Assessment of I-Section Member LTB Resistances Considering Experimental Test Data and Practical Inelastic Buckling Design Calculations

LAKSHMI SUBRAMANIAN, WOO YONG JEONG, RAJA YELLEPEDDI and DONALD W. WHITE

ABSTRACT

The current AASHTO and AISC *Specification* equations characterizing the lateral-torsional buckling (LTB) resistance of steel I-section members are the same, with minor exceptions, and are based in large part on unified provisions calibrated to experimental data. This paper takes a fresh look at the correlation of the flexural strength predictions from these equations with a large experimental data set compiled from research worldwide. To account fully for the moment gradient and end restraint effects present in the physical tests, the study employs practical buckling calculations using inelastic stiffness reduction factors (SRFs) based on the design resistance equations. The study focuses on uniform bending tests as well as moment gradient tests in which the transverse loads are applied at braced locations. Reliability indices are estimated in the context of building design. It is shown that a proposed modified form of the current resistance equations provides a more uniform level of reliability, as a function of the LTB slenderness, consistent with the target intended in the AISC *Specification*. The paper also calls attention to the limited experimental data pertaining to the inelastic LTB resistances in certain cases. The paper concludes by providing additional recommendations for LTB strength calculations in routine design, including illustrative plots conveying the impact of the proposed changes.

Keywords: lateral torsional buckling, inelastic buckling, experimental tests, AASHTO, AISC *Specification*, reliability indices.

INTRODUCTION

The AASHTO Specification (2016) and AISC *Specification* (2016) I-section member lateral-torsional buckling (LTB) strength curves are based largely on the so-called unified provisions (White, 2008). The unified provisions were developed given an extensive assessment of several large experimental datasets encompassing a wide range of member types and strength limit states (White and Jung, 2008; White and Kim, 2008; White et al., 2008). The unified AISC and AASHTO provisions differ in only a few minor technical details, which are discussed in the next section. They also differ in the form in which their rules are presented—the AISC *Specification* emphasizes the simplified design of compact section members, the AASHTO Specifications feature the simplified design of slender-web members, and the unified provisions present the resistance equations as a single set of flowcharts covering all I-section member types.

Various researchers have observed that finite element (FE) test simulations using idealized boundary conditions and commonly employed deterministic residual stresses and initial geometric imperfections tend to exhibit smaller strengths than indicated by experimental data (Greiner et al., 2001; Kim, 2010; Lokhande and White, 2014; Subramanian and White, 2017a). This disconnect between test simulations and experimental test data has led to wide variations among the results from predictor equations derived predominantly from test simulations, such as the LTB equations defined in CEN (2005), versus those obtained from predictor equations derived predominantly from experimental tests, such as the AASHTO and AISC *Specification* equations. Figure 1, adapted from Ziemian (2010), illustrates the stark differences among the LTB strength predictions employed in various current international steel design standards. Engineers who work on both U.S. and international projects can observe predicted nominal strengths that differ by more than a factor of 2 in the most extreme cases. The reader is referred to Ziemian (2010) for a detailed discussion of the origins and nature of the curves in this plot.

It should be noted that if the members considered in Figure 1 are only slightly singly symmetric, the AISC

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Specification singly symmetric curve applies. According to the unified provisions, this is the appropriate LTB resistance curve for both doubly- and singly-symmetric sections. In addition, the reader should note that the curve for welded I-section members recommended by MacPhedran and Grondin (2009) is essentially identical to the Standards Association of Australia (SAA) (1998) strength curve. Lastly, it should be noted that the relatively optimistic predictions by MacPhedran and Grondin (2009) (compared to the majority of the other equations) is due to a lack of consideration of end restraint from adjacent test unbraced lengths and, hence, the use of a lateral-torsional buckling effective length factor of 1.0 in the prediction calculations (MacPhedran and Grondin, 2011). The CSA Group (2014) curve is based largely on the statistical analysis conducted by Baker and Kennedy (1984) of Dibley's (1969) rolled I-section member LTB tests, considering the effective lengths reported by Dibley; however, these same tests are included in the analyses by White and Jung (2004, 2008), providing validation of the unified provisions. Therefore, the conclusions from White and Jung (2004, 2008) and from Baker and Kennedy (1984) are at odds. The CSA Group (2014) curve appears to be more related to MacPhedran and Grondin's recommended curve, based on the use of $K = 1.0$. From White and Jung (2004), the unified provisions, using elastic LTB K factors per Nethercot and Trahair (1976), predict Dibley's 30 rolled I-section test resistances with a mean M_{test}/M_n of 1.017 and a coefficient of variation of 0.064. In addition, a substantially larger number of rolled I-section tests are considered in White and Jung (2004, 2008) and White and Kim (2004, 2008), as well as in this paper.

Subramanian and White (2017a) discuss the need to resolve the disconnects between FE test simulations, the AASHTO Specification (generally referred to in this paper as simply AASHTO) design strength equations, and experimental test data. It is essential to resolve these disconnects so that engineers can properly apply refined methods that move beyond traditional effective length and moment modification factors, K and C_b, where merited, to better quantify member LTB strengths. AISC *Specification* Appendix A provides guidance for conducting test simulations for design assessment; however, engineers who employ these methods typically will find that their calculated strengths are substantially smaller than strengths estimated using the ordinary AISC *Specification* Chapter F equations.

Subramanian and White (2017a, 2017b, 2017c) explain that the preceding disconnect can be resolved partly by the use of smaller nominal residual stresses and geometric imperfections in FE test simulations. This finding is based on correlation with experimental data as well as evaluation of sensitivity studies using test simulations (Subramanian and White, 2017a). However, these investigators also recommend the following modifications to the unified LTB resistance equations:

1. The plateau length, $L_p = 0.63 r_t \sqrt{E/F_{yc}}$, should be employed for all cross-section types (Subramanian and White, 2017b), as opposed to the use of this equation with a coefficient of 1.1 in the unified provisions. In addition to the findings by Subramanian and White, Greiner and Kaim (2001) and Kim (2010) have shown that test simulations suggest a smaller plateau length

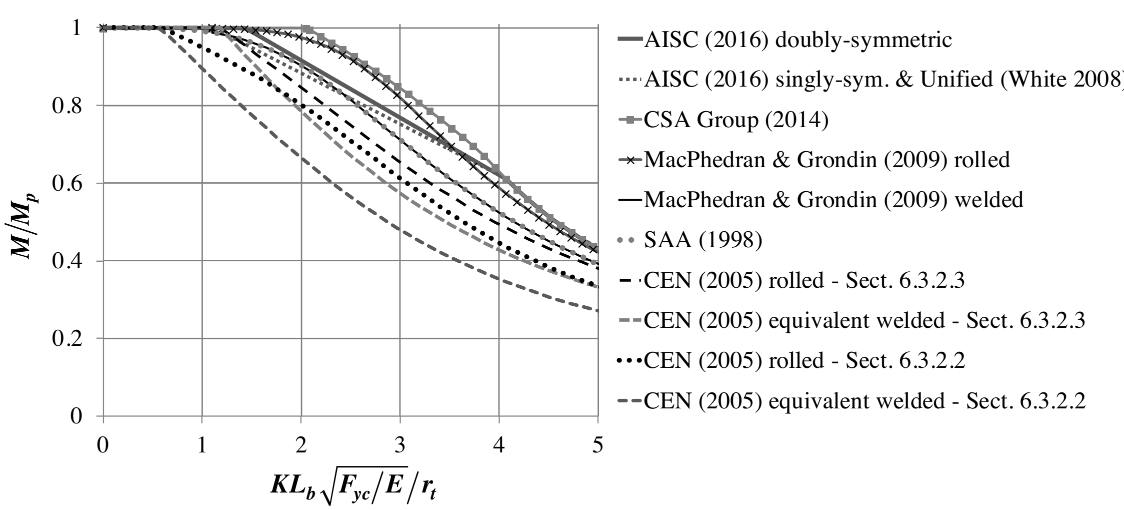


Fig. 1. Comparison of nominal LTB resistances for W27×84 beams (F_y = 50 ksi) and equivalent section welded beams subjected to uniform bending moment.

than indicated by the unified provisions (as well as by many current design specifications).

2. A smaller maximum stress level for elastic LTB of $F_{yr} = 0.5F_{yc}$ ($F_L = 0.5F_{yc}$ in the context of the AISC *Specification*) should be utilized for all cross-section types, including hybrid girders (Subramanian and White, 2017b). This change recognizes the fact that the LTB resistance can be affected significantly by the combined influence of residual stresses, unavoidable geometric imperfections, and second-order lateral-torsional displacement amplifications at unbraced lengths corresponding to the current F_{yr} (F_L) value of $0.7F_{yc}$.

3. The noncompact web slenderness limit, λ_{rw}, should be modified to

$$\lambda_{rw} = \left(3.1 + 2.5\frac{A_{fc}}{A_{wc}}\right)\sqrt{\frac{E}{F_{yc}}} \tag{1}$$
$$\geq 4.6\sqrt{\frac{E}{F_{yc}}}$$
$$\leq 5.7\sqrt{\frac{E}{F_{yc}}}$$

(Subramanian and White, 2017b, 2017d). This change is based on observations, from physical tests and test simulations, that I-girders with relatively small flanges compared to the web area exhibit some reduction in their flexural resistances when their webs are near the current noncompact web limit. That is, noncompact-web girders of this nature tend to perform more like slender-web girders.

Given the preceding recommendations, the proposed LTB resistance curve for the W27×84 and equivalent welded section members is as illustrated by the dark bold curve in Figure 2. It should be noted that the proposed L_p is comparable to the effective L_p values from the CEN (2005) Section 6.3.2.2 equations and that the proposed inelastic LTB curve is approximately tangent to the theoretical elastic LTB curve at the proposed L_r.

This paper takes a fresh look at the predictions relative to the experimental data for uniform moment and moment gradient tests in the context of the unified flexural resistance equations (White, 2008) as well as the above-proposed modifications to these resistance equations. With respect to moment gradient cases, the paper focuses on tests in which the transverse loads are applied at braced locations. Moment gradient tests considering loads applied at nonbraced locations as well as load height effects are addressed by Toğay et al. (2016). The experimental results considered in this paper include the prior data from White and Jung (2008) and White and Kim (2008) plus additional data from Kusuda et al. (1960) and Righman (2005).

In this paper, the nominal design resistances are determined using inelastic buckling calculations based on inelastic stiffness reduction factors (SRFs) obtained from corresponding design LTB resistance equations (White et al., 2016a). This approach allows for a practical, yet reasonably rigorous, accounting for (1) continuity effects across braced points, including the restraint of more heavily yielded member segments by adjacent unbraced lengths that

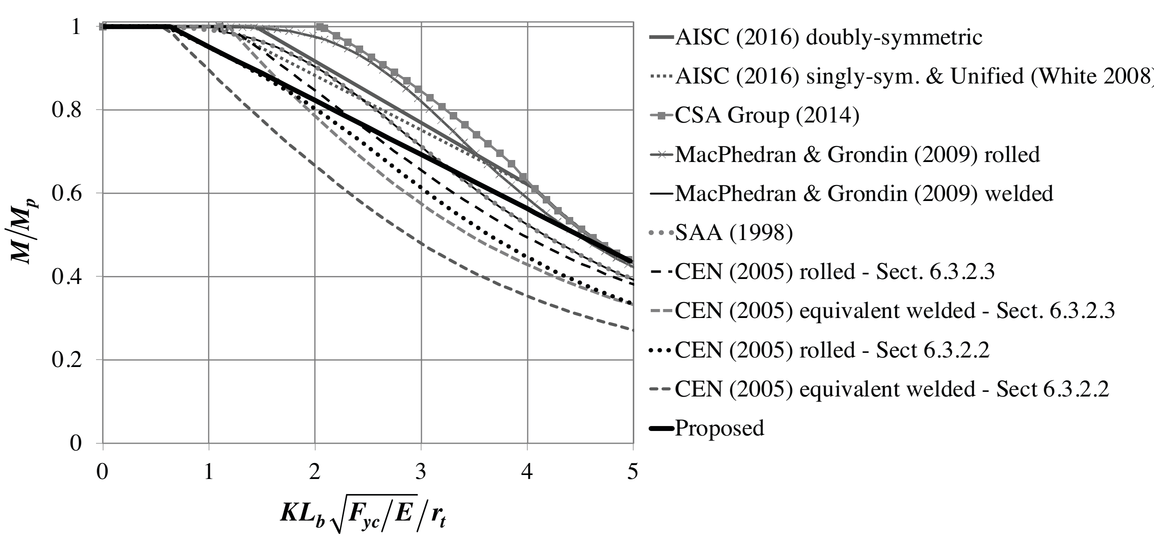


Fig. 2. Comparison of proposed LTB resistance for W27×84 beams (F_y = 50 ksi) and equivalent section welded beams subjected to uniform bending moment to current LTB resistance curves.

are relatively elastic (i.e., elastic and inelastic LTB effective length effects), and (2) variations in the spread of yielding along the member lengths due to moment gradient effects (i.e., effects approximated by C_b factors in common manual calculations). The subject inelastic buckling calculations are implemented within the SABRE2 software (White et al., 2016a, 2016b). SABRE2 applies the SRFs associated with the selected design LTB resistance equations along with thin-walled, open-section beam theory for the member strength assessment. SABRE2 allows the choice of either the unified LTB provisions or the proposed modifications to the design strength equations.

