidance for the calculation of the ultimate strength of unstiffened plate girder webs subjected to concentric edge loads. Specifications consider three categories: local web yielding, web crippling, and sideway web buckling. Based on previous studies, the presence of longitudinal stiffeners in the web has not been considered in the calculation procedures. Longitudinal stiffeners in steel plate girders are primarily used to increase bending. In the last two decades, a number of projects regarding the positive effect of longitudinal stiffening on the strength of plate girder webs to concentrated load have been conducted around the world. The results have shown that this type of stiffening enhances ultimate strength for web crippling, depending on the position of the stiffener that modifies the slenderness of the directly loaded panel and flexural and torsional rigidities of the stiffener. This paper presents a methodology for the consideration of longitudinal stiffening on the ultimate strength of plate girders webs subjected to concentrated loads. The methodology is based on the plastic collapse mechanism observed experimentally, in which plastic hinges are formed in the loaded flange and yield lines result in the portion of the web limited by the loaded flange and stiffener. Then, a closed-form solution accounting for the influence of the stiffener is developed following the current expression available in the AISC *Specification*. Theoretical predictions are compared with available test results, showing that the predicted ultimate loads are in good agreement with experimental results.

Keywords: web buckling, longitudinal stiffeners, ultimate resistance, concentrate load, steel girders.

INTRODUCTION

In the last two decades, a number of research projects regarding the positive effect of longitudinal stiffening on the strength of plate girder webs to concentrated load have been conducted around the world. The results have shown that this type of stiffening enhances ultimate strength for web crippling, depending on the position of the stiffener that modifies the slenderness of the directly loaded panel and flexural and torsional rigidities of the stiffener.

Currently, in the Eurocode (EC3 Part 1-5, 2006), the resistance of steel girder webs to concentrated load is calculated using an χ-λ approach. Lagerqvist and Johansson (1996), after conducting an extensive literature review, proposed a design procedure to calculate the resistance of transversally stiffened girder webs subjected to a concentrated force. Afterward, Graciano (2002) included the effect of longitudinal stiffening into this design procedure.

Thereafter, further investigations have been conducted, particularly in Europe. Seitz (2005) conducted a series of experimental tests on longitudinally stiffened girders to investigate the influence of the patch loading length and the presence of closed section stiffeners. At the same time, Davaine (2005) performed an extensive numerical investigation on both critical load and resistance of longitudinally stiffened webs considering very deep girders, beyond the ranges studied experimentally. Continuing the investigation carried out by Lagerqvist and Johansson (1996), Gozzi (2007) numerically investigated the resistance to concentrated loads of unstiffened plate girders at ultimate and serviceability limit states. In parallel, Clarin (2007) evaluated various ultimate-strength approaches and incorporated these into a calibrated formulation for longitudinally stiffened girder webs. Considering the flange-to-web yield strength inhomogeneities present in the design of bridge girders, Chacón (2009) numerically and experimentally investigated the resistance of hybrid plate girders subjected to concentrated forces. Concerning the use of multiple longitudinal stiffeners, Dall'Aglio (2011) performed a numerical investigation to evaluate the influence of two longitudinal stiffeners in the compression zone on the ultimate strength of girder webs under concentrated loading. Figure 1 shows the nomenclature used for the longitudinal stiffened steel plate girder studied herein.

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In spite of the number of research projects demonstrating that longitudinal stiffeners enhance the ultimate strength of plate girder webs subjected to concentrated forces, the latest edition of the AISC *Specification* (AISC, 2016) presents only guidance for the calculation of the ultimate strength of longitudinally unstiffened plate girder webs and webs with vertical stiffeners. Therefore, this paper is aimed at presenting a methodology for the consideration of longitudinal stiffening on the ultimate strength of plate girders webs subjected to concentrated loads. The methodology is based on the plastic collapse mechanism observed experimentally, in which plastic hinges are formed in the loaded flange and yield lines result in the portion of the web limited by the loaded flange and stiffener. The results are compared with various approaches taken from the literature.

