ERRATA

Stability of Rectangular Connection Elements

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The design examples have been revised based on the errata to Dowswell and Whyte (2014), where Equation 25 was revised to:

$$C_b = \left[3 + \ln\left(\frac{L_b}{d}\right)\right]\left(1 - \frac{d_{ct}}{d}\right) \geq 1.84 \qquad (25)$$

REFERENCE

Dowswell, B. and Whyte, R. (2014), "Local Stability of Double-Coped Beams," *Engineering Journal*, AISC, Vol. 51, No. 1.

DESIGN EXAMPLES

Coped Beam Example 1

In this example, the cope buckling strength will be calculated for a double-coped W18×50 beam subjected to shear and axial compression. The cope is 42 in. long × 12 in. deep at both flanges as shown in Figure 16. The beam material is ASTM A992, and the beam is braced laterally by a floor slab at the face of the top-flange cope.

ASTM A992: $F_y = 50$ ksi
W18×50: $t_w = 0.355$ in. $d = 18.0$ in.

Cope length: $c = c_t = c_b = 4\frac{1}{2}$ in.
Cope depth: $d_c = d_{ct} = d_{cb} = 1\frac{1}{2}$ in.

Distance from the face of the cope to the end reaction: $e = 4\frac{1}{2}$ in.

Reduced depth of web, $h_o = 18.0\text{ in.} - (2)(1\frac{1}{2}\text{ in.}) = 15.0$ in.

The vertical and horizontal reactions are:

LRFD	ASD
$R_u = 90$ kips $F_{tu} = 120$ kips	$R_a = 60$ kips $F_{ta} = 80$ kips

The moment at the face of the cope is:

LRFD	ASD
$M_u = R_u e = (90\text{ kip})(4\frac{1}{2}\text{ in.})$ $= 405$ kip-in.	$M_a = R_a e = (60\text{ kip})(4\frac{1}{2}\text{ in.})$ $= 270$ kip-in.

Flexural Strength

From Dowswell and Whyte (2014), for beams with equal cope lengths at the top and bottom flange, C_b is

$$\begin{aligned} C_b &= \left[3+\ln\left(\frac{L_b}{d}\right)\right]\left(1-\frac{d_{ct}}{d}\right) \geq 1.84 \\ &= \left[3+\ln\left(\frac{4\tfrac{1}{2}\text{ in.}}{18.0\text{ in.}}\right)\right]\left(1-\frac{1\tfrac{1}{2}\text{ in.}}{18.0\text{ in.}}\right) \geq 1.84 \\ &= 1.84 \end{aligned}$$

AISC *Specification* Section F11 is used with $C_b = 1.84$, $L_b = c_t = 4\tfrac{1}{2}$ in., $t = t_w = 0.355$ in., and $d = h_o = 15.0$ in. (Dowswell and Whyte, 2014).

$$\begin{aligned} M_y &= (50\text{ ksi})\left[\frac{(0.355\text{ in.})(15.0\text{ in.})^2}{6}\right] \\ &= 666\text{ kip-in.} \end{aligned}$$

$$\begin{aligned} M_p &= (50\text{ ksi})\left[\frac{(0.355\text{ in.})(15.0\text{ in.})^2}{4}\right] \\ &= 998\text{ kip-in.} \end{aligned} \tag{35}$$

$$\begin{aligned} \frac{L_b d}{t^2} &= \frac{(4\tfrac{1}{2}\text{ in.})(15.0\text{ in.})}{(0.355\text{ in.})^2} \\ &= 536 \end{aligned}$$

$$\begin{aligned} \frac{0.08E}{F_y} &= \frac{(0.08)(29{,}000\text{ ksi})}{(50\text{ ksi})} \\ &= 46.4 \end{aligned}$$

$$\begin{aligned} \frac{1.9E}{F_y} &= \frac{(1.9)(29{,}000\text{ ksi})}{(50\text{ ksi})} \\ &= 1{,}100 \end{aligned}$$

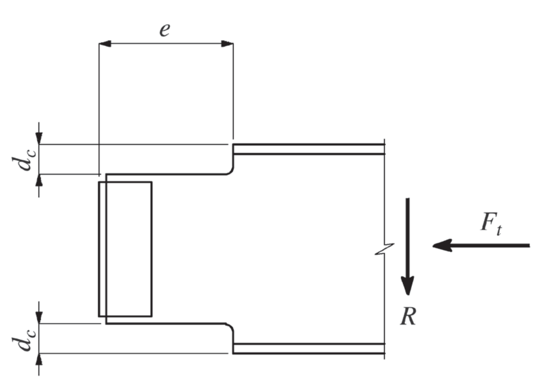

Fig. 16. Coped beam for Examples 1 and 2.

