Buckling of Conventional and High-Strength Vanadium Steel Double-Angle Compression Members: Computational Parametric Evaluation of Slenderness Modification Factors

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ABSTRACT

High-strength, low-alloy vanadium (HSLA-V) steel offers higher strength and toughness than conventional steel. The resulting lighter weight and more slender structural components are more susceptible to buckling in compression. Of particular interest to this study are open-web joists, which utilize double-angle sections—typically for chord members and often for web members. Design specification treatment for both global and specifically local buckling of double-angle compression members is evaluated in this study. Specification equation predictions of the buckling load strength for a wide range of specimens and material strengths are examined and compared to analytical simulations. This paper proposes two alternative modifications to the so-called *Q*-factor formulation in order to address the nonconservative buckling strength predictions for double-angle compression members with low *Q* factors. This study concludes that the adoption of a modified *Q*-factor formulation for local elements of compression members in the element elastic buckling region produces consistent predictions of the buckling strength. This finding is equally applicable to both HSLA-V and conventional steels. For design and other applications where a lower-bound estimate of the strength is required, this combination of proposed *Q*-factor formulation and AISC built-up member slenderness modification is recommended.

Keywords: high-strength vanadium steel, compression, computational parametric study, modification factors, buckling analysis, *Q* factor.

INTRODUCTION

A long-term research project sponsored by the Army Research Laboratory (ARL) under Cooperative Agreement DAAD 19-03-2-0036 and executed by the Advanced Technology Institute (ATI) was initiated in 2003 to assess the impact of high-strength, low-alloy microalloyed vanadium (HSLA-V) steels on a wide variety of different applications. HSLA-V steels can have specified yield strengths of up to 90 ksi and thus provide the opportunity both for weight reduction and enhanced sustainabiltiy.

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This paper evaluates the treatment for both global and specifically local buckling of double-angle compression members in the Steel Joist Institute (SJI) Specification and proposes modifications to improve the specification equation buckling strength results for a wide range of parameters and material strength. It presents results from a computational correlation and parametric study the authors performed on double-angle compression components (SGH, 2011, 2012) and complements a companion paper by the authors (Webster et al., 2017).

In *Correlation and Sensitivity Study on the Buckling of HSLA-V Steel in Single and Double Angle Members* (SGH, 2011), the authors described the successful use of nonlinear finite element (FE) analysis to closely match buckling failure modes and strengths observed in 20 double-angle compression member tests with a range of properties. Based on the success of this correlation study and verification of the modeling and simulation approach, a parametric study was executed to extend the range of parameters beyond those in the physical test program.

In the companion paper (Webster et al., 2017), the authors described the validity of applying the buckling equations in the SJI Design Specification (SJI, 2010) to double-angle compression members manufactured using higher strength HSLA-V steel. Present SJI specifications are applicable only for steel with specified yield stress of 50 ksi or less.

The design equations for compression buckling in the SJI Specification were also compared to the 2010 American Institute of Steel Construction (AISC) *Specification*. For both conventional and HSLAV steel members, the Webster et al. study found that the flexural-torsional buckling equation in AISC produces highly conservative estimates of nominal buckling strength and that the 2010 AISC slenderness modification for built-up members enhances the accuracy of the SJI buckling equations, which do not incorporate the AISC flexural-torsional buckling provisions. When comparing the analytical results to the nominal buckling strengths calculated using SJI equations, the observed differences showed a pattern dependence on element local slenderness—that is, the Q factor. The use of AISC built-up member slenderness modification improved the accuracy of the buckling equation on average yet did not significantly reduce this dependence on element slenderness. [Note: SJI adopted the modified slenderness ratio for built-up web members in its 2015 Specification (SJI, 2015)]

This paper extends the analytical parametric study to examine the effect of element slenderness (Q factor) on the accuracy of the specification buckling equations. The analytical and design specification nominal buckling strengths of double-angle members with Q factors ranging from 0.7 to 1.0 are compared. The impacts of introducing two proposed reformulations of the Q factor are examined. The two alternative Q-factor formulations are based on the American Iron and Steel Institute (AISI) 1968 *Specifications for the Design of Cold-Formed Steel Structural Members*. The influence of each alternative Q factor is examined with and without the modified built-up section slenderness ratio as defined in the 2010 AISC *Specification*.