ULTIMATE STRENGTH MODELS FOR CONCENTRATED LOADING

Failure Mechanism Proposed by Roberts

Roberts (1981) developed a failure mechanism model for the estimation of the ultimate load of an unstiffened slender I-girder subjected to concentrated forces (Figure 2). The model considers that the external load at plastic collapse is similar to the internal dissipation of plastic energy during a small variation of displacement, δ. This mechanism describes the plastic collapse of the loaded flange subjected to, and the portion of the web beneath, the load. Four plastic hinges are used in this model to represent the mode of failure in the flange and the crippling effect produced in the web panel.

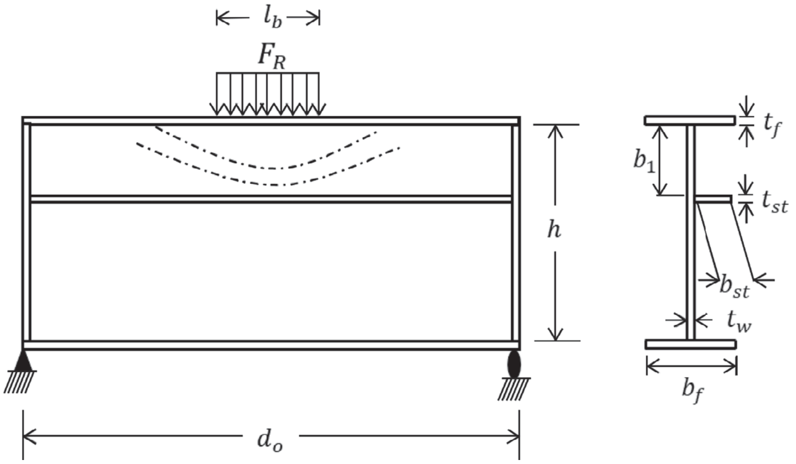


Fig. 1. Notation used for longitudinally stiffened steel plate girder webs subjected to concentrated loading.

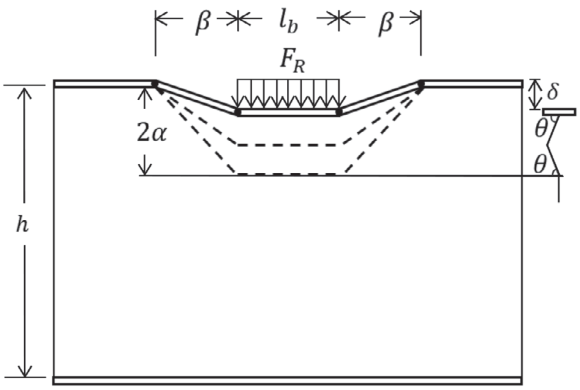


Fig. 2. Failure mechanism of four plastic hinges for longitudinally unstiffened webs.

After several mathematical operations, an expression for the ultimate load, F_R, is found:

$$F_R = 2\sqrt{2}\, t_w^2 \sqrt{\frac{EF_{yw}^2 t_f}{\alpha F_{yf}} + \frac{l_b E F_{yw}^2 t_w^4}{F_{yf}^2 b_f t_f \alpha}} \tag{1}$$

Correspondingly, the following hypotheses are considered:

- Roberts (1981), based on the observation of its experimental results, determined that the distance α between yield lines in the web measured from the loaded flange (see Figure 2) is a function of the web thickness $\alpha = 25t_w$, then Equation 1 becomes:

$$F_R = \frac{2\sqrt{2}}{5} t_w^2 \left[1 + k l_b \left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{EF_{yw}^2 t_f}{t_w F_{yf}}} \tag{2}$$

where

$$k = \left(\frac{1}{2\sqrt{2}} \sqrt{\frac{E}{F_{yw}}}\right) \frac{1}{b_f}$$

- Thereafter, both yield strengths for web and flange were assumed equal, $F_{yf} = F_{yw}$, and simplifying the factor k to $3/h$, the ultimate strength to concentrated forces F_R is, therefore:

$$F_R = \frac{2\sqrt{2}}{5} t_w^2 \left[1 + 3\left(\frac{l_b}{h}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \tag{3}$$

- Finally, as a safe approximation, the number $2\sqrt{2}/5$ was rounded off to 0.5.

$$F_R = 0.50\, t_w^2 \left[1 + 3\left(\frac{l_b}{h}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \tag{4}$$

It should be noticed that after some experimental comparisons, Equation 4 is valid only for short concentrated lengths $l_b/h \le 0.2$ and a flange-to-web thickness ratio of $t_f/t_w \ge 3$. For a detailed derivation of these formulas, the readers are encouraged to see Roberts (1981).