Because $\dfrac{0.08E}{F_y} < \dfrac{L_b d}{t^2} < \dfrac{1.9E}{F_y}$, the nominal flexural strength is:

$$M_n = C_b \left[1.52 - 0.274\left(\frac{L_b d}{t^2}\right)\frac{F_y}{E}\right] M_y \le M_p \tag{36}$$

$$= (1.84)\left[1.52 - (0.274)(536)\left(\frac{50 \text{ ksi}}{29{,}000 \text{ ksi}}\right)\right](666 \text{ kip-in.}) \le 998 \text{ kip-in.}$$

$$= 998 \text{ kip-in.}$$

The available flexural strength is:

LRFD	ASD
$\phi M_n = (0.90)(998 \text{ kip-in.})$ $= 898 \text{ kip-in.}$	$\dfrac{M_n}{\Omega} = \dfrac{998 \text{ kip-in.}}{1.67}$ $= 598 \text{ kip-in.}$

Axial Strength

$$A_g = h_o t_w$$

$$= (15.0 \text{ in.})(0.355 \text{ in.})$$

$$= 5.33 \text{ in.}^2$$

$$\frac{KL}{r} = \frac{(0.5)(4\tfrac{1}{2} \text{ in.})}{(0.355 \text{ in.})/\sqrt{12}}$$

$$= 22.0 < 25$$

Therefore, according to Specification Section J4.4, the nominal compressive strength is:

$$P_n = F_y A_g \qquad \text{(}\textit{Spec}\text{. Eq. J4-6)}$$

$$= (50 \text{ ksi})(5.33 \text{ in.}^2)$$

$$= 267 \text{ kips}$$

And the available compressive strength is:

LRFD	ASD
$\phi P_n = (0.90)(267 \text{ kips})$ $= 240 \text{ kips}$	$\dfrac{P_n}{\Omega} = \dfrac{267 \text{ kips}}{1.67}$ $= 160 \text{ kips}$

Stability Interaction

When $0.12 < \lambda_y \le 0.33$ ($KL/r \le 25$ for $F_y = 50$ ksi) and $M_n = M_p$, the effects of stability can be neglected, and the interaction equation in AISC *Specification* Section H1.1 is applicable.

LRFD	ASD
$\frac{P_r}{P_c} = \frac{P_u}{\phi P_n}$ $= \frac{120 \text{ kips}}{240 \text{ kips}}$ $= 0.500$	$\frac{P_r}{P_c} = \frac{P_a}{P_n/\Omega}$ $= \frac{80 \text{ kips}}{160 \text{ kips}}$ $= 0.500$

Because $\frac{P_r}{P_c} > 0.2$, *Specification* Section H1.1(a) is applicable,

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) \leq 1.0 \tag{53}$$

LRFD	ASD
$0.500 + \frac{8}{9}\left(\frac{405 \text{ kip-in.}}{898 \text{ kip-in.}}\right) = 0.901 < 1.0$	$0.500 + \frac{8}{9}\left(\frac{270 \text{ kip-in.}}{598 \text{ kip-in.}}\right) = 0.901 < 1.0$

Therefore, the cope is adequate for the limit state of local stability.

Coped Beam Example 2

In this example, the cope buckling strength will be calculated for a double-coped W18×50 beam subjected to shear and axial compression. The cope is 18 in. long × 1½ in. deep at both flanges as shown in Figure 16. The beam material is ASTM A992, and the beam is braced laterally by a floor slab at the face of the top-flange cope.

ASTM A992: $F_y = 50$ ksi
W18×50: $t_w = 0.355$ in. $d = 18.0$ in.

Cope length: $c = c_t = c_b = 18$ in.
Cope depth: $d_c = d_{ct} = d_{cb} = 1½$ in.

Distance from the face of the cope to the end reaction: $e = 18$ in.

Reduced depth of web, $h_o = 18.0 \text{ in.} - (2)(1½ \text{ in.}) = 15.0$ in.