PROVISIONS FOR BUCKLING IN SJI AND AISC *SPECIFICATION*

The estimation of the critical buckling load depends on the mode and type (i.e., elastic or inelastic) of buckling. Critical buckling load is computed for several possible buckling modes depending on the compression member profile. The lowest critical load for the associated buckling mode is assumed to represent the governing buckling phenomenon. The SJI Specification mostly follows the AISC *Specification* but ignores the provisions for built-up member slenderness modification and flexural-torsional buckling. The details of these equations are discussed in the authors' companion paper, mentioned earlier (Webster et al., 2017). This paper focusses on an assessment of various formulations for the Q-factor definition.

Element Slenderness Modification

Compression members with slender cross-section elements have a reduced inelastic buckling strength due to local instabilities that can be evaluated based on plate buckling theory. To capture this local effect, a reduction factor, Q, is introduced to the buckling equations as a reduction multiplier to the material yield strength, with values ranging from 0.7 to 1.0 for common angle sizes.

The local slenderness of an element of the compression member (i.e., angle leg) is determined by the ratio of element size to thickness (b/t) and material yield stress. More slender elements have lower Q-factor values (Figure 1) and thus greater reduction in the buckling strength.

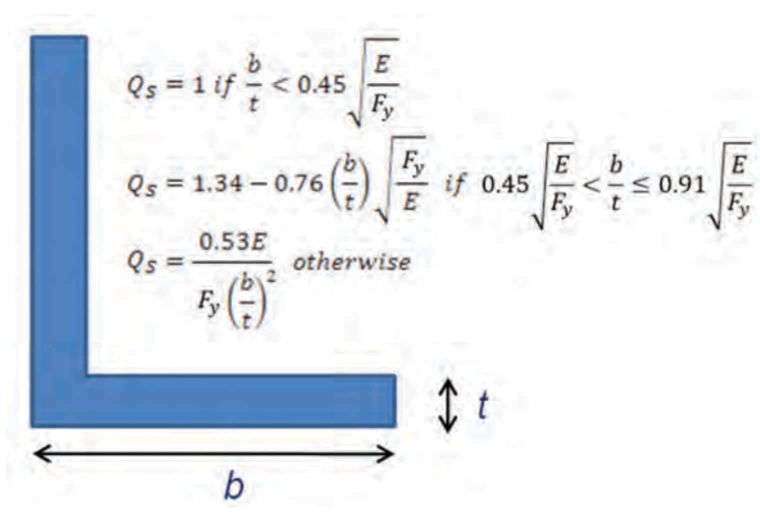


Fig. 1. Angle local element slenderness ratio.

Modified Flexural Buckling for Built-Up Sections

The member slenderness term may be modified if the compression member is built up from two or more sections and interconnected by bolted or welded elements. The modification in slenderness accounts for the impact on the buckling strength of the relative displacement due to shear forces in the connectors between the individual components forming the member. For double angles with welded spacers, the 2010 AISC *Specification* modifies the slenderness ratio as follows:

$$\text{For } \frac{a}{r_i} \leq 40 \qquad (KL/r)_{y,m} = (KL/r)_o \tag{1}$$

$$\text{For } \frac{a}{r_i} > 40 \qquad (KL/r)_{y,m} = \sqrt{(KL/r)_o^2 + (K_i a/r_i)^2} \tag{2}$$

where $(KL/r)_{y,m}$ is the modified slenderness ratio of the built-up member, $(KL/r)_o$ is the slenderness ratio of the built-up member acting as a unit, $K_i = 0.50$ for back-to-back angles, a is the connector spacing along the length of the compression member, and r_i is the minimum radius of gyration of an individual component.