It is important to note that accounting for moment gradient effects, as well as end restraint effects from adjacent unbraced lengths and/or end connections, is essential to obtain any meaningful correlation between test and/or test simulation results and LTB strength predictions. The inelastic buckling solutions using the approach implemented in SABRE2 provide, in effect, exact member LTB effective lengths (inelastic or elastic, as applicable) based on the selected design resistance equations. These capabilities provide an unprecedented capability for engineers to account accurately for the combined influence of moment gradient and end restraint effects on the inelastic LTB strength limit states. However, the accuracy of this approach depends critically on the proper definition of the underlying LTB strength curve.

To evaluate the quality of the current unified and recommended modified LTB strength equations, this paper provides estimates of reliability indices, in the context of building design, using the preceding SRF-based approach for the prediction of the LTB test resistances. The reliability indices are estimated across a full range of LTB slenderness values. An important aim of this paper is to compare the reliability indices determined using the unified LTB provisions (which are the basis for the current AASHTO Specifications) to those obtained using the proposed modified LTB equations.

Lastly, the paper provides additional recommendations for LTB strength calculations in routine design, including a set of illustrative plots conveying the impact of the proposed changes.

OVERVIEW OF AISC, AASHTO AND UNIFIED LATERAL-TORSIONAL BUCKLING EQUATIONS

The lateral-torsional buckling (LTB) curves for I-section members in AASHTO (2016) and the AISC *Specification* (2016) consist of three distinct regions: the plateau region, the inelastic LTB region, and the elastic LTB region. The plateau resistance is equal to the plastic flexural strength for compact sections, while for noncompact web sections, it is the yield moment multiplied by the web plastification factor, R_{pc}. The plateau strength for slender-web sections is the yield moment reduced by the web bend-buckling factor, R_b (the parameter R_b in AASHTO is the same as the parameter R_{pg} in the AISC *Specification*). Members in uniform bending with effective unbraced lengths (KL_b) greater than L_r are designed using the theoretical elastic LTB strength, where L_r is the limiting unbraced length at which residual stresses, geometric imperfections, and second-order amplification of the lateral-torsional displacements start to influence the nominal resistance for this type of loading. Members with KL_b between L_p (the limiting length at which a member can attain the plateau strength) and L_r are designed using the inelastic LTB resistance, obtained by linearly interpolating between the plateau and the elastic LTB anchor points at L_p and L_r. It should be noted that both AASHTO and the AISC *Specification* give their LTB equations in terms of just the unsupported length between the braced points, L_b, recognizing the fact that the LTB resistance can be assessed conservatively and practically by assuming a LTB effective length factor of $K = 1$ in most situations. However, the commentaries of both specifications explain that KL_b may be used in place of L_b to obtain a more refined estimate of the LTB resistance when this beneficial refinement is merited.

The LTB equations in AASHTO and the AISC *Specification* follow the unified provisions (White, 2008) with minor exceptions. The differences among the unified provisions, the AISC *Specification*, AASHTO, and proposed modified provisions (Subramanian and White, 2017b) are shown in Table 1. The reader is referred to the AISC *Specification*, AASHTO, or White (2008) for all other equations required to calculate the LTB strengths.

This paper focuses on the differences in the strength predictions between the unified and the proposed modified LTB equations. These differences are quantified directly as well as via estimated reliability indices associated with the two sets of provisions. The paper also provides additional recommendations for LTB strength calculations in routine design, including a discussion of the impact of the proposed modifications.

SHORTCOMINGS OF THE UNIFIED PROVISIONS

The studies conducted in this research are an improvement over the prior calibrations (White and Jung, 2008; White and Kim, 2008) with respect to several key aspects discussed next.

Inelastic versus Elastic Effective Length Effects

The prior calibration of the unified flexural resistance provisions to experimental data (White and Jung, 2008; White and Kim, 2008) was conducted using approximate elastic effective length factors, $K_{elastic}$, calculated per Nethercot and Trahair (1976). When the critical unbraced length experiences some yielding, the restraint provided by adjacent

Table 1. LTB Equation Comparisons				
Parameter	**Unified Provisions**	**AISC *Specification***	**AASHTO**	**Proposed Equations**
L_p (doubly-symmetric, compact-web members with compact or noncompact flanges)	$1.1r_t\sqrt{E/F_{yc}}$	$1.76r_y\sqrt{E/F_y}$	$1.0r_t\sqrt{E/F_{yc}}$	$0.63r_t\sqrt{E/F_{yc}}$
L_p (all other I-section members)	$1.1r_t\sqrt{E/F_{yc}}$	$1.1r_t\sqrt{E/F_{yc}}$	$1.0r_t\sqrt{E/F_{yc}}$	$0.63r_t\sqrt{E/F_{yc}}$
$F_{yr}{}^*$	$\min\begin{bmatrix}0.7F_{yc}\\ R_hF_{yt}\dfrac{S_{xt}}{S_{xc}}\\ F_{yw}\end{bmatrix} \geq 0.5F_{yc}$	$\min\begin{bmatrix}0.7F_y\\ F_y\dfrac{S_{xt}}{S_{xc}}\end{bmatrix} \geq 0.5F_{yc}$	$\min\begin{bmatrix}0.7F_{yc}\\ R_hF_{yt}\dfrac{S_{xt}}{S_{xc}}\\ F_{yw}\end{bmatrix} \geq 0.5F_{yc}$	$0.5F_{yc}$
λ_{rw}	$5.7\sqrt{\dfrac{E}{F_{yc}}}$	$5.7\sqrt{\dfrac{E}{F_{yc}}}$	$5.7\sqrt{\dfrac{E}{F_{yc}}}$	$\left(3.1+2.5\dfrac{A_{fc}}{A_{wc}}\right)\sqrt{\dfrac{E}{F_{yc}}} \geq 4.6\sqrt{\dfrac{E}{F_{yc}}} \leq 5.7\sqrt{\dfrac{E}{F_{yc}}}$

* F_{yr} is denoted by F_L in the AISC *Specification*.

segments is typically more effective than when the critical segment is assumed to remain elastic (Subramanian and White, 2017b; Trahair and Hancock, 2004). This results in the theoretical $K_{elastic}$ being larger than the true effective length factor. Subramanian and White (2017b) show that the resulting larger estimated plateau length, L_p, in the unified provisions is due to the implicit inelastic effective length factor, $K_{inelastic}$, being smaller than $K_{elastic}$.

Due to the calibration to experimental test strengths using a larger elastic effective unbraced length ($K_{elastic}L_b$) instead of the smaller and true inelastic effective length ($K_{inelastic}L_b$), the test data are shifted toward the right in LTB strength plots. That is, for a given experimentally determined test strength, the corresponding elastic effective length is generally larger than the true inelastic effective length. The test strengths are, hence, incorrectly taken to be higher at longer lengths, resulting in a falsely optimistic calibration to the data. The use of $K_{inelastic}$ in the calibration to the test data correctly shifts the data toward the left in strength versus effective unbraced length plots compared to the prior calibrations.

Inelastic LTB effective length effects can be considered quite accurately and efficiently using tools such as SABRE2 (White et al., 2016b), which perform buckling analyses using inelastic stiffness reduction factors (SRFs) based directly on the specified design resistance equations. Similar approaches have been proposed by Trahair and Hancock (2004) in the context of the Australian standard (SAA, 1998) and by Kucukler et al. (2015a, 2015b) in the context of Eurocode 3 (CEN, 2005). The inelastic buckling calculations strictly do not require the calculation of any effective length factors. One can simply use the inelastic buckling analysis results directly. However, effective lengths are a convenient way of quantifying the LTB design resistances as a function of the end restraint (warping, lateral bending, and/or lateral displacement) provided to the critical unbraced length of a member.

The use of computational tools such as SABRE2 provides a major advantage over approximate $K_{elastic}$ or $K_{inelastic}$ calculations, in that the calculations are fast. Furthermore, the restraint from adjacent segments is implicitly accounted for, without the need for simplifying assumptions inherent in manual computations. For example, uniform moment tests conducted by Richter (1998) with several unbraced lengths within the test specimen, and with test fixtures providing restraint at the member ends, were assumed to have a K of 1.0 within the critical unbraced segments in the prior research by White and Jung (2004). [As noted earlier, the elastic LTB K factor estimates in these studies used the approach forwarded by Nethercot and Trahair (1976); this approach gives an estimate of $K = 1$ in Richter's tests.] However, the authors have found that K is significantly smaller than 1.0 for the critical unbraced lengths when there are only three to five unbraced lengths (such as in Richter's tests) due to restraint from the end fixtures. It is only in the presence of a large number of adjacent unbraced segments subjected to uniform moment that $K_{inelastic}$ approaches 1.0. SABRE2 implicitly accounts for the general elastic–inelastic end restraint effects in the design calculation of the test strengths.

Inelastic versus Elastic Moment Gradient Effects

AASHTO and the AISC *Specification* apply a multiplicative moment gradient modification factor, C_b, to the elastic and inelastic LTB regions of the design curves in the case of moment gradient loading. That is, the unified provisions and both of these specifications simply multiply (i.e., scale) the elastic or inelastic LTB resistance by C_b, while limiting the corresponding resistance to the plateau strength. Numerous expressions for C_b exist in the literature—all of which are based on elastic buckling solutions. Subramanian and White (2017c) discuss the implications of using this elastically derived C_b in the inelastic LTB equations and report an "inelastic C_b" effect. That is, the basic C_b factor approach in the current AASHTO Specification and the AISC *Specification* tends to overpredict the moment gradient effects due to partial yielding in the members. This effect is relatively small when the maximum moment occurs at a braced point, as observed in the original developments by Yura et al. (1978). However, it can be more significant for transversely loaded cases, where the maximum moment occurs within an unbraced length. Computational tools such as SABRE2 implicitly account for this "inelastic C_b" effect. Thus, the calculated nominal strengths presented in this paper are expected to be more representative of the true member strengths.

EXPERIMENTAL TEST DATABASE

The experimental tests discussed in this paper are focused on noncomposite I-section members in which LTB is the controlling flexural limit state. Tests governed by flange local buckling (FLB) and tension flange yielding (TFY) limit states are addressed in the referenced prior studies. With the exception of the additional tests included from Kusuda et al. (1960) and Righman (2005), details of the test configurations, cross-section dimensions, and member properties are provided in White and Jung (2004) and White and Kim (2004). The data from the additional tests may be found in the corresponding reports. The prior datasets were a central focus in the development of the unified provisions (White, 2008), which serve as one basis for the current AISC and AASHTO flexural resistance provisions.

In the current study, the flexural resistances are calculated using inelastic buckling analysis procedures (White et al., 2016a) implemented in SABRE2 (White et al., 2016b). Inelastic SRFs for LTB are calculated using the unified provisions as well as the proposed modified provisions. The corresponding resistances are referred to respectively as $M_{nUnified}$ and $M_{nProposed}$. The test members are modeled using the measured geometry and separate measured flange and web yield strengths, where these data are available. The elastic modulus of the steel is taken as $E = 29{,}000$ ksi for all members. For rolled beams, the web-to-flange fillet areas are included in the models and in the underlying resistance calculations. Although this practice does not greatly affect the predicted strength of the members, it has been observed to give the best correlation with the test results (White and Jung, 2008). The web-to-flange fillet areas are taken as zero for welded sections.

The following detailed classifications of the test members are the same as in White and Jung (2004) and White and Kim (2004). Members for which the flange dimensions or the web depths are reported only as nominal values, or where these dimensions are reported to less than three significant digits, are considered as "nominal/approximate geometry." All other tests, including those in which the web thicknesses are reported as nominal values, or where only a single cross-section yield strength is provided, are considered as "accurate measured geometry." This is because of the minor influence of the web yield strength and thickness on the flexural resistance compared with the flange dimensions and the web depths. For rolled sections where the cross-section properties listed in the test reports do not include the web-to-flange fillet areas, the web-to-flange fillet areas are taken equal to the difference between the area listed in nominal property tables and the area calculated from the nominal plate dimensions. The web-to-flange fillet yield strengths are taken to be equal to the yield strength of the flange material.

The results in the following sections are delineated according to the normalized slenderness, c, defined as $\left(KL_b\sqrt{F_{yc}/E}\right)/r_t$. The effective length factor, K, is back-calculated as the value that, when substituted into the proposed LTB equations, yields the nominal resistance obtained from SABRE2 (based on the proposed equations). In the cases where the members attain the plateau resistance per SABRE2, the corresponding K factor is undetermined. In this case, K is calculated using the approximate elastic effective length procedure given by Nethercot and Trahair (1976), which is the procedure utilized in the prior development of the unified provisions (White, 2008). In the event that the plateau strength is not obtained using this $K_{elastic}$ value, K is determined as the largest value for which the member attains the plateau strength. Although this is a coarse estimate of K, it is reasonable considering that the K factors are employed only to classify the experimental tests into different ranges based on the "LTB slenderness" in this work.