The vertical and horizontal reactions are:

LRFD	ASD
$R_u = 15$ kips	$R_a = 10$ kips
$F_{tu} = 45$ kips	$F_{ta} = 30$ kips

The moment at the face of the cope is:

LRFD	ASD
$M_u = R_u e$ $= (15 \text{ kip})(18 \text{ in.})$ $= 270$ kip-in.	$M_a = R_a e$ $= (10 \text{ kip})(18 \text{ in.})$ $= 180$ kip-in.

Flexural Strength

From Dowswell and Whyte (2014), for beams with equal cope lengths at the top and bottom flange, C_b is:

$$
\begin{aligned}
C_b &= \left[3+\ln\left(\frac{L_b}{d}\right)\right]\left(1-\frac{d_{ct}}{d}\right) \geq 1.84 \\
&= \left[3+\ln\left(\frac{18\text{ in.}}{18.0\text{ in.}}\right)\right]\left(1-\frac{1\tfrac{1}{2}\text{ in.}}{18.0\text{ in.}}\right) \geq 1.84 \\
&= 2.75
\end{aligned}
$$

AISC *Specification* Section F11 is used with $C_b = 2.75$, $L_b = c_t = 18$ in., $t = t_w = 0.355$ in., and $d = h_o = 15.0$ in. (Dowswell and Whyte, 2014).

$$
\begin{aligned}
S_x &= \frac{(0.355\text{ in.})(15.0\text{ in.})^2}{6} \\
&= 13.3\text{ in.}^3
\end{aligned}
$$

$$
\begin{aligned}
M_p &= (50\text{ ksi})\left[\frac{(0.355\text{ in.})(15.0\text{ in.})^2}{4}\right] \\
&= 998\text{ kip-in.}
\end{aligned}
$$

$$
\begin{aligned}
\frac{L_b d}{t^2} &= \frac{(18\text{ in.})(15.0\text{ in.})}{(0.355\text{ in.})^2} \\
&= 2{,}140
\end{aligned}
$$

$$
\begin{aligned}
\frac{1.9E}{F_y} &= \frac{(1.9)(29{,}000\text{ ksi})}{(50\text{ ksi})} \\
&= 1{,}100
\end{aligned}
$$

Because $\frac{1.9E}{F_y} < \frac{L_b d}{t^2}$, the critical stress is:

$$
\begin{aligned}
F_{cr} &= \frac{1.9EC_b}{\frac{L_b d}{t^2}} \qquad (38) \\
&= \frac{(1.9)(29{,}000\text{ ksi})(2.75)}{2{,}140} \\
&= 70.8\text{ ksi}
\end{aligned}
$$

And the nominal flexural strength is:

$$
\begin{aligned}
M_n &= F_{cr} S_x \leq M_p \qquad (37) \\
&= (70.8\text{ ksi})(13.3\text{ in.}^3) \leq 998\text{ kip-in.} \\
&= 942\text{ kip-in.}
\end{aligned}
$$

The available flexural strength is:

LRFD	ASD
$\phi M_n = (0.90)(942 \text{ kip-in.}) = 848 \text{ kip-in.}$	$\dfrac{M_n}{\Omega} = \dfrac{942 \text{ kip-in.}}{1.67} = 564 \text{ kip-in.}$

Axial Strength

$$A_g = h_o t_w$$
$$= (15.0 \text{ in.})(0.355 \text{ in.})$$
$$= 5.33 \text{ in.}^2$$

$$\frac{KL}{r} = \frac{(0.5)(18 \text{ in.})}{(0.355 \text{ in.})/\sqrt{12}}$$
$$= 87.8 > 25$$

Therefore, the axial strength will be calculated according to AISC *Specification* Chapter E. The elastic buckling stress is:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad (\textit{Spec}\text{. Eq. E3-4})$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(87.8)^2}$$
$$= 37.1 \text{ ksi}$$

The slenderness parameter is:

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 113$$

Because $\dfrac{KL}{r} < 4.71\sqrt{\dfrac{E}{F_y}}$, the critical buckling stress is calculated using AISC *Specification* Equation E3-2:

$$F_{cr} = \left[0.658^{\left(\frac{F_y}{F_e}\right)}\right] F_y$$
$$= \left[0.658^{\left(\frac{50 \text{ ksi}}{37.1 \text{ ksi}}\right)}\right](50 \text{ ksi})$$
$$= 28.4 \text{ ksi}$$