Nominal Strength of the Web Against Crippling

Using Equation 4, the AISC *Specification* (AISC, 2016) provides a modified formulation for the nominal strength of an unstiffened slender I-girder subjected to concentrated forces. Several equations are proposed in AISC *Specification* Section J10, depending on where the load is applied. From *Specification* Equation J10-4, when the concentrated force is applied at a distance from the member end greater than or equal to $d/2$, the nominal strength is calculated as

$$F_R = 0.80\, t_w^2 \left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \tag{5}$$

and when a concentrated force is applied at a distance from the member end less than $d/2$:

For $l_b/d \le 0.2$

$$F_R = 0.40\, t_w^2 \left[1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \tag{6a}$$

For $l_b/d > 0.2$

$$F_R = 0.40\, t_w^2 \left[1 + \left(4\frac{l_b}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right] \sqrt{\frac{EF_{yw} t_f}{t_w}} \tag{6b}$$

Equation 5 is very similar to the one proposed by Roberts (1981). Furthermore, the influence of longitudinal stiffeners is not considered in the AISC *Specification* for concentrated forces (AISC, 2016).

Resistance to Transverse Forces: EC3 Part 1-5

The Eurocode EC3 Part 1-5 (2006) rules for plated structural elements provides another approximation for the resistance to concentrated forces of slender girders. In contrast to the AISC *Specification* (AISC, 2016), the EC3 Part 1-5 (2006) incorporates the influence of a longitudinal stiffener in the calculation of the resistance to concentrated forces. This design procedure follows a harmonized technique developed by Lagerqvist and Johansson (1996) that consists of calculating the yield resistance, F_y, and the critical buckling load, F_{cr}, of the web panel. Currently, the EC3 Part 1-5 (2006) rules are under review (Chacón et al., 2010; Graciano, 2015), and the following amendments have been suggested:

- First, the yield resistance, F_y, is obtained from a four–plastic hinge mechanism developed by Lagerqvist and Johansson (1996):

$$F_y = F_{yw} t_w l_y \tag{7}$$

  where l_y is the effective load length and is computed using the expression recommended by Chacón et al. (2010) for hybrid girders (plate girders with a yield strength ratio $F_{yf}/F_{yw} \ne 1$), which states that flange-to-web yield resistance ratio should be considered equal to 1 ($F_{yf}/F_{yw} = 1$), due to its diminished influence on the ultimate load:

$$l_y = \left[l_b + 2t_f\left(1 + \sqrt{b_f/t_w}\right)\right] \tag{8}$$

- Next, the critical buckling load is obtained with Equation 9 proposed by Davaine (2005):

$$\frac{1}{F_{cr}} = \frac{1}{F_{cr1}} + \frac{1}{F_{cr2}} \tag{9}$$

where Equation 9 is an expression that considers an interaction between the critical buckling load, F_{cr1}, established by Graciano and Lagerqvist (2003), and the critical buckling load, F_{cr2}, of the upper web panel was developed by Davaine (2005). First, the critical buckling load F_{cr1} is computed according to classical buckling theory:

$$F_{cr1} = k_{f1} \frac{\pi^2 E}{12(1-\nu^2)} \frac{t_w^3}{h} \tag{10}$$

where k_{f1} is a buckling coefficient obtained from a linear buckling analysis of plate girders subjected to a fixed concentrated force length of $l_b/h = 0.2$ (Graciano and Lagerqvist, 2003). This expression is found in EC3 Part 1-5 (2006) as

$$k_{f1} = 6 + 2\left[\frac{h}{d_o}\right]^2 + \left[5.44\frac{b_1}{d_o} - 0.21\right]\sqrt{\gamma_s} \tag{11}$$

$$\gamma_s = 10.9\frac{I_{st}}{ht_w^3} \le 13\left[\frac{d_o}{h}\right]^3 + 210\left[0.3 - \frac{b_1}{d_o}\right] \tag{12}$$

where γ_s is the relative flexural rigidity of the stiffener and I_{st} is the moment of inertia of the longitudinal stiffener calculated with respect to its centroidal axis parallel to the web plate. Considering the composed area of stiffener and two portions of the web plate with a width of $15t_w$ on each side of the stiffener weld, Figure 3 illustrates the effective cross-section of open section stiffeners.