There is one exception to the preceding procedure. In the tests by Suzuki and Kubodera (1973), elaborate test fixtures were employed that provided ideal, torsionally simply supported end conditions on the test unbraced length while developing uniform bending moment within the test. In these cases, K is equal to 1.0. When these tests are modeled in SABRE2, LTB resistances precisely corresponding to the nominal resistance equations with an unbraced length $KL_b = L_b$ are obtained.

In the prior unified resistance calculations (White, 2008), the LTB plateau length is recommended as $L_p = 1.1 r_t \sqrt{E/F_y}$ for all uniform bending cases, including rolled members. This equation is specified by AISC for all cross sections with the exception of doubly symmetric I-sections with compact webs and nonslender (i.e., compact or noncompact) flanges. AISC specifies $L_p = 1.76 r_y \sqrt{E/F_y}$ for these section types. White and Jung (2008) and White (2008) explain that the latter of these AISC *Specification* equations provides an optimistic estimate of the plateau length and is, in fact, developed by (1) assuming that the design calculations will always use $K = 1$ and (2) dividing by an implicit $K < 1$ in the expression for L_p. The authors submit that a better approach is to allow engineers to apply an explicit $K < 1$ to L_b rather than hide the LTB effective length factor in the L_p equation. Alternatively, a more rigorous approach is to determine the LTB resistance directly, accounting for end restraint effects, via tools such as SABRE2 (White et al., 2016a, 2016b). In either of these situations, the use of $L_p = 1.76 r_y \sqrt{E/F_y}$ amounts to a double counting of the end restraint effects, and therefore, this equation is not appropriate.

The unified L_p value is slightly larger than the value employed by AASHTO (2016). AASHTO uses $L_p = 1.0 r_t \sqrt{E/F_y}$.

As noted by White and Kim (2008), although the collected experimental data sets are quite extensive, they are not sufficient to encompass the multitude of cross-section types and loading and displacement boundary conditions that form the complete design space. The acute scarcity of the experimental test data in certain inelastic LTB cases is discussed further in the section just prior to the conclusions of this paper. The proposed LTB strength curve (key parameters of which are summarized in Table 1) is based on the experimental data as well as a wider range of test simulations discussed by Subramanian and White (2017b, 2017c).

ESTIMATION OF RELIABILITY INDEX, β

The reliability indices presented in this paper are calculated as detailed in White and Jung (2008) and White and Kim (2008). These calculations, in turn, are based on the prior procedures established by Galambos et al. (1982). Based on the assumption that the resistance, R, and the load effect, Q, are log-normally distributed, the reliability index in the context of LRFD of steel building members is given by the expression

$$\beta = \frac{\ln\left(\frac{\overline{R}}{\overline{Q}}\right)}{\sqrt{V_R^2 + V_Q^2}} \tag{2}$$

where $\overline{R}$ and $\overline{Q}$ are the mean values of the resistance and load effects and V_R and V_Q are the respective coefficients of variation (Ellingwood et al., 1980; Ellingwood et al., 1982; Galambos, 2004; Galambos et al., 1982). The determination of $\overline{R}$, $\overline{Q}$, V_R and V_Q is described in detail in White and Jung (2008) and White and Kim (2008). The same procedures are adopted in this paper.

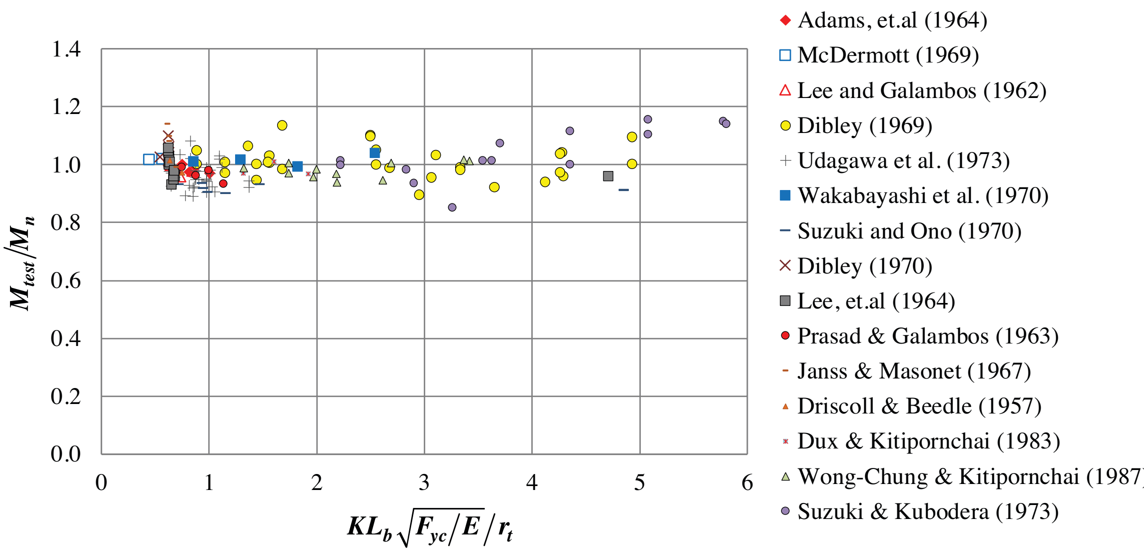
ASSESSMENT OF UNIFORM MOMENT TESTS

Figures 3 and 4 show the professional factors, M_{test}/M_n, from SABRE2 for the proposed and the unified equations considering the rolled and welded member uniform bending tests from the experimental database. The normalized resistances are plotted versus the normalized slenderness $c = \left(KL_b \sqrt{F_{yc}/E}\right)/r_t$. The reader should note that $c = 1$ corresponds to the length L_p in the AASHTO Specification, whereas $c = 1.1$ corresponds to the length L_p in all cases with the exception of doubly-symmetric compact-web members having nonslender flanges in the AISC *Specification* (2016).

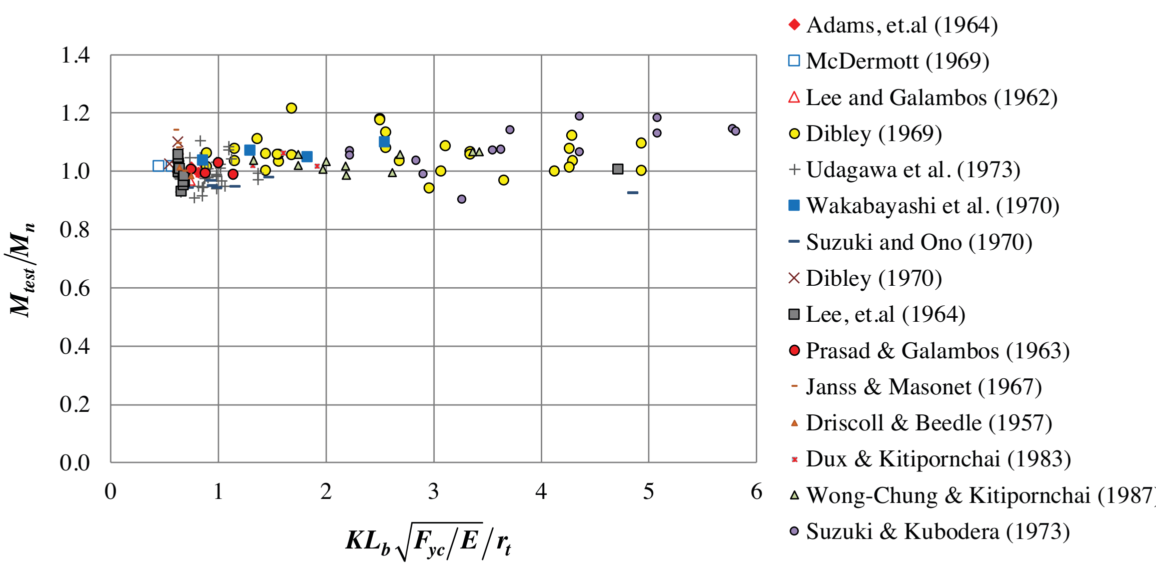
Tables 2 and 3 show the corresponding statistics on M_{test}/M_n. The number of different quantities presented in the tables is substantial; however, this is necessary to assess the ability of the resistance equations to predict the LTB strength limit states for all potential I-section member geometries. The test configurations and characteristics are discussed in detail by White and Jung (2004). Among the tests in the prior database, cases with cover plates and tests with a web depth-to-compression flange width ratio, D/b_{fc}, greater than 7.5 are not considered. A similar restriction on D/b_{fc} was employed by White and Jung (2004). The flexural resistance equations do not perform as well for the limited number of tests with D/b_{fc} larger than this limit. Table 2 summarizes the results for all the tests, including both accurate and nominal/approximate geometry. Table 3 shows the results only for the tests with accurate measured geometry. For c values less than or equal to 0.63, the predicted strengths are equal to the plateau flexural resistance both using the proposed as well as the unified provisions.

It can be observed from Figures 3 and 4 that there is generally a minor increase in the mean and minimum of M_{test}/M_n for the proposed equations compared to the unified equations. It is also observed that the dispersion of the test data in the different regions of c is largely the same for both calculations. This is corroborated by the coefficient of variation (COV) values reported in Tables 2 and 3. In these tables, the variables N and V denote the number of tests and coefficient of variation of the strength ratio M_{test}/M_n. The following observations can be gleaned from Tables 2 and 3:

1. The statistics for $c \le 0.63$ indicate that the experimental tests have no trouble attaining the plateau strength at small unbraced lengths. The COV considering all the tests is approximately 4%.
2. The statistics for $0.63 < c \le 1$ are largely similar for the proposed and unified equations, except for welded noncompact-web sections. For the tests of this type with accurate measured geometry, the proposed provisions give a slightly larger mean, M_{test}/M_n, of 1.04 versus 1.02 and a slightly smaller COV of 6.28% versus 7.71%. The tests listed in this category are all doubly-symmetric cross sections.
3. Rolled members in the range of $1 < c \le 2$ show an increase of 0.05 in the mean and minimum values of M_{test}/M_n using the proposed equations compared to the unified equations in Table 2. Welded members show an increase of 0.05 and 0.06 for these values within this



(a)

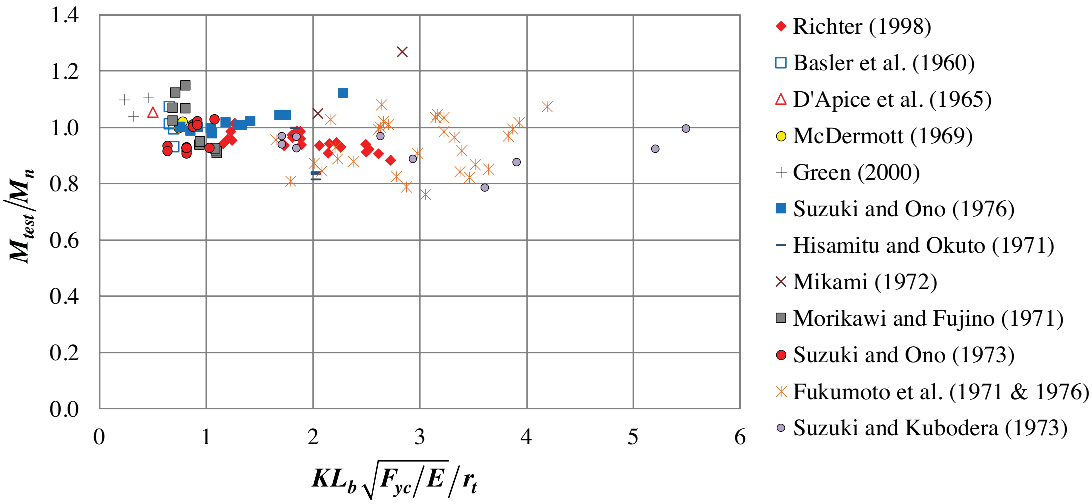


(b)

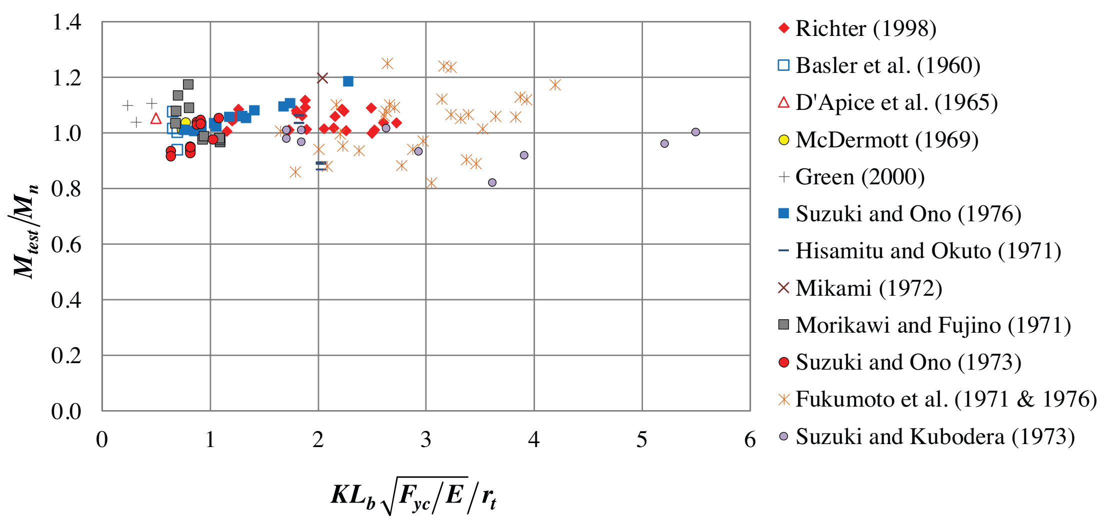
Fig. 3. Uniform moment professional factors M_{test}/M_n *for rolled members: (a) unified equations; (b) proposed equations.*

range, considering the cases with nominal/approximate geometry. The trend is similar when only the tests with accurate measured geometry are considered (Table 3). Welded members with compact, noncompact and slender webs show increases in these values ranging from 0.05 to 0.09 in Table 2. The mean values change from slightly less than 1.0 to slightly greater than 1.0 for many of the cases.