And the nominal compressive strength is:

$$P_n = F_{cr} A_g \qquad (\textit{Spec}\text{. Eq. E3-1})$$
$$= (28.4 \text{ ksi})(5.33 \text{ in.}^2)$$
$$= 151 \text{ kips}$$

The available compression strength is:

LRFD	ASD
$\phi P_n = (0.90)(151 \text{ kips})$ $= 136 \text{ kips}$	$\dfrac{P_n}{\Omega} = \dfrac{151 \text{ kips}}{1.67}$ $= 90.4 \text{ kips}$

Stability Interaction

When $0.33 < \lambda_y$ ($KL/r > 25$ for 50 ksi) or $M_n < M_p$, the effects of stability must be included in the design, and the interaction equation in AISC *Specification* Section H2 is applicable. Due to the high flexural stiffness in the strong-axis direction, the strong-axis second-order moment is neglected.

$$\frac{P_r}{P_c} + \frac{M_{rx}}{M_{cx}} \leq 1.0 \qquad \text{(from Eq. 57)}$$

LRFD	ASD
$\dfrac{45 \text{ kip}}{136 \text{ kip}} + \dfrac{270 \text{ kip-in.}}{848 \text{ kip-in.}} = 0.649 < 1.0$	$\dfrac{30 \text{ kip}}{90.4 \text{ kip}} + \dfrac{180 \text{ kip-in.}}{564 \text{ kip-in.}} = 0.651 < 1.0$

Therefore, the cope is adequate for the limit state of local stability.

Coped Beam Example 3

In this example, the cope buckling strength will be calculated for a double-coped W18×50 beam subjected to shear and axial tension. The cope is 18 in. long × 1½ in. deep at both flanges as shown in Figure 17. The beam material is ASTM A992, and the beam is braced laterally by a floor slab at the face of the top-flange cope.

ASTM A992: $F_y = 50$ ksi
W18×50: $t_w = 0.355$ in. $\quad d = 18.0$ in.

Cope length: $c = c_t = c_b = 18$ in.
Cope depth: $d_c = d_{ct} = d_{cb} = 1\frac{1}{2}$ in.

Distance from the face of the cope to the end reaction: $e = 18$ in.

Reduced depth of web, $h_o = 18.0 \text{ in.} - (2)(1\frac{1}{2} \text{ in.}) = 15.0$ in.

The vertical and horizontal reactions are:

LRFD	ASD
$R_u = 21$ kips $F_{tu} = 120$ kips	$R_a = 14$ kips $F_{ta} = 80$ kips

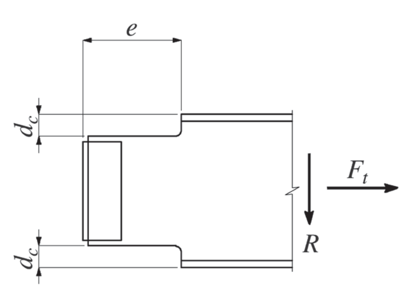

Fig. 17. Coped beam for Example 3.

The moment at the face of the cope is:

LRFD	ASD
$M_u = R_u e$ $= (21\text{ kip})(18\text{ in.})$ $= 378\text{ kip-in.}$	$M_a = R_a e$ $= (14\text{ kip})(18\text{ in.})$ $= 252\text{ kip-in.}$

Flexural Strength

From Dowswell and Whyte (2014), for beams with equal cope lengths at the top and bottom flange, C_b is

$$\begin{aligned}C_b &= \left[3+\ln\left(\frac{L_b}{d}\right)\right]\left(1-\frac{d_{ct}}{d}\right) \geq 1.84\\ &= \left[3+\ln\left(\frac{18\text{ in.}}{18.0\text{ in.}}\right)\right]\left(1-\frac{1\tfrac{1}{2}\text{ in.}}{18.0\text{ in.}}\right) \geq 1.84\\ &= 2.75\end{aligned}$$

AISC *Specification* Section H1.2 will be used to calculate the effect of the tension load on the lateral-torsional buckling strength.

$$\begin{aligned}I_y &= \frac{h_o t_w^3}{12}\\ &= \frac{(15.0\text{ in.})(0.355\text{ in.})^3}{12}\\ &= 0.0559\text{ in.}^4\end{aligned}$$

$$P_{ey} = \frac{\pi^2 E I_y}{L_b^2} \tag{56}$$

$$\begin{aligned}&= \frac{\pi^2(29{,}000\text{ ksi})(0.0559\text{ in.}^4)}{(18\text{ in.})^2}\\ &= 49.4\text{ kips}\end{aligned}$$