This modification addresses the ability of the built-up section to act compositely in the direction(s) where the radius of gyration of a single component is significantly less than the distance between the centroids of the individual components and, consequently, than the composite radius of gyration of the built-up member (Aslani and Goel, 1991). In the case of double-angle compression members, this modification applies only to strong-axis buckling because the spacers have no influence in the weak-axis buckling case.

Effect of Modification Factors on Strong-Axis Flexural Buckling Strength

The critical stress for flexural buckling given in both the SJI Specification and the 2010 AISC *Specification* is as follows:

$$F_{cr} = \min_i \begin{cases} \left(0.658^{\frac{QF_y}{F_{e,i}}}\right) QF_y \\ 0.877 F_{e,i} \end{cases} \tag{3}$$

$$(KL/r)_i \leq 4.71\sqrt{\frac{E}{QF_y}}, \quad \frac{QF_y}{F_e} \leq 2.25$$

$$(KL/r)_i > 4.71\sqrt{\frac{E}{QF_y}}, \quad \frac{QF_y}{F_e} > 2.25$$

where F_{cr} is the critical buckling stress, Q is the section element slenderness reduction factor for unstiffened elements, $F_{e,i} = \dfrac{\pi^2 E}{(KL/r)_i^2}$ is the theoretical member elastic buckling stress, E is the material's Young's modulus of elasticity, and KL/r is the slenderness ratio of the compression member (modified for built-up sections in AISC). Subscript i refers to the two buckling axes: strong and weak axis.

Hence, the material yield stress is multiplied directly by the Q factor and reduced to account for local buckling of slender elements, while the modified global slenderness indirectly reduces the nominal buckling strength by reducing the value of the elastic buckling stress to account for influence of the built-up section.

BACKGROUND AND PROPOSED MODIFICATIONS FOR *Q* FACTOR

The companion paper by the authors that presented the computational parameter study (Webster et al., 2017) reviewed past research on single- and double-angle buckling behavior, published over the course of nearly three decades. The following sections will present the theoretical and historical development of the Q factor.

Theoretical Development of Element Slenderness Factor

The 2010 AISC *Specification* identifies two types of elements when considering local buckling depending on the element disposition within the overall member (Figure 2). An element bounded on both edges along its length with other elements, such as the web of an I-beam, is defined as a *stiffened element*. An element that has one free edge along its length, such as the leg of an angle, is defined as an *unstiffened element*.

The 2010 AISC slender element reduction factor for unstiffened slender member elements (e.g., angle legs), Q_s, is directly based on a simple-element, elastic buckling model of a plate subjected to uniform in-plane compression. The reduction factor is simply the ratio of the critical (local) buckling stress determined by this simple plate model to the material yield strength. This reduction factor is directly applied to the material yield strength when specifying the overall member nominal buckling strength.

The plate elastic buckling stress relationship is given in Equation 4, where μ is the material Poisson ratio and k is a plate buckling coefficient dependent on the plate boundary conditions and aspect ratio. For unstiffened slender-angle elements, the buckling coefficient, k converges to 0.425:

$$F_{cr} = k\frac{\pi^2 E}{12(1-\mu^2)(b/t)^2} \tag{4}$$

Figure 3 plots the ratios of critical buckling stress to yield stress versus compression member slenderness for plates and columns. Three distinct regions of behavior are observed in this figure:

- For very slender elements with a slenderness parameter, λ_c greater than 1.4 the buckling is essentially elastic.
- For very stout elements (λ_c less than e.g. 0.46 for plates), the entire cross-section under compression can reach and exceed the yielding stress of the material by hardening. However, for design purposes, hardening in the material is ignored, and the cross-section strength is limited to the material yield stress.
- Finally, there is a transition zone between the element elastic buckling region and the full yielding region, where the buckling strength is lower than the theoretical critical load envelope (solid line in Figure 3) due to residual stress and geometric imperfections.