4. The statistics for rolled members in the range of $2 < c \leq 3$ show an increase of 0.06 in the mean M_{test}/M_n using the proposed equations. This result is observed considering all tests as well as tests with only accurately measured geometry. The minimum value for the tests with accurate geometry increases from 0.90 with the unified equations to 0.94 when the proposed equations are used. The increase in these values is larger for welded members. The mean and minimum of the test data for all the welded members increase by 0.08 by using the proposed equations (see Table 2). The increase is 0.06 for compact-web welded, 0.09 for noncompact-web welded, and 0.15 for slender-web welded members, considering all the available tests (Table 2). If only tests with accurate geometry are considered, the overall welded test mean and minimum M_{test}/M_n values increase by 0.09 for $2 < c < 3$. There are no compact-web welded members in this range



(a)



(b)

Fig. 4. Uniform moment professional factors M_{test}/M_n *for welded members: (a) unified equations; (b) proposed equations.*

Table 2. M_{test}/M_n Statistics for Unified and Proposed LTB Equations—Uniform Bending Tests with Accurate and Nominal/Approximate Geometry

(a) Rolled Members

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	16	61		34		17		11		12	
Min	0.99	0.89	0.91	0.90	0.95	0.88	0.93	0.85	0.90	0.91	0.92
Median	1.03	0.98	0.99	0.98	1.03	0.99	1.04	1.01	1.07	0.98	1.03
Max	1.14	1.08	1.11	1.13	1.22	1.10	1.18	1.07	1.14	1.12	1.19
Mean	1.04	0.97	0.99	0.98	1.03	0.99	1.05	0.99	1.05	1.00	1.05
V (%)	4.07	4.46	4.00	4.86	5.05	6.14	6.78	6.08	6.20	6.16	6.56

(b) All Welded Members

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	4	27		34		33		16		1	
Min	1.04	0.90	0.92	0.81	0.86	0.79	0.87	0.76	0.82		
Median	1.08	1.00	1.01	0.97	1.04	0.93	1.02	0.94	1.06		
Max	1.11	1.15	1.17	1.04	1.12	1.27	1.50	1.05	1.24		
Mean	1.07	0.99	1.01	0.97	1.03	0.94	1.03	0.92	1.03	1.07	1.17
V (%)	3.06	6.46	6.35	4.78	5.02	10.37	12.17	10.25	12.63		

(c) Rolled and Welded Members Combined

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	20	88		68		50		27		13	
Min	0.99	0.89	0.91	0.81	0.86	0.79	0.87	0.76	0.82	0.91	0.92
Median	1.04	0.98	1.00	0.97	1.03	0.94	1.04	0.98	1.07	1.00	1.04
Max	1.14	1.15	1.17	1.13	1.22	1.27	1.50	1.07	1.24	1.12	1.19
Mean	1.05	0.98	1.00	0.97	1.03	0.96	1.04	0.95	1.04	1.01	1.06
V (%)	4.01	5.21	4.93	4.84	5.00	9.34	10.56	9.17	10.32	6.20	7.06

(d) Welded Members with Compact Webs

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	3	16		20		18		10		0	
Min	1.04	0.90	0.92	0.81	0.86	0.81	0.87	0.76	0.82		
Median	1.10	0.99	1.01	0.99	1.04	0.90	0.96	0.92	0.98		
Max	1.11	1.02	1.05	1.04	1.11	1.12	1.19	1.04	1.12		
Mean	1.08	0.97	0.99	0.98	1.03	0.93	0.99	0.91	0.98		
V (%)	3.40	4.56	4.83	5.42	5.34	9.46	9.71	11.06	12.19		

Table 2. M_{test}/M_n Statistics for Unified and Proposed LTB Equations—Uniform Bending Tests with Accurate and Nominal/Approximate Geometry (cont'd)											
(e) Welded Members with Noncompact Webs											
	$c \leq 0.63$	$0.63 < c \leq 1$		$1 < c \leq 2$		$2 < c \leq 3$		$3 < c \leq 4$		$c \geq 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	0	5		5		4		0		0	
Min		0.94	0.98	0.94	1.01	0.91	1.00				
Median		1.02	1.03	0.94	1.01	0.93	1.01				
Max		1.12	1.13	1.01	1.09	0.94	1.02				
Mean		1.02	1.04	0.95	1.03	0.92	1.01				
V (%)		7.71	6.28	3.59	3.28	1.13	0.68				
(f) Welded Members with Slender Webs											
	$c \leq 0.63$	$0.63 < c \leq 1$		$1 < c \leq 2$		$2 < c \leq 3$		$3 < c \leq 4$		$c \geq 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	1	6		9		11		6		1	
Min		0.93	0.94	0.91	0.97	0.79	0.94	0.85	1.02		
Median		1.04	1.05	0.96	1.06	0.94	1.08	0.96	1.10		
Max		1.15	1.17	0.98	1.12	1.27	1.50	1.05	1.24	1.07	1.17
Mean	0.99	1.04	1.05	0.95	1.04	0.97	1.12	0.95	1.13		
V (%)		7.15	7.76	2.89	5.39	13.05	13.61	8.92	8.48		

considering only accurate geometry. The increases are 0.09 and 0.14 for both the minimum and mean values respectively for noncompact- and slender-web sections with accurate measured geometry. The mean values are closer to 1.0 with the proposed equations for several groups where the unified equations give values significantly less than 1.0.

5. The statistics for rolled members in the range of $3 < c \leq 4$ show an increase of 0.05 in the mean when the proposed equations are used for all the tests, as well as for the tests having only accurate measured geometry. The mean of the data increases by 0.11 from 0.92 with the unified equations to 1.03 when the proposed equations are used for the welded test specimens. There are no welded tests in this range with accurately measured geometry.
6. The statistics for rolled members show an increase of 0.05 and 0.04 in the mean of the data for nominal/approximate and accurate geometry in the range $c \geq 4$. There is only one welded member test in this range.
7. The COV for all the tests is largely similar for the proposed and unified equations in both Tables 2 and 3. It is observed that the COV is larger for tests in the middle of the inelastic LTB region in Table 2. When only the tests with accurate geometry are considered (Table 3), this COV is reduced. For example, in the range of $2 < c \leq 3$, the COV for welded members is reduced from 9.71% for all tests to 3.19% for accurately measured tests.
8. It is observed that the largest unconservatism of the unified equations is in the middle to end of the inelastic LTB region ($2 < c \leq 4$). The unified equations overpredict the experimental test data by as much as 14% for welded members with accurate geometry, and by as much as 32% for tests with nominal/approximate geometry (the minimum values of $M_{test}/M_{nUnified}$ are 0.88 and 0.76, respectively; therefore, $M_{nUnified}/M_{test}$ is equal to 1.14 and 1.32, respectively). This is consistent with the observations by Subramanian and White (2017a, 2017b, 2017c) that the unified equations tend to overpredict the finite element test simulation data within the inelastic LTB region. These observations are a key reason the proposed modifications should be implemented in the AISC *Specification* and AASHTO.

Figure 5 shows the reliability indices, estimated as explained in the previous section. The target reliability index in the AISC LRFD *Specification* is 2.6 for statically

Table 3. M_{test}/M_n Statistics for Unified and Proposed LTB Equations— Uniform Bending Tests with Accurate Measured Geometry

(a) Rolled Members

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	13	28		20		11		7		8	
Min	1.00	0.93	0.93	0.93	0.99	0.90	0.94	0.92	0.97	0.94	1.00
Median	1.05	0.99	1.00	0.99	1.04	0.99	1.04	0.99	1.07	0.99	1.02
Max	1.14	1.08	1.08	1.13	1.22	1.10	1.18	1.03	1.09	1.10	1.12
Mean	1.05	0.99	1.00	1.00	1.05	1.00	1.06	0.99	1.04	1.00	1.04
V (%)	3.91	3.28	3.13	4.46	4.68	6.42	7.29	3.91	4.14	5.34	4.63

(b) All Welded Members

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	2	13		16		11		0		0	
Min	1.05	0.93	0.94	0.91	0.97	0.88	1.00				
Median		1.02	1.03	0.96	1.05	0.93	1.04				
Max	1.11	1.15	1.17	1.01	1.12	0.95	1.09				
Mean	1.08	1.03	1.04	0.95	1.04	0.92	1.04				
V (%)		6.55	6.29	3.06	4.55	2.14	3.19				

(c) Rolled and Welded Members Combined

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	15	41		36		22		7		8	
Min	1.00	0.93	0.93	0.91	0.97	0.88	0.94	0.92	0.97	0.94	1.00
Median	1.05	1.00	1.01	0.97	1.04	0.94	1.04	0.99	1.07	0.99	1.02
Max	1.14	1.15	1.17	1.13	1.22	1.10	1.18	1.03	1.09	1.10	1.12
Mean	1.06	1.00	1.01	0.98	1.04	0.96	1.05	0.99	1.04	1.00	1.04
V (%)	3.84	4.80	4.77	4.41	4.52	6.22	5.59	3.91	4.14	5.34	4.63

(d) Welded Members with Compact Webs

	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	1	2		2		0		0		0	
Min		1.00	1.01	0.98	1.05						
Median		1.01	1.02	0.99	1.06						
Max		1.02	1.04	0.99	1.06						
Mean	1.11	1.01	1.02	0.99	1.06						
V (%)		1.59	1.68	0.16	0.71						

Table 3. M_{test}/M_n Statistics for Unified and Proposed LTB Equations— Uniform Bending Tests with Accurate Measured Geometry (cont'd)										
(e) Welded Members with Noncompact Webs										
	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	0	5		5		4		0		0	
Min		0.94	0.98	0.94	1.01	0.91	1.00				
Median		1.02	1.03	0.94	1.01	0.93	1.01				
Max		1.12	1.13	1.01	1.09	0.94	1.02				
Mean		1.02	1.04	0.95	1.03	0.92	1.01				
V (%)		7.71	6.28	3.59	3.28	1.13	0.68				
(f) Welded Members with Slender Webs											
	$c \le 0.63$	$0.63 < c \le 1$		$1 < c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	M_{test}/M_n	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	1	6		9		7		0		0	
Min		0.93	0.94	0.91	0.97	0.88	1.02				
Median		1.04	1.05	0.96	1.06	0.94	1.06				
Max		1.15	1.17	0.98	1.12	0.95	1.09				
Mean	1.00	1.04	1.05	0.95	1.04	0.92	1.06				
V (%)		7.15	7.76	2.89	5.39	2.64	2.67				

determinate compact-section beams under uniform moment, based on a live load–to–dead load ratio (L/D) of 3. Bartlett et al. (2003) and Galambos (2004) have shown that 2.6 is a reasonable lower-bound reliability index for these member types when discretization error is not considered. In addition, White (2008) explains that the reliability index is 2.6 corresponding to the ASCE 7 load model and elastic LTB of general statically determinate beams. Rolled beams in general have been observed to have higher reliability than welded members (Galambos, 2004; White and Jung, 2008; White and Kim, 2008).

Figure 5 shows the reliability indices for various live load–to–dead load ratios, given a resistance factor, ϕ_b, of 0.9. The results presented in this paper do not consider discretization error (Bartlett et al., 2003). In cases where there are fewer than four tests, the reliability estimates are very coarse due to the sparsity of the test data. The evaluation of the reliability index for elastic LTB is discussed in White and Jung (2008) and is not reproduced here.