Second, the critical buckling load F_{cr2} is obtained from a model proposed by Davaine (2005), in which only a portion of the web panel is studied. This part of the panel has a height of b_1, and it is simply supported, with opposite concentrated forces of lengths $l_b + 2t_f$ and $l_b + 2t_f + 2b_1$ applied to both the upper and lower ends as shown in Figure 4. The purpose of this modification was to correct the increase of ultimate load values found in EC3 Part 1-5 (2006) when the position of the stiffener increases with respect to the loaded flange. In this case, the critical buckling load, F_{cr2}, is calculated, replacing the depth of web panel, D, with the position b_1 of the stiffener

$$F_{cr2} = k_{f2} \frac{\pi^2 E}{12(1-\nu^2)} \frac{t_w^3}{b_1} \tag{13}$$

After performing an eigenvalue analysis, the buckling coefficient k_{f2} is expressed as

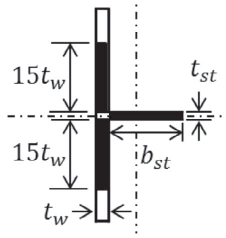


Fig. 3. Effective cross area used for calculating I_{st}.

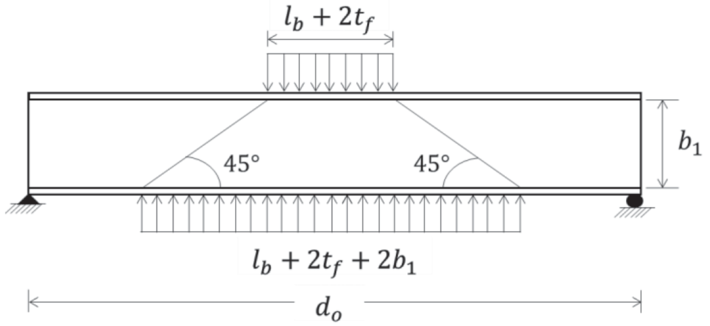


Fig. 4. Simply supported model proposed by Davaine (2005).

$$k_{f2} = \left[0.8\left(\frac{l_b + 2t_f}{d_o}\right) + 0.6\right]\left(\frac{d_o}{b_1}\right)^{\left[0.6\left(\frac{l_b+2t_f}{d_o}\right)+0.5\right]} \tag{14}$$

- Finally, the ultimate load, F_R, is calculated with the χ_F-λ approach, an estimation that reduces the yield resistance, F_y. This reduction is obtained multiplying the resistance function, χ_F, with the aforementioned resistance, F_y.

$$F_R = F_y \chi_F(\overline{\lambda}_F) \tag{15}$$

with the resistance function χ_F equal to

$$\chi_F = \frac{1}{\phi + \sqrt{\phi^2 - \lambda}} \leq 1 \tag{16}$$

and the slenderness parameter λ

$$\overline{\lambda}_F = \sqrt{F_y / F_{cr}} \tag{17}$$

It should be pointed out that Equation 16 was developed by Müller (2003), in which ϕ is a function that depends on the slenderness parameter λ, the imperfection factor α_0, and the plateau length λ_0—values that can be found in different resistance models (Davaine, 2005; Müller, 2003; Gozzi, 2007; Clarin, 2007; Chacón et al., 2012).

$$\phi = 0.5\left[1 + \alpha_0(\lambda - \lambda_0) + \lambda\right] \tag{18}$$

Proposed Failure Mechanism for Longitudinal Stiffened Plate Girders

In order to consider the influence of a longitudinal stiffener, Graciano and Edlund (2003) presented a reviewed version of the plastic failure mechanism developed by Roberts and Rockey (1979). In this mechanism, the buckling behavior is affected significantly by the presence of a longitudinal stiffener, mainly because the distance to yield lines in the web α is restricted by the position of the stiffener b_1, as shown in Figure 5. Figure 6 shows the deformed shape obtained in experimental results of longitudinal stiffened webs subjected to concentrated forces (Rockey et al., 1978).