For unstiffened plates under uniform compression, the transition zone is idealized as a linear transition as shown in Figure 4a, which also compares the analytical buckling

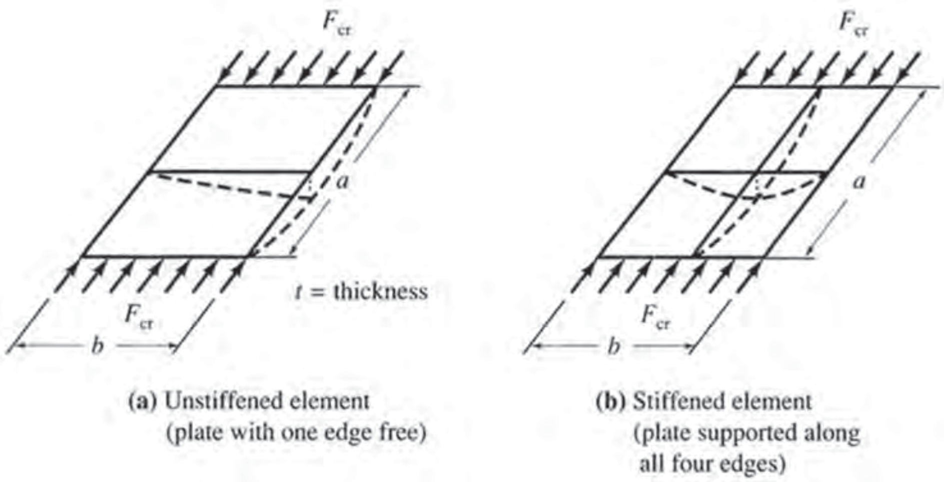


Fig. 2. Plate buckling configuration for unstiffened and stiffened compression elements [AISI S100-2007-C (AISI, 2007)].

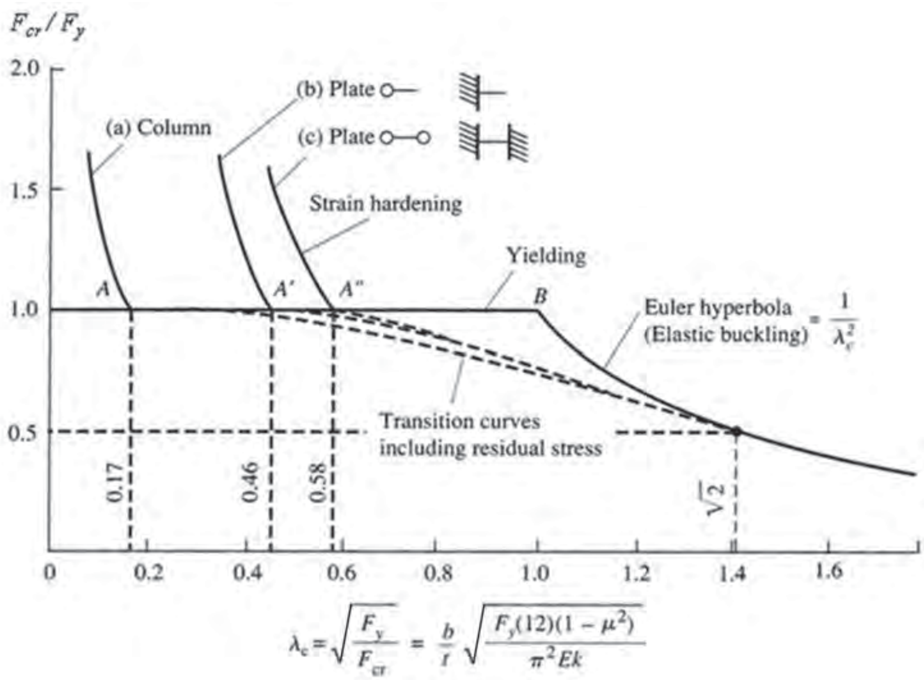


Fig. 3. Comparison of critical buckling stress to yield stress ratio versus member slenderness for (a) columns, (b) unstiffened plates, and (c) stiffened plates (Salmon and Johnson, 1990).