Figure 5 shows the reliability indices for both the unified and the proposed equations. The values for the unified equations are different in this paper compared to those shown in White and Jung (2008) because of the more accurate consideration of end restraint effects in the calculation of the ordinate values and the use of the more rigorous K factors in the calculation of the abscissa ($K_{inelastic}$ vs. $K_{elastic}$).

The following can be gleaned from Figure 5:

1. The reliability in the inelastic and less slender elastic LTB regions is increased by using the proposed equations. The target reliability is based on a live load–to–dead load ratio (L/D) of 3. For rolled members, the corresponding reliability index is as low as 2.4 for the unified equations in the region c ≥ 4. This value is improved to 2.7.

2. The reliability index for welded members is improved from 2.2 and 2.1 to 2.5 and 2.5 for $L/D = 3$ in the ranges of $2 < c \le 3$ and $3 < c \le 4$ for the tests that include nominal/approximate geometry. For tests with accurate geometry, both values increase from 2.5 to 2.9 for $L/D = 3$.

3. With the proposed equations, one obtains a more uniform reliability across all the LTB slenderness ranges.

ASSESSMENT OF MOMENT GRADIENT TESTS

Figures 6 and 7 show how the moment gradient experimental test results compare with results from SABRE2 for the unified and the proposed equations. Tables 4 and 5 show the results for the rolled and welded cross sections from the experimental database. The test configurations are detailed in White and Kim (2004), except for the tests from Kusuda et al. (1960) and Righman (2005). Among the tests in the prior database, tests containing cover plates and tests with a ratio of web depth to compression flange width $D/b_{fc} > 7.5$ are not considered in the statistics presented in this paper.

As in the case of the uniform moment tests, the parameter c is calculated based on the $K_{inelastic}$ value that yields the same theoretical strength as the inelastic buckling solution in SABRE2. In determining the $K_{inelastic}$ to be used in the expression for c, C_b is calculated from the equation developed by Salvadori (1955),

$$C_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3 \tag{3}$$

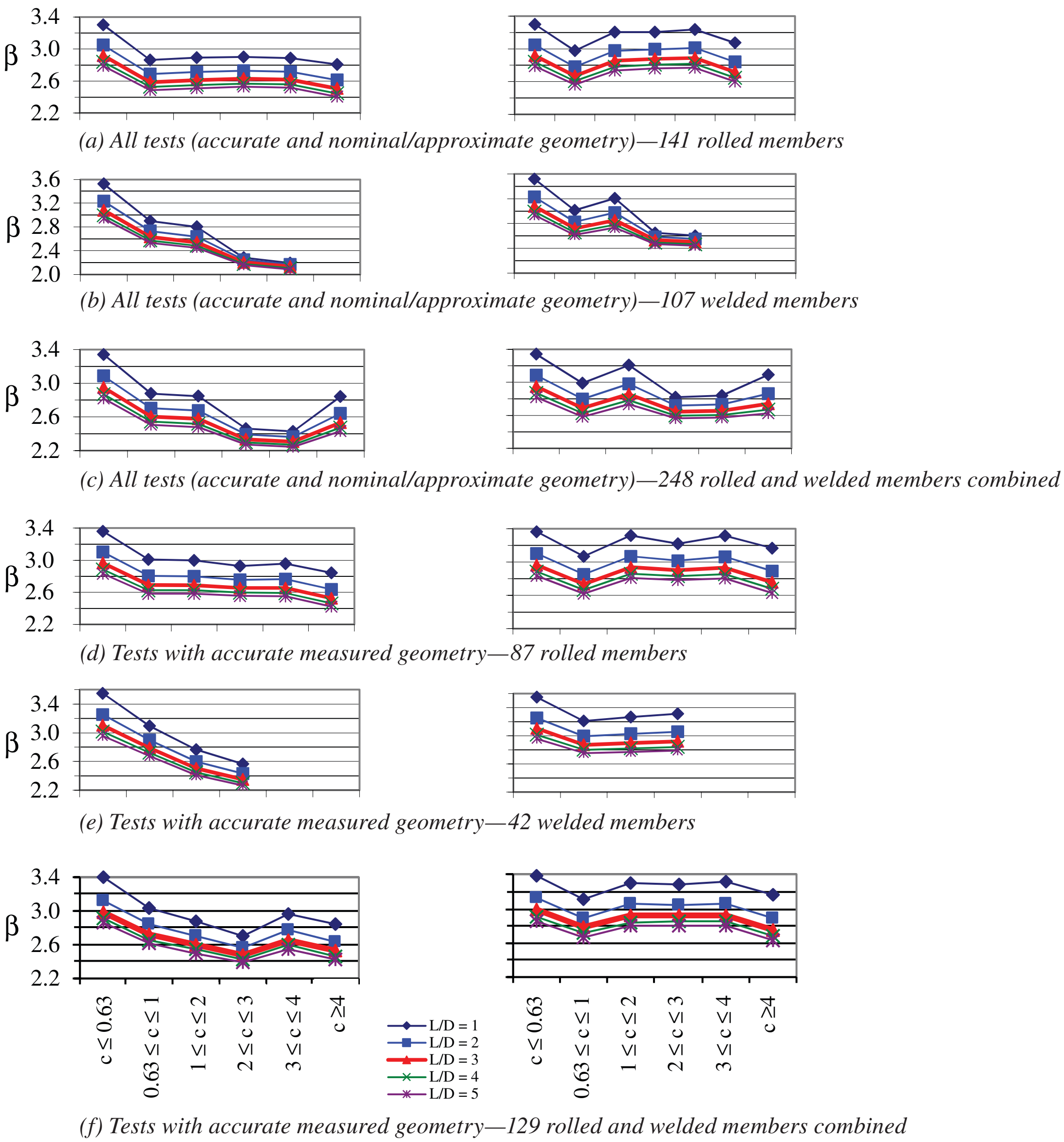
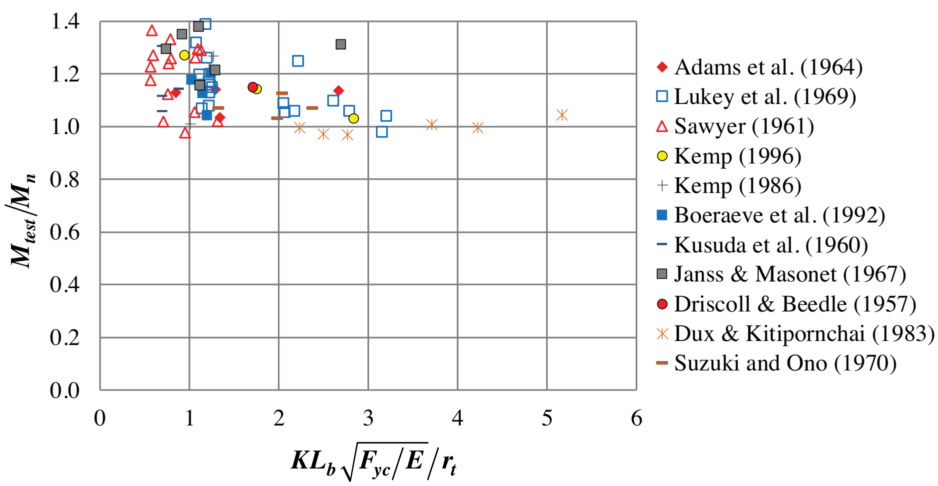


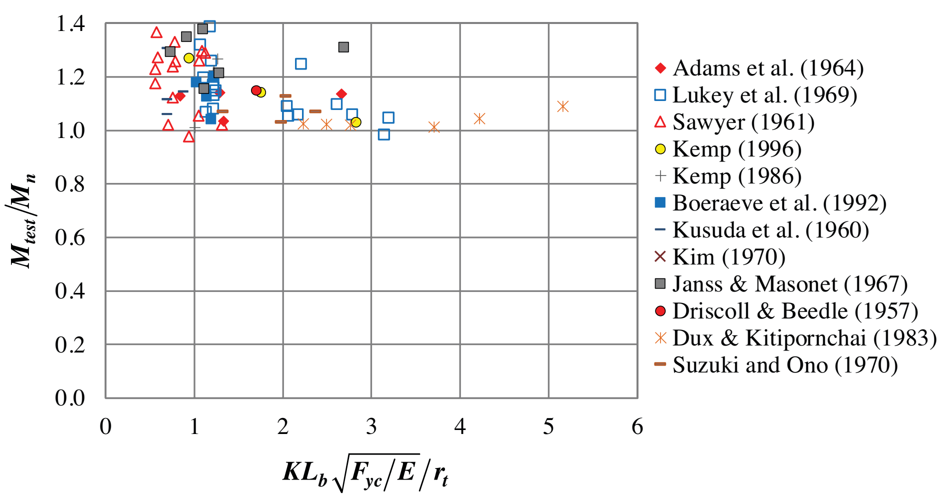
Fig. 5. Reliability indices for uniform moment tests at various ranges of $c = \left(KL_b\sqrt{F_{yc}/E}\right)/r_t$ *and live load–to–dead load ratios* (L/D), $\phi = 0.9$*; unified provisions (left) and proposed equations (right).*

which is Equation C-F1-1 in the AISC *Specification* Commentary, where M_1 and M_2 are the smaller and larger moments at the ends of the unbraced lengths, respectively, and M_1/M_2 is positive for reversed curvature bending. This expression gives a better lower-bound elastic C_b factor compared to AISC *Specification* Equation F1-1 for cases where the moment diagram is linear between braced points. For example, given an unbraced length with zero moment at one end and maximum moment at the other end, Equation 3 gives C_b = 1.75 versus C_b = 1.67 using AISC *Specification* Equation F1-1. All the loading cases discussed in this paper fall under this category. Test cases where transverse loads are applied away from the brace points, and where load-height effects are predominant, are evaluated by Toğay et al. (2016). It should be noted that the preceding C_b approximation influences only the abscissa within the plots and the categorization of the tests in terms of their LTB slenderness in the tables because Equation 3 is used only in estimating $K_{inelastic}$. The ordinate values are determined directly using SABRE2 (White et al., 2016a, 2016b).

Similar to the uniform bending tests, there is one exception to the earlier calculation of $K_{inelastic}$. A number of the moment gradient tests involve three-point bending with equal unbraced lengths on each side of the braced point at the member midspan. In these cases, K = 1. When these tests are modeled in SABRE2, rigorous LTB resistances are



(a)



(b)

Fig. 6. Moment gradient professional factors M_{test}/M_n *for rolled members: (a) unified provisions; (b) proposed equations.*

obtained corresponding to the nominal resistance equations with an unbraced length $KL_b = L_b$, including an "inelastic C_b" effect.

The following can be observed from Figures 6 and 7:

1. Both the unified equations and the proposed equations tend to be conservative for smaller values of c. This is due to strain hardening effects, which are a predominant feature of moment gradient tests of compact-section members with short unbraced lengths.

2. Similar to the trends observed for the uniform moment tests, the proposed equations result in smaller predicted flexural resistances, resulting in larger professional factors M_{test}/M_n. This changes the M_{test}/M_n for rolled members from values that are, in some cases, less than 1.0 to values that are predominantly 1.0 or higher.

3. An increase in the professional factors is also evident for the welded member tests, especially at longer unbraced lengths [e.g., see the data points from Frost and Schilling (1964) and Righman (2005) in Figure 7]. However, a number of these M_{test}/M_n values are still less than 1.0.