As a result of this behaviour, Graciano and Edlund (2003) proposed a mechanical model, which uses the same mechanism developed by Roberts and Rockey (1979):

$$F_R = 8F_{yw}t_w^2\sqrt{\frac{Et_f}{8\alpha F_{yf}}} + \frac{2(C_e - \eta)M_w}{\alpha\cos\theta} \tag{19}$$

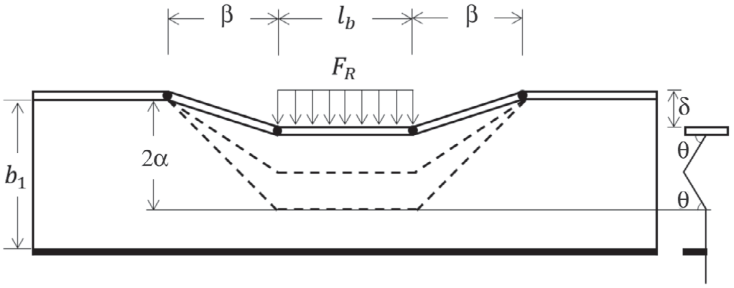


Fig. 5. Failure mechanism of four plastic hinges for longitudinal stiffened webs.

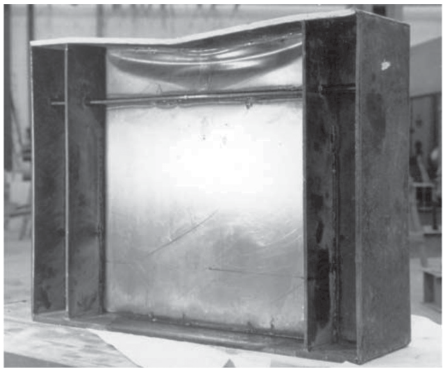


Fig. 6. Experimental results of web crippling in a longitudinal stiffened girder (Rockey et al., 1978).

The following geometrical parameters are basically the same:

$$\eta = \frac{(4\beta + 2C_e)M_w}{2M_w + F_{yw}t_w \alpha \cos\theta} \tag{20}$$

$$\beta = \left(\frac{M_f \alpha \cos\theta}{M_w} \right)^{1/2} \tag{21}$$

$$\cos\theta = \frac{M_f^2}{6EI_f M_w} \tag{22}$$

and the plastic moments of the web and flange are:

$$M_w = \frac{F_{yw} t_w^2}{4} \tag{23}$$

$$M_f = \frac{F_{yw} b_f t_f^2}{4} \tag{24}$$

As seen in Figures 5 and 6, the position of the yield lines α are restricted by the position of the stiffener b_1. Hence, Graciano and Edlund (2003) conservatively proposed the following values:

$$\alpha = 0.5b_1 \qquad \text{if } b_1/t_w \le 40 \tag{25a}$$

$$\alpha = 20t_w F_{yw}/F_{yf} \qquad \text{if } b_1/t_w > 40 \tag{25b}$$

Equation 25b was initially proposed by Roberts and Newark (1997); therefore, the limits to consider the influence of the longitudinal stiffener are $b_1/t_w \le 40$. Otherwise, the stiffener is unable to enhance the load carrying capacity of the girder under concentrated loading.

However, as mentioned earlier, Chacón et al. (2010) demonstrated that the flange-to-web yield strength ratio has no influence on the resistance to concentrated forces for hybrid girders. Consequently, Equation 19 can be rewritten as

$$F_R = 8t_w^2 \sqrt{\frac{Et_f F_{yw}}{8\alpha}} + \frac{2(C_e - \eta)M_w}{\alpha \cos\theta} \tag{26}$$

By means of regression analysis, the position of yield lines α is adjusted herein to obtain a good correlation between experimental ultimate load and theoretical predictions:

$$\alpha = 0.42b_1 \qquad \text{if } b_1/t_w \le 40 \tag{27a}$$

$$\alpha = 17t_w \qquad \text{if } b_1/t_w > 40 \tag{27b}$$

RESULTS

In the previous section, various ultimate-strength models were explained. In this section, a comparative analysis is performed in order to contrast the experimental loads, F_{exp}, with theoretical predictions, F_R. Simple statistics for the ratio F_{exp}/F_R are used for this purpose: maximum and minimum values; mean, m; standard deviation, s; and coefficient of variation, v. Table 1 summarizes 45 experimental test results taken compiled in the literature (Graciano, 2005).