stresses defined in the three regions to test results. For each test, two data points are plotted in the figure: the local buckling stress and the member failure stress. For stout compression members with low slenderness parameters, the difference between the two plotted points is small, whereas as the slenderness parameter increases the difference between the two points becomes greater. This is attributed to the post-buckling strength where the section stress increases as the applied displacement increases.

The additional post-buckling strength available in slender elements is accounted for by increasing the element elastic buckling stress as shown in Figure 4b. Line A marks the strength of elements that are stout enough to reach the full-section yield strength. Line B is the linear transition zone between the element yielding and elastic buckling zones. Line D is the plate elastic buckling curve, which is increased by a factor to account for the post-buckling strength mentioned earlier and is shown with line E.

The 2010 AISC *Specification* accounts for post-buckling strength of unstiffened elements as depicted by line E in Figure 4b. The 1968 AISI Specification ignored this additional strength for angle components, stating the following in its Commentary: "There is a type of cross-section composed entirely of unstiffened elements which shows little or no post-buckling strength. This is the angle section when used for compression struts. This is because, when an equal leg, thin angle reaches the buckling stress of the two equal, component plates (legs), both of them buckle in the same direction; this results in a twisting distortion of the angle as a whole, leading to early collapse…" The compression strength is thus distinguished for angle components, as shown in Figure 5a. The 2010 AISC *Specification* does not distinguish the behavior of slender elements and slender angle legs, as shown in Figure 5b.

Proposed Q Factor Modifications

Two alternative redefinitions for the slender cross-section element buckling reduction factor, Q, are proposed and assessed in this study. Both proposals are based on a return to the 1968 AISI Specification that did not consider post-buckling strength for angle components. The two alternative Q-factor formulations use the plate elastic buckling formulation for calculating the local buckling strength of slender elements. Alternative 1, Q_{A1}, shown in Figure 6, uses the plate elastic buckling relationship up to the point where it intersects the linear transition zone as defined in the 2010 AISC *Specification*. Alternative 2, Q_{A2}, shown in Figure 7, uses the plate elastic buckling relationship until it reaches the full cross-section yield strength.

COMPUTATIONAL PARAMETER STUDY

The parameter study consisted of analytical buckling simulations on double-angle specimens made of steel materials with 50-, 65- and 80-ksi nominal yield strengths (Candas et al., 2008). The angle sizes included in the study were LL8×8, LL6×6, LL4×4 and LL2×2. The parameters assessed were the member slenderness (KL/r), element slenderness (Q factor), number of spacers, the imperfection magnitude, and the end conditions about the weak and strong axes. A total of 3552 cases were analyzed, resulting in a database of 1776 buckling strengths.