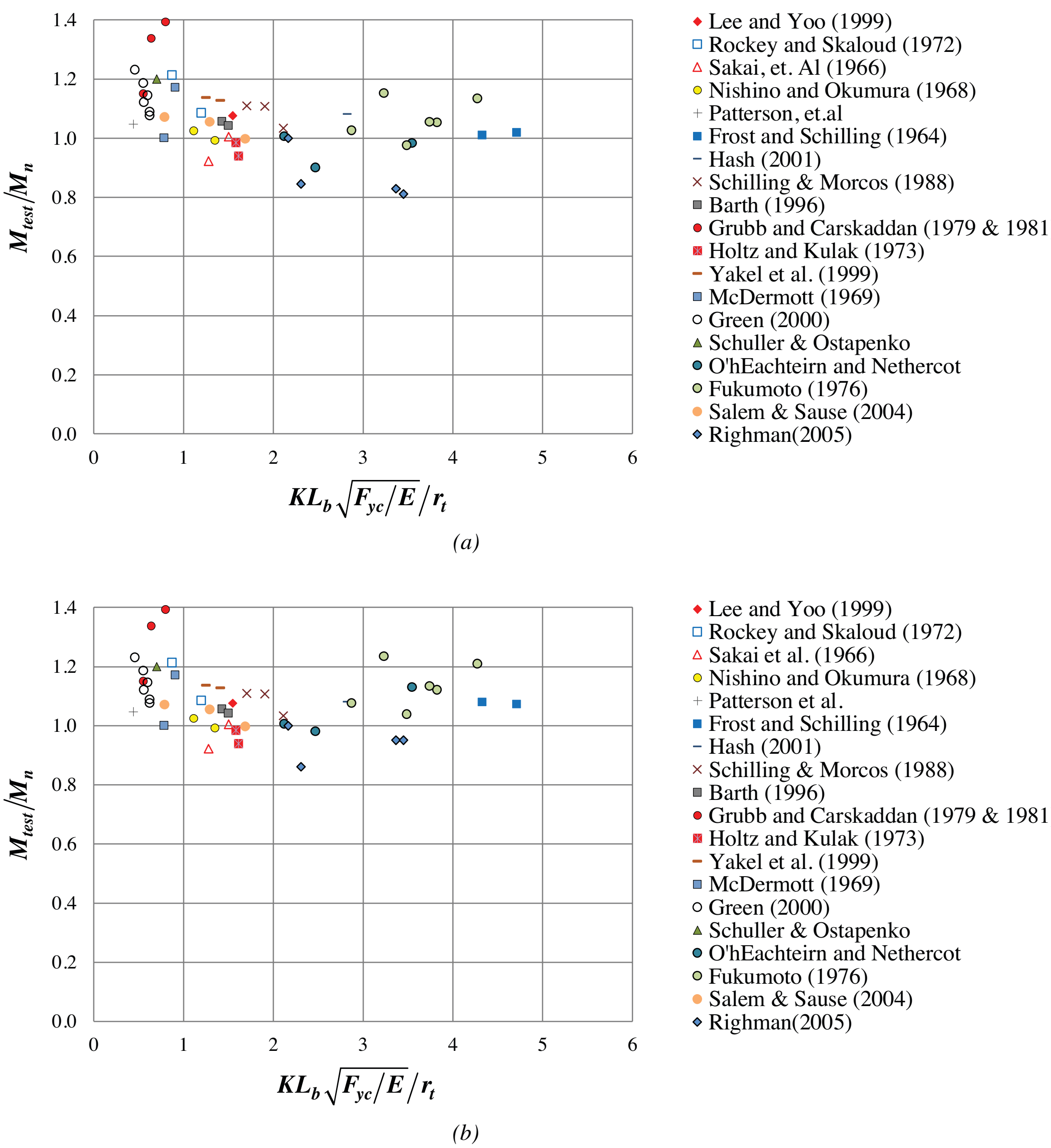


Fig. 7. Moment gradient professional factors M_{test}/M_n *for welded members: (a) unified provisions; (b) proposed equations.*

Table 4. M_{test}/M_n Test Statistics for Unified and Proposed LTB Equations—Moment Gradient Tests with Accurate and Nominal/Approximate Geometry

(a) Rolled Members								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	54		15		3		2	
Min	0.98	0.98	0.97	1.01	0.98	0.98	0.99	1.04
Median	1.20	1.20	1.06	1.06	1.01	1.01	1.02	1.07
Max	1.48	1.48	1.31	1.31	1.04	1.05	1.05	1.09
Mean	1.21	1.21	1.08	1.09	1.01	1.01	1.02	1.07
V (%)	10.24	10.24	8.81	7.92	3.00	3.18	3.52	3.12
(b) All Welded Members								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	53		7		7		3	
Min	0.92	0.92	0.85	0.86	0.81	0.95	1.01	1.07
Median	1.19	1.19	1.01	1.01	0.98	1.12	1.02	1.08
Max	1.62	1.62	1.08	1.08	1.15	1.24	1.13	1.21
Mean	1.18	1.18	0.98	1.01	0.98	1.08	1.05	1.12
V (%)	11.74	11.73	8.33	7.37	12.67	9.73	6.61	6.91
(c) Rolled and Welded Members Combined								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	107		22		10		5	
Min	0.92	0.92	0.85	0.86	0.81	0.95	0.99	1.04
Median	1.20	1.20	1.04	1.06	1.00	1.04	1.02	1.08
Max	1.62	1.62	1.31	1.31	1.15	1.24	1.13	1.21
Mean	1.19	1.19	1.05	1.06	0.99	1.06	1.04	1.10
V (%)	10.98	10.98	9.58	8.50	10.45	8.76	5.35	5.87
(d) Welded Members with Compact Webs								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	37		1		4		3	
Min	0.92	0.92			0.98	1.04	1.01	1.07
Median	1.21	1.21			1.06	1.13	1.02	1.08
Max	1.36	1.36			1.15	1.24	1.13	1.21
Mean	1.18	1.18	1.03	1.08	1.06	1.13	1.05	1.12
V (%)	10.99	10.99			6.81	7.09	6.61	6.91

Table 4. M_{test}/M_n Test Statistics for Unified and Proposed LTB Equations—Moment Gradient Tests with Accurate and Nominal/Approximate Geometry (cont'd)								
(e) Welded Members with Noncompact Webs								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	8				0		0	
Min	1.04	1.04						
Median	1.08	1.08						
Max	1.15	1.15						
Mean	1.09	1.09						
V (%)	3.49	3.51						
(f) Welded Members with Slender Webs								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	8		6		3		0	
Min	1.05	1.05	0.85	0.86	0.81	0.95		
Median	1.21	1.21	1.00	1.00	0.83	0.95		
Max	1.62	1.62	1.08	1.08	0.98	1.13		
Mean	1.25	1.25	0.98	0.99	0.87	1.01		
V (%)	15.43	15.41	8.96	7.40	10.86	10.29		

4. Two data points [one from Rockey and Skaloud (1972) and one from Righman (2005)] have essentially the same M_{test}/M_n with both the proposed and unified equations. This is because these tests achieve the plateau resistance in both predictions.

Table 4 shows the results for all tests, including cases with nominal/approximate geometry, and Table 5 shows the results for tests with only accurate geometry. Values of c that are less than or equal to 2.0 fall on the plateau of the LTB curves for both the unified and proposed provisions.

The following can be gleaned from Tables 4 and 5:

1. The test statistics for $c \le 2$ indicate that the experimental tests have no trouble attaining the plateau strength and that both the unified and proposed equations are conservative. The mean M_{test}/M_n is as high as 1.21 for rolled members, both when considering all the tests and when considering only accurate geometry. The mean M_{test}/M_n for welded members is 1.11 for tests with accurate geometry and 1.18 for tests that include members with approximate geometry. The COV is similar for the unified and proposed equations. The COV is between 10 and 11% when the rolled and welded members are combined as one data set.

2. The test statistics for rolled members in the range of $2 < c \le 3$ show a small increase in the mean M_{test}/M_n from 1.08 to 1.09 when using the proposed versus the unified equations. This is observed for all tests as well as tests with accurate measured geometry in this range. For rolled members, the minimum value of M_{test}/M_n for accurate geometry increases from 0.97 (unified equations) to 1.01 (proposed equations). The mean of the test data for all welded members (including approximate geometry) increases from 0.98 to 1.01 when using the proposed equations within this range of c. The M_{test}/M_n increases from 1.03 to 1.08 for a single compact-web welded member, and the mean M_{test}/M_n increases from 0.98 to 0.99 for slender-web welded members. There are no noncompact-web members in this range. All the slender-web welded members considered here have accurate measured geometry. There are no tests with compact or noncompact webs with accurate geometry.

3. The maximum M_{test}/M_n for rolled members in the range of $3 < c \le 4$ increases from 1.04 to 1.05 when using the proposed equations for all tests as well as tests with accurately measured geometry. The mean of the data increases from 0.98 (unified equations)

Table 5. M_{test}/M_n Statistics for Unified and Proposed LTB Equations— Moment Gradient Tests with Accurate Measured Geometry

(a) Rolled Members								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	52		13		3		2	
Min	0.97	0.98	0.97	1.01	0.98	0.98	0.99	1.04
Median	1.21	1.21	1.06	1.06	1.01	1.01	1.02	1.07
Max	1.48	1.48	1.31	1.31	1.04	1.05	1.05	1.09
Mean	1.21	1.21	1.08	1.09	1.01	1.01	1.02	1.07
V (%)	10.05	10.05	9.45	8.49	3.00	3.18	3.52	3.12
(b) All Welded Members								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	30		6		3		2	
Min	0.92	0.92	0.85	0.86	0.81	0.95	1.01	1.07
Median	1.11	1.11	1.00	1.00	0.83	0.95	1.01	1.08
Max	1.27	1.27	1.08	1.08	0.98	1.13	1.02	1.08
Mean	1.11	1.11	0.98	0.99	0.87	1.01	1.01	1.08
V (%)	7.71	7.71	8.96	7.40	10.86	10.29	0.62	0.53
(c) Rolled and Welded Members Combined								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	82		19		6		4	
Min	0.92	0.92	0.85	0.86	0.81	0.95	0.99	1.04
Median	1.16	1.16	1.03	1.03	0.98	1.00	1.01	1.08
Max	1.48	1.48	1.31	1.31	1.04	1.13	1.05	1.09
Mean	1.18	1.18	1.05	1.06	0.94	1.01	1.02	1.07
V (%)	10.16	10.16	10.21	9.10	10.29	6.81	2.10	1.89
(d) Welded Members with Compact Webs								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	18		0		0		2	
Min	0.92	0.92					1.01	1.07
Median	1.12	1.12					1.01	1.08
Max	1.27	1.27					1.02	1.08
Mean	1.12	1.12					1.01	1.08
V (%)	9.07	9.07					0.62	0.53

Table 5. M_{test}/M_n Statistics for Unified and Proposed LTB Equations—Moment Gradient Tests with Accurate Measured Geometry (cont'd)								
(e) Welded Members with Noncompact Webs								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	7		0		0		0	
Min	1.04	1.04						
Median	1.09	1.09						
Max	1.15	1.15						
Mean	1.09	1.09						
V (%)	3.73	3.76						
(f) Welded Members with Slender Webs								
	$c \le 2$		$2 < c \le 3$		$3 < c \le 4$		$c \ge 4$	
	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}	$M_{test}/M_{nUnified}$	M_{test}/M_{nPr}
N	5		6		3		0	
Min	1.05	1.05	0.85	0.86	0.81	0.95		
Median	1.12	1.12	1.00	1.00	0.83	0.95		
Max	1.21	1.21	1.08	1.08	0.98	1.13		
Mean	1.13	1.13	0.98	0.99	0.87	1.01		
V (%)	6.65%	6.64%	8.96	7.40	10.86	10.29		

to 1.08 (proposed equations) for welded test sections with nominal/approximate geometry included within this range of c. The mean of the data increases from 0.87 to 1.01 for welded members when only tests with accurate geometry are considered. Clearly, 0.87 is a low value for the mean of the data. The minimum M_{test}/M_n for these section types in this range is only 0.81 using the unified equations. It increases to 0.95 using the proposed equations. However, only three tests, each with accurate geometry, are available for each of the rolled and welded member categories in this LTB region.

4. The test statistics for rolled members show an increase in the mean of the data from 1.02 to 1.07 for both nominal/approximate and accurate geometry in the range $c \ge 4$. The mean M_{test}/M_n for the welded members increases from 1.05 to 1.12 for the tests with accurate and nominal/approximate geometry and 1.01 to 1.08 for the tests with accurate measured geometry in this range. However, there are only two welded and two rolled member tests in this range that have accurate measured geometry.

5. The COV for all the tests is largely similar for the proposed and unified equations in Tables 4 and 5. This is the same as the trend observed for the uniform moment tests.

6. For the moment gradient tests, it is observed that the largest unconservatism in the unified equations is in the inelastic LTB region ($3 < c \le 4$), similar to the behavior for the uniform moment tests. The unified equations overpredict the experimental test data by as much as 23% for welded members with accurate and nominal/approximate geometry, while the proposed equations overpredict the data by as much as 5% (the minimum of M_{test}/M_n in this range is 0.81 for the unified provisions and 0.95 for proposed equations; therefore, M_{test}/M_n = 1.23 and 1.05, respectively). This overprediction of the test data is manifested clearly in the low reliability indices presented in Figure 8 and is discussed in detail next. The predictions by the current AISC *Specification* equations are identical to the unified provisions for all of these tests.

Figure 8 shows the reliability indices for the moment gradient tests, estimated as explained in the previous section on uniform moment tests. The following can be gleaned from this figure:

1. The reliability with respect to LTB is increased across all the ranges of LTB slenderness by using the proposed

equations. For rolled members, the target reliability of 2.6 for $L/D = 3$ is achieved for all ranges of c with the unified provisions. The estimated minimum reliability index is increased to 2.8 by using the proposed equations.

2. For tests that include nominal/approximate geometry and $L/D = 3$, the reliability index for welded members is improved from 2.5 and 2.3 to 2.7 and 2.9 in the ranges $2 < c \leq 3$ and $3 < c \leq 4$. For tests with accurate geometry, the values respectively increase from 2.4 and 1.9 to 2.6 and 2.5. When welded and rolled members are considered together, the proposed equations give a reliability index estimate of 2.8 and 2.7 in these ranges of c.