Figure 7 displays the values for the ratio F_{exp}/F_R versus N° of test, corresponding to each mechanism studied. The results have been separated in terms of the type of stiffener, open-section (flat) stiffener, or closed-section (trapezoidal and triangular) stiffener. As expected, the failure mechanism proposed by Roberts (1981) is the most conservative of all, with a mean value $m = 1.85$; see also Table 2. This model also presents a large standard deviation $s = 0.34$, which makes it an unreliable prediction for longitudinally stiffened girder webs. Results obtained with the AISC *Specification* (AISC, 2016) for nominal strength attained a mean value $m = 1.16$, despite that fact that this approach is similar to the one proposed by Roberts (1981). However, it is important to notice that the standard deviation and coefficient of variation are significantly high when taking into account the mean value of predicted load ratio as seen in Table 2. Additionally, it can be observed in Figures 7(a) and 7(b) that predictions based upon Roberts's (1981) estimation of the ultimate load are quite conservative for closed-section stiffeners.

On the other hand, the predictions obtained with the revised EC3 Part 1-5 (2006) for longitudinal stiffened webs are still conservative ($m = 1.82$). Nevertheless, it must be mentioned that the range of the predicted load ratio F_{exp}/F_R is acceptable (max = 2.31; min = 1.17) [Figure 7(c)]. At the same time, Figure 7(d) shows that the predicted strengths using the model proposed herein display a good agreement with experimental test results. As observed in Table 2, the mean value for the ratio F_{exp}/F_R is around $m = 1.23$, and the standard deviation is $s = 0.15$.

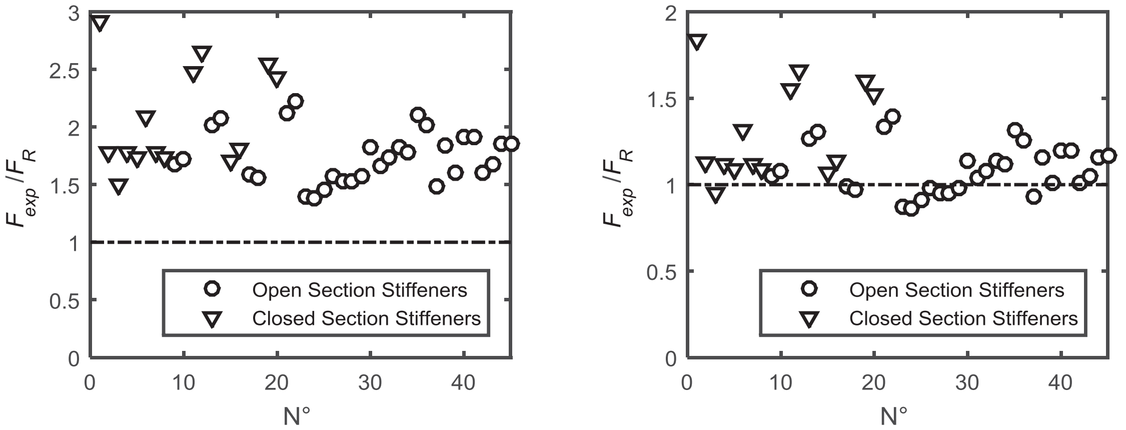
Particularizing the results of the proposed model, Figure 8 shows the predicted load ratio F_{exp}/F_R as a function of various geometrical parameters. The results plotted in Figure 8 show a reduced scatter in the ratio F_{exp}/F_R for all values of slenderness ratio, b_1/t_w, and load length-to-width ratio, l_b/d_o, and it slightly increases with the flexural rigidity of the stiffener, γ_s. It is important to mention the proposed model implicitly considers that the stiffener is rigid enough to form a nodal line at the stiffener location.