According to Equation 55, $C_b' = C_b\sqrt{1+\dfrac{\alpha P_r}{P_{ey}}}$:

LRFD	ASD
$C_b' = 2.75\sqrt{1+\dfrac{(1.0)(120\text{ kips})}{49.4\text{ kips}}}$ $= 5.09$	$C_b' = 2.75\sqrt{1+\dfrac{(1.6)(80\text{ kips})}{49.4\text{ kips}}}$ $= 5.21$

AISC *Specification* Section F11 is used with $C_b = 5.09$ (LRFD), $C_b = 5.21$ (ASD), $L_b = c_t = 18$ in., $t = t_w = 0.355$ in., and $d = h_o =$ 15.0 in. (Dowswell and Whyte, 2014).

$$\begin{aligned}S_x &= \frac{(0.355\text{ in.})(15.0\text{ in.})^2}{6}\\ &= 13.3\text{ in.}^3\end{aligned}$$

$$M_p = (50 \text{ ksi})\left[\frac{(0.355 \text{ in.})(15.0 \text{ in.})^2}{4}\right]$$
$$= 998 \text{ kip-in.}$$

$$\frac{L_b d}{t^2} = \frac{(18 \text{ in.})(15.0 \text{ in.})}{(0.355 \text{ in.})^2}$$
$$= 2{,}140$$

$$\frac{1.9E}{F_y} = \frac{(1.9)(29{,}000 \text{ ksi})}{(50 \text{ ksi})}$$
$$= 1{,}100$$

Because $\dfrac{1.9E}{F_y} < \dfrac{L_b d}{t^2}$, the critical stress is:

LRFD	ASD
$F_{cr} = \dfrac{1.9EC'_b}{\dfrac{L_b d}{t^2}}$ $= \dfrac{(1.9)(29{,}000 \text{ ksi})(5.09)}{2{,}140}$ $= 131 \text{ ksi}$	$F_{cr} = \dfrac{1.9EC'_b}{\dfrac{L_b d}{t^2}}$ $= \dfrac{(1.9)(29{,}000 \text{ ksi})(5.21)}{2{,}140}$ $= 134 \text{ ksi}$

The nominal flexural strength is:

$$M_n = F_{cr} S_x \le M_p \tag{37}$$
$$= (131 \text{ ksi})(13.3 \text{ in.}^3) \le 998 \text{ kip-in.}$$
$$= 998 \text{ kip-in.}$$

The available flexural strength is:

LRFD	ASD
$\phi M_n = (0.90)(998 \text{ kip-in.})$ $= 898 \text{ kip-in.}$	$\dfrac{M_n}{\Omega} = \dfrac{998 \text{ kip-in.}}{1.67}$ $= 598 \text{ kip-in.}$

Axial Strength

$$A_g = h_o t_w$$
$$= (15.0 \text{ in.})(0.355 \text{ in.})$$
$$= 5.33 \text{ in.}^2$$

The nominal tensile strength is:

$$P_n = F_y A_g$$
$$= (50 \text{ ksi})(5.33 \text{ in.}^2)$$
$$= 267 \text{ kips}$$

The available tensile strength is:

LRFD	ASD
$\phi P_n = (0.90)(267 \text{ kips})$ $= 240 \text{ kips}$	$\dfrac{P_n}{\Omega} = \dfrac{267 \text{ kips}}{1.67}$ $= 160 \text{ kips}$

Interaction

For axial tension loads with $M_n = M_p$, plastic interaction according to Equation 62 is applicable.

$$\left(\frac{P_r}{P_c}\right)^2 + \frac{M_{rx}}{M_{cx}} \le 1.0$$

LRFD	ASD
$\left(\dfrac{120 \text{ kip}}{240 \text{ kip}}\right)^2 + \dfrac{378 \text{ kip-in.}}{898 \text{ kip-in.}} = 0.671 < 1.0$	$\left(\dfrac{80 \text{ kip}}{160 \text{ kip}}\right)^2 + \dfrac{252 \text{ kip-in.}}{598 \text{ kip-in.}} = 0.671 < 1.0$

Therefore, the cope is adequate for the limit state of local stability.