3. For $L/D = 3$, the reliability index obtained using the unified equations is particularly low ($\beta = 1.9$) for welded members with accurate geometry in the region $3 < c \leq 4$. Figures 9 and 10 show the reliability indices for the welded members with accurate test geometry, considering two different ranges of C_b. Figure 9 summarizes the results for the unified provisions, whereas Figure 10 corresponds to the proposed

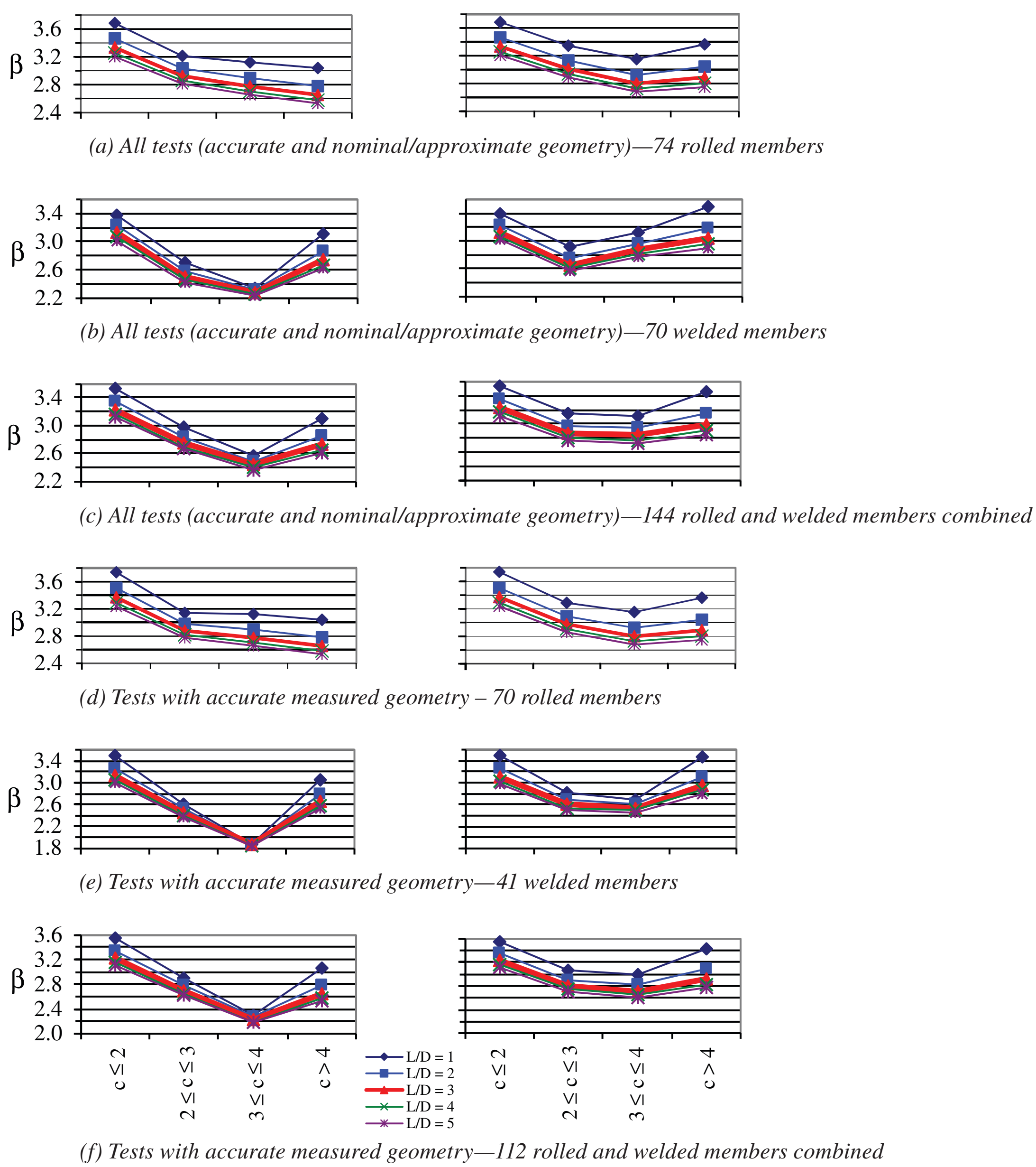


Fig. 8. Reliability indices for moment gradient tests at various ranges of $c = \left(KL_b\sqrt{F_{yc}/E}\right)/r_t$ *and live load–to–dead load ratios* (L/D), $\phi = 0.9$*; unified provisions (left) and proposed equations (right).*

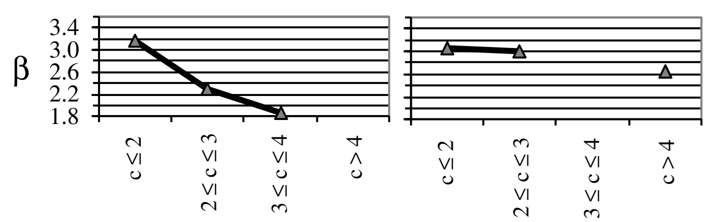
equations. The plots on the left show the results for all the tests that range from $C_b = 1.0$ to $C_b = 1.3$. The plots on the right are for tests with $C_b = 1.75$. It is observed that the moment gradient tests with the smaller C_b yield a lower estimated reliability index.

4. From Table 5, it can be seen that there are only three welded members in the range $3 < c \leq 4$, all of which have slender webs. Two out of these three tests are from Righman (2005) and were not included in the prior database calibration by White and Kim (2008). These test cross sections are extremely singly symmetric ($I_{yc}/I_{yt} < 0.3$) in addition to having slender webs ($\lambda_w > \lambda_{rw}$). Clearly, the current AASHTO equations are an inadequate predictor for these tests.

IMPACT OF PROPOSED MODIFICATIONS ON ROUTINE DESIGN STRENGTH CALCULATIONS

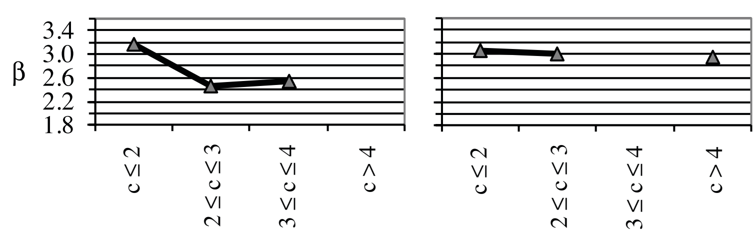
The preceding sections show that the proposed LTB equations result in a clear improvement in achieving a more uniform level of reliability across all ranges of LTB slenderness within the plastic and inelastic buckling ranges, consistent with the AISC LRFD *Specification* target of 2.6 for statically determinate beams and a live load–to–dead load ratio of 3. These reliability estimates are based on refined inelastic/elastic buckling solutions associated with the inelastic stiffness reductions implied by the LTB design equations (White et al., 2016a, 2016b). As noted in the Introduction, accurate accounting for the moment gradient and end restraint effects on critical unbraced lengths is essential to achieving any meaningful correlation among experimental test data, test simulation results, and LTB strength predictions.

In routine practice, designers commonly assume $K = 1.0$ when calculating member LTB resistances. In these situations, when considering shorter and shorter critical unbraced lengths within the inelastic LTB range, the more extensive yielding within the critical unbraced length commonly results in a "true" LTB K factor that can be significantly less than 1.0. Related to this attribute, Yura et al. (1978) stated in the context of compact-section beams, "For uniform moment, the theory indicates that a very small bracing spacing of 2 ft. is required just to reach M_p … The apparent disagreement at the M_p level is due mainly to the torsionally pinned boundary conditions assumed in the theory. A laboratory beam that models actual conditions in practice must have adjacent spans to generate the moments. These spans will then also offer some restraint. Also, for beams with small unbraced lengths, the effects of boundary conditions are more dominating." Based on this behavior, the AISC *Specifications* have traditionally divided by an implicit effective length factor when setting the limiting length, L_p, for the plateau of the LTB resistance curves (White, 2008). With the advent of the 2005 AISC *Specification* (AISC, 2005), this practice has been limited to the L_p for doubly-symmetric nonslender flange members (i.e., $1.76\ r_y\sqrt{E/F_y}$). As demonstrated by



(a) Tests with $C_b \leq 1.3$ *(b) Tests with* $C_b = 1.75$

Fig. 9. Estimated reliability indices for the unified provisions, $L/D = 3$ *and* $\phi = 0.9$*; moment gradient tests of welded members with accurate test geometry.*



(a) Tests with $C_b \leq 1.3$ *(b) Tests with* $C_b = 1.75$

Fig. 10. Estimated reliability indices for the proposed equations, $L/D = 3$ *and* $\phi = 0.9$*; moment gradient tests of welded members with accurate test geometry.*

the correlations of the unified and proposed LTB equations with the experimental data, the use of this L_p equation is overly optimistic if employed with any LTB calculations that accurately account for end restraint effects. The use of this equation along with any other accounting for $K < 1$ amounts to a double counting of the end restraint effects. In addition, the equation $L_p = 1.76\ r_y\sqrt{E/F_y}$ is inappropriate if one is comparing to the results of test simulations conducted using ideal torsionally simply supported end conditions (because this equation implicitly assumes that the unbraced length has significant end restraint).

In the context of the routine use of $K = 1.0$ in LTB calculations, and based on all of the earlier considerations, it is recommended that the expression for L_p can be divided by $K = 0.8$ for all types of I-section members. Even in beams that have physical end conditions that are very close to ideal torsionally simply supported, it is common to observe some incidental restraint. Furthermore, it can be inferred from Figures 5 and 8 (and Figures 3, 4, 6 and 7) that a small increase in L_p of this magnitude can be tolerated in terms of its influence on the estimated reliability.

Figures 11 through 15 show the impact of the recommended modifications relative to the corresponding unified resistance equations, including division of the proposed L_p equation by $K = 0.8$, resulting in the use of $L_p = 0.63r_t\sqrt{E/F_{yc}}\,/\,0.8 = 0.8r_t\sqrt{E/F_{yc}}$. The moment capacity ordinate is normalized by the section plateau resistance, M_{max}, for the homogeneous slender-web member cases, where $M_{max} = R_{pg}M_{yc}$ using the AISC *Specification* notation or R_bM_{yc} using the AASHTO notation. The ordinate is normalized by the section yield moment M_y for the compact rolled-section member cases. The abscissa is the normalized unbraced length $L_b\sqrt{F_y/E}\,/\,r_t$ in all the plots.

The following observations can be gleaned from Figures 11 through 15:

1. Figure 11 conveys the relationship between the unified and proposed LTB resistance curves for uniform bending of all types of slender-web members. One can observe that the recommended reduction relative to the unified resistance curve, which is identical to the AISC *Specification* LTB strength curve for these member types, ranges from 6.2% at the current L_p limit (i.e., at $L_b\sqrt{F_y/E}\,/\,r_t = 1.1$) to 15.7% at the current unified/AISC *Specification* L_r limit. The proposed and current curves are coincident at normalized lengths larger than 4.44 (i.e., at lengths larger than L_r based on the proposed $F_L = 0.5F_{yc}$). The inelastic LTB portion of the recommended strength curve is nearly tangent to the theoretical elastic LTB curve at the proposed values of L_r.

2. Figure 12 shows comparable slender-web member curves for a representative moment gradient case with $C_b = 1.3$. In this case, there is no difference between the current and the proposed strength curves for normalized lengths smaller than 2.48 or larger than 4.44. The maximum difference between the curves is again 15.7%, corresponding to the current L_r.

3. Figure 13 shows the current and proposed LTB resistance curves for W36×150 members, which are representative of relatively lightweight, wide-flange rolled beams, subjected to uniform bending. It should be noted that for normalized lengths smaller than 1.8, the moment capacity of these members is larger than the yield moment M_y, and thus the member compression flange is extensively yielded throughout

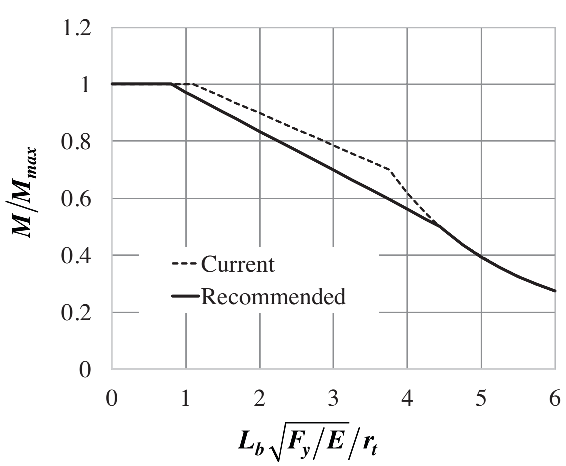


Fig. 11. Current (unified) and proposed LTB strength curves for slender-web I-section members subjected to uniform bending moment.