Table 1. Experimental Results of Stiffened Web Panels

Author(s)	Test	Numbers of Tests	Type of Stiffener
Carretero and Lebet (1998)	Panel 1-2, Panel 2-2 Panel 4-4, Panel 4-6 Panel 5-1, Panel 6-2	6	All trapezoidal stiffeners
Dubas and Tschamper (1990)	VT07-2, VT07-3 VT07-5, VT07-6 VT08-2, VT08-3 VT08-5, VT08-6 VT09-2, VT09-3 VT09-5, VT09-6 VT10-2, VT10-3 VT10-5, VT10-6	16	8 flat stiffeners 8 V-shaped stiffeners
Bergfelt (1983)	731, 732, 733 734, 735, 736	6	All flat stiffeners
Rockey et al. (1978)	R2, R4 R22 ss, R42 ss	4	All flat stiffeners
Bergfelt (1979)	A12 s, A14 s A16 s, A22 s A24 s, A26 s A32 s, A34 s A36 s	9	All flat stiffeners
Dogaki et al. (1990)	Model 4, Model 5	2	All flat stiffeners
Galea et al. (1987)	P2, P3	2	All flat stiffeners

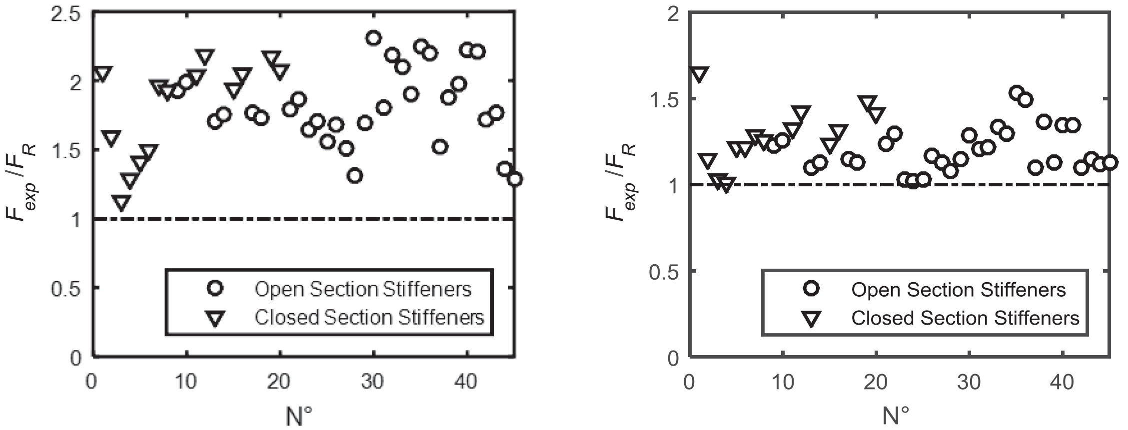
Table 2. Statistics of F_{exp}/F_R

Methodology	Min	Max	*m*	*s*	*v*
Roberts (1981) (Eq. 4)	1.38	2.92	1.85	0.34	0.18
AISC *Specification* (AISC, 2016) (Eq. 5)	0.86	1.83	1.16	0.22	0.19
Revised EC3 Part 1-5 (2006) (Eq. 15)	1.13	2.31	1.82	0.30	0.17
Proposed mechanism (Eq. 26)	1.01	1.65	1.23	0.15	0.12



(a) Failure mechanism proposed by Roberts (1981)

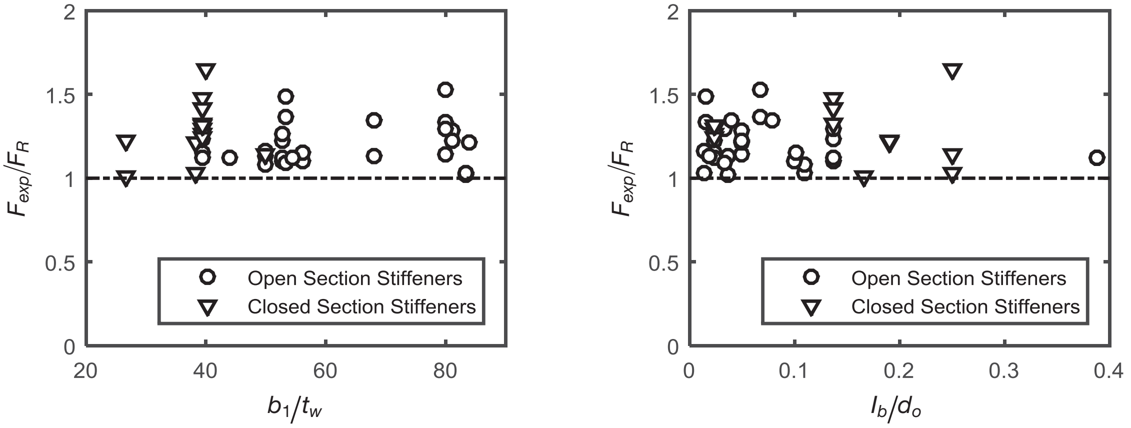
(b) AISC Specification *(AISC, 2016)*



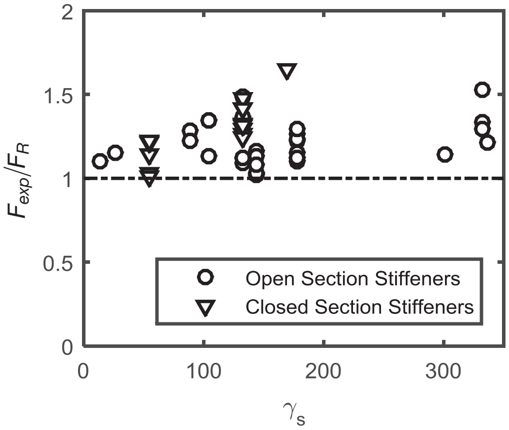
(c) Revised EC3 Part 1-5 (2006)

(d) Proposed mechanism

Fig. 7. Experimental and predicted ultimate load ratio F_{exp}/F_R *for longitudinal stiffened webs.*



(a) F_{exp}/F_R *vs.* b_1/t_w *(b)* F_{exp}/F_R *vs.* s_s/a



(c) F_{exp}/F_R *vs.* γ_s

Fig. 8. Experimental and predicted ultimate load ratio F_{exp}/F_R *vs. the slenderness ratio* b_1/t_w*, load length-to width ratio* l_b/d_o*, and flexural rigidity of the stiffener* γ_s *(proposed mechanism).*

CONCLUSIONS

In this paper, a methodology for the ultimate-strength prediction of longitudinal stiffened plate girders subjected to concentrated loads is presented. The results of the proposed methodology are compared with three approaches available in the literature. Based on those results, the conclusions are:

- Predicted strengths are conservative when the influence of longitudinal stiffeners is not considered in the prediction model.
- For all types of longitudinal stiffeners studied, the proposed model has a good correlation with the experimental results.

SYMBOLS

C_e	Effective length of the concentrated load $= l_b + 2t_f$
D	Flexural rigidity of unit width of the web plate $= Et_w^3/12(1-\nu^2)$
E	Young's modulus
F_{yw}	Web yield strength
F_{yf}	Web yield strength
F_{cr}	Critical buckling load
F_{exp}	Experimental ultimate load
F_R	Predicted ultimate load
F_y	Yield resistance
I_f	Moment of inertia of the flange $= b_f t_f^3/12$
I_{st}	Effective moment of inertia of the stiffener
M_f	Plastic moment of the flange
M_w	Plastic moment per unit length of the web
b_f	Width of flange
b_{st}	Width of longitudinal stiffener
b_1	Position of longitudinal stiffener
d	Total height of web panel $= h + 2t_f$
d_o	Length of web panel
h	Depth of web panel between the inner surfaces of the flanges
k_f	Buckling coefficient
k_{fl}	Buckling coefficient for longitudinally stiffened plate girders
k_{sl}	Contribution of a longitudinal stiffener to the buckling coefficient, k_{fl}
l_b	Length of concentrated loading
l_y	Effective load length
t_f	Flange thickness
t_{st}	Longitudinal stiffener thickness
t_w	Web thickness
α	Distance to yield lines in the web
α_0	Plateau length
β	"Spread" of plastic hinges in loaded flange
γ_s	Relative flexural rigidity of the longitudinal stiffener $= EI_{st}/Dh$
γ^t	Transition rigidity
δ	Vertical displacement of the concentrated load at the instant of collapse
η	Yielded length of web plate
θ	Angle defining deformation of web prior to collapse
λ_0	Plateau length
λ_F	Slenderness parameter
ν	Poisson's ratio
χ_F	Resistance function

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