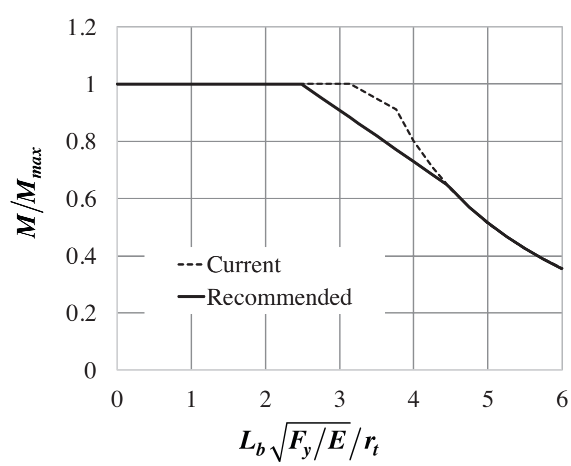


Fig. 12. Current (unified) and proposed LTB strength curves for slender-web I-section members subjected to moment gradient loading with C_b *= 1.3.*

the unbraced length. The reduction in the resistance relative to the current unified values varies from 6.0% at the normalized current unified L_p value of 1.1 to 8.9% at the current unified (and AISC *Specification*) L_r, which corresponds to a normalized length of 4.12. The curves become coincident for normalized lengths larger than 5.05, corresponding to the proposed L_r.

4. Figure 14 shows the results for the current and proposed W36×150 strength curves for a moment gradient case with $C_b = 1.3$. The curves are coincident for normalized lengths less than 2.53 and greater than 5.05. The largest reduction relative to the current unified resistance is again 8.9%.

5. Lastly, Figure 15 shows the results for W14×257 column-type rolled members. This is a representative intermediate-weight column-type section. In this case, the recommended and current unified curves are practically coincident throughout the lengths shown in the plot. One should note that a normalized length of 7.2 corresponds to $L_b/r_y = 200$; therefore, it is expected that normalized lengths larger than 7.2 would be rare.

SHORTAGE OF EXPERIMENTAL DATA

It is evident from Tables 2 through 5 that despite the large total number of experimental tests used in the calibration of the AASHTO Specification LTB curves, there is a paucity of data in the inelastic LTB region. This is particularly the case for welded members with unbraced lengths close to L_r as defined by the proposed equations. For example, from Tables 2 and 3, it is seen that there is only one welded

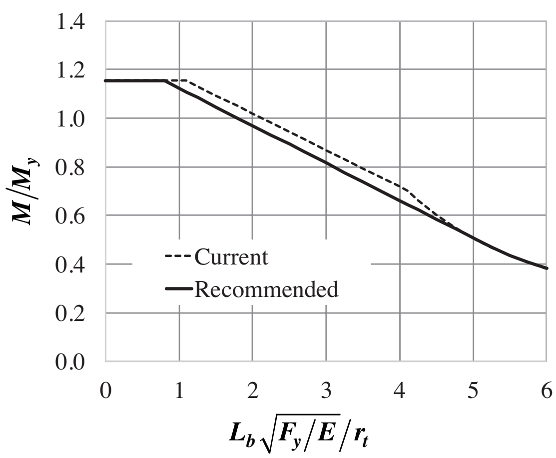


Fig. 13. Current (unified) and proposed LTB strength curves for W36×150 members subjected to uniform bending moment.

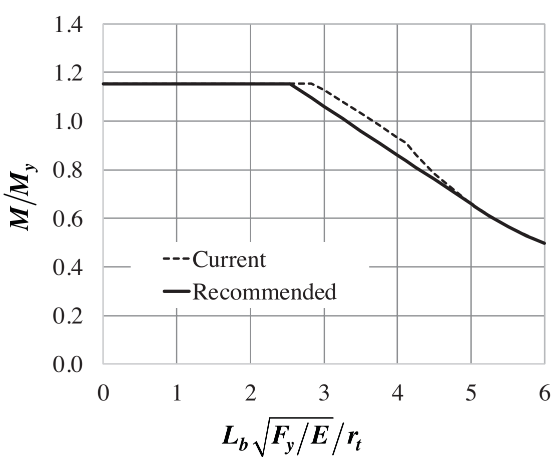


Fig. 14. Current (unified) and proposed LTB strength curves for W36×150 members subjected to moment gradient loading with $C_b = 1.3$.

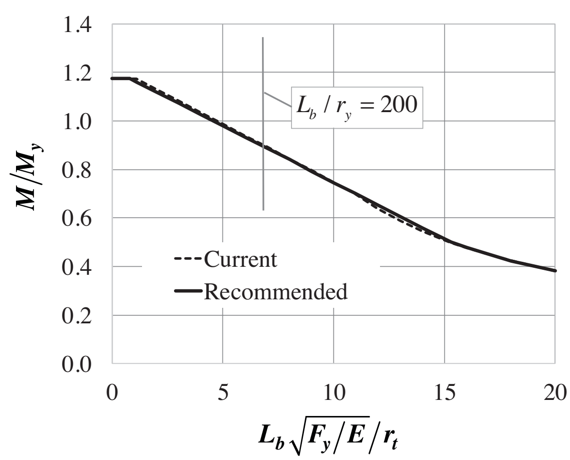


Fig. 15. Current (unified) and proposed LTB strength curves for W14×257 members subjected to uniform bending moment.

member experimental test in the region $c \geq 4$ subjected to uniform moment. This test is of a welded member with a slender web and nominal/approximate geometry. There are no experimental test data for welded members with accurate geometry or for members with compact and noncompact webs in this region. Table 3 shows further that there are no test data for welded members with compact webs and accurate measured geometry in the region $2 < c \leq 3$ and that there are only two tests in the region $1 < c \leq 2$. The number of noncompact-web welded member tests in these regions is five or less, with only a slightly higher number of available tests for slender-web members.

Tables 4 and 5 show that there is a scarcity of experimental test data in certain regions for both rolled and welded I-section members subjected to moment gradient loading. There are only three tests in the region $3 < c \leq 4$ and two in the region $c \geq 4$, for both rolled and welded members with accurate geometry. There are no welded member tests with compact webs in the regions $2 < c \leq 3$ and $3 < c \leq 4$ and only two tests in the region $c \leq 4$. There are no test data for welded members with noncompact webs for $c > 2$. While there are three slender-web welded member tests in the region $3 < c \leq 4$, it is important to note that two of these tests are from Righman (2005) and were not included in the prior calibration (White, 2008). Figures 9 and 10 show that the reliability is substantially improved in this region using the proposed equations. The unified equations are extremely unconservative in estimating the strengths observed in Righman's tests.

The scarcity of experimental data and the improved reliability in the inelastic LTB region using the proposed equations highlights the need to consider a larger database of tests for acceptable reliability computations. Subramanian and White (2017b, 2017c) propose the modified LTB equations based on FE test simulations that encompass a wide variety of cross sections, while simultaneously ensuring a fit to the available experimental data.

CONCLUSIONS

This paper presents a comprehensive analysis of the correlation between nominal strength predictions, obtained based on practical LTB calculations using inelastic stiffness reduction factors (SRFs), with a large suite of experimental data compiled from research worldwide. Both current design resistance equations as well as modified equations recommended by Subramanian and White (2017b, 2017c) are considered. The following are the key conclusions from this study:

1. The equations proposed herein and in Subramanian and White (2017a, 2017b, 2017c, 2017d) are shown to provide a more uniform reliability index compared with the unified provisions, based on the available experimental test data. The proposed equations also provide estimates of the reliability index that are all approximately equal to or greater than the intended values for statically determinate members in the AISC *Specification*. The sparsity of experimental tests in certain regions of the design space is countered by a large number of additional finite element test simulation studies in Subramanian and White (2017b, 2017c). Several important experimental tests are included that were not available at the time of the calibrations by White and Jung (2008) and White and Kim (2008). These tests include extreme singly symmetric slender-web cross sections with unbraced lengths in the inelastic LTB region. These tests indicate a relatively low reliability index for the unified provisions within the intermediate inelastic LTB range. The proposed equations address this shortcoming.
2. The use of inelastic SRFs in LTB calculations provides a practical means of accurately representing the restraining effects from adjoining unbraced lengths, as well as the moment gradient effects associated with partial yielding. The calculations for the effective length and moment gradient factors (K and C_b) in the prior calibration efforts have involved various simplifying assumptions. These simplifications, along with the availability of new test data subsequent to the prior calibration efforts, result in smaller levels of reliability than intended for the LTB strength curve in certain cases. This is particularly the case when the unified provisions are employed in the context of accurate accounting of end restraint and moment gradient effects, such as can be accomplished with practical inelastic buckling analysis methods.

Although the inelastic buckling calculations provide better estimates of the true strengths, engineers may also calculate the LTB design resistance using theoretical elastic effective length factors for the unbraced length, as given by Nethercot and Trahair (1976), or using other estimates. Elastic estimates provide larger values of K than those determined from inelastic buckling calculations. The LTB resistances thus calculated tend to be conservative relative to the "true" solutions. However, it should be noted that when end warping and/or lateral bending restraint are accounted for in the buckling calculations of members subjected to moment gradient, the combined effects of the commonly used K and C_b factors can lead to higher strength predictions than obtained using tools such as SABRE2 or refined finite element test simulations (Subramanian, 2015). That is, the C_b factor equations, which are commonly derived assuming torsionally simply supported end conditions, are not necessarily a good representation of the moment gradient effects in unbraced lengths having significant end warping and/or lateral bending restraint.

It is recommended that in routine design practice, when $K = 1$ is assumed in the LTB strength calculations, the proposed L_p value may be divided by an implicit K value of 0.8, thus providing some liberalization of the more restrictive L_p equation recommended when end restraint effects are addressed directly within the design calculations.

Additional experimental data and inelastic buckling predictions involving transverse loading on members with no intermediate brace points, including load height effects, are discussed by Toğay et al. (2016).

SYMBOLS

A_{fc} Area of compression flange

A_{wc} Area of web in compression

C_b Moment modification factor

D Dead load

D_c Depth of web in compression measured from the inside of the compression flange

E Modulus of elasticity

F_y Yield strength of steel

F_{yc} Yield strength of compression flange

F_{yr} Compression flange stress at nominal onset of yielding including the effects of residual stresses, taken as $0.7F_y$ for homogeneous doubly symmetric I-sections in the AISC *Specification* and AASHTO, denoted by F_L in the AISC *Specification*.

K Effective length factor for lateral-torsional buckling

$K_{elastic}$ Elastic effective length factor for lateral-torsional buckling

$K_{inelastic}$ Inelastic effective length factor for lateral-torsional buckling

L Live load

L_b Unbraced length of beam or girder

L_p Limiting effective unbraced length below which the strength under uniform bending is characterized by the plateau resistance

L_r Limiting effective unbraced length above which the strength under uniform bending is characterized by the theoretical elastic lateral-torsional buckling resistance

M_{max} Maximum possible flexural resistance obtained for short member unbraced lengths, equal to M_p for compact section members and equal to $R_{pg}M_{yc}$ (AISC) = R_bM_{yc} (AASHTO) for homogeneous slender-web members

$M_{n\ Unified}$ Moment calculated using the unified provisions

M_{nPr} Moment calculated using the proposed changes to the AISC *Specification*, AASHTO, and unified LTB resistance equations

M_p Plastic moment

M_{test} Maximum moment obtained from experimental tests as reported by authors

M_y Nominal yield moment

M_{yc} Yield moment corresponding to the compression flange

M_{yt} Yield moment corresponding to the tension flange

N Number of experimental tests considered in the statistical analysis

Q Load effects on member

$\overline{Q}$ Mean of load effects

R Resistance of cross-section

$\overline{R}$ Mean of resistance effects

R_b Web bend-buckling factor, which accounts for the typical decrease in the LTB plateau strength of slender-web sections due to load shedding to the compression flange caused by web bend-buckling, denoted by R_{pg} in the AISC *Specification*.

R_h Hybrid factor, which accounts for early web yielding when the member has a lower yield strength web as compared to the tension and/or compression flange

R_{pc} Web plastification or cross-section effective shape factor for the compression flange, which accounts for the typical increase in the LTB plateau strength above M_{yc} for noncompact and compact web sections

R_{pt} Web plastification or cross-section effective shape factor for the tension flange, which accounts for an increase in the tension flange yield strength over M_{yt}

V Coefficient of variation

V_Q Coefficient of variation of load effects

V_R Coefficient of variation of resistance effects

b_{fc} Width of compression flange

c Normalized LTB slenderness, equal to $KL_b\sqrt{F_{yc}/E}$)/ r_t; $c = 1$ corresponds to the length L_p in the AASHTO Specifications (2016), whereas $c = 1.1$ corresponds to the length L_p in all cases in the AISC *Specification* (2016), with the exception of doubly-symmetric compact-web members having nonslender (i.e., compact or noncompact) flanges

r_t Effective radius of gyration for LTB (in), approximated by $r_t = \dfrac{b_{fc}}{\sqrt{12\left(1+\dfrac{1}{3}\dfrac{D_c t_w}{b_{fc}t_{fc}}\right)}}$

r_y Radius of gyration of a steel I-section with respect to its minor axis

t_{fc} Thickness of compression flange

t_w Thickness of web

β Reliability index

λ_{rw} Noncompact web slenderness limit

λ_w Web slenderness ratio

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