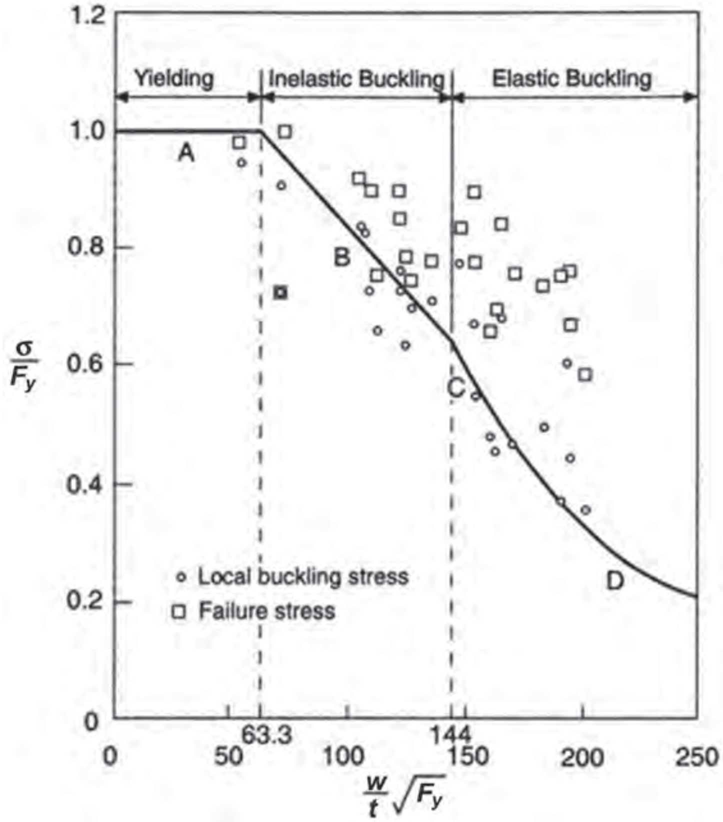
The generation of the set of parametric models was automated using customized scripts and the mesh generation program Truegrid (XYZ Scientific Computing, n.d.) The buckling analyses of the double angles were carried out using the general-purpose, nonlinear, FE software ABAQUS. ABAQUS has extensive capabilities for modeling continuum mechanics, including contact, and for solving elastic buckling as well as unstable post-buckling problems. The nonlinear buckling analyses were solved using the Modified Riks algorithm, which is available in ABAQUS for loading regimes with geometrically unstable phases. Further details are discussed in the companion paper (Webster et al., 2017).

Analysis Results

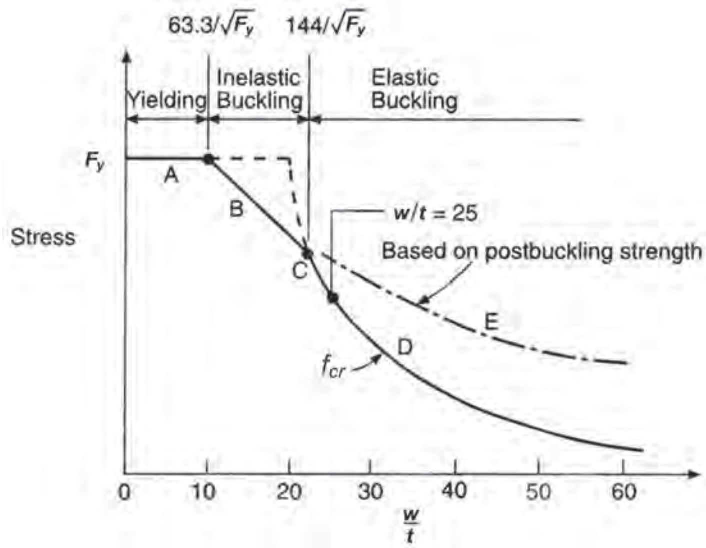
Table 1 summarizes the ratios of buckling strengths from the analysis results to the nominal buckling strengths calculated using the SJI Specification equations for the different parameter study variables. The comparison results are shown for modeled geometric imperfection magnitude values of code-basis L/1500 as well as L/500. The results for weak-axis buckling show that the SJI buckling equations are adequate and appropriately conservative for nearly all the cases included in the study. The results for strong-axis buckling show that the SJI buckling equations are nonconservative for many cases in the study. This nonconservatism is particularly pronounced for cases where $Q = 0.7$ and is present for all three steel grades.

While the average strong-axis results for $Q = 1.0$ and $Q = 0.85$ cases are above unity when the code-basis L/1500 imperfection magnitude is used (note that the ratios for $Q = 0.85$ are slightly yet consistently more conservative than $Q = 1.0$), the results for several individual specimens are nonconservative, as can be seen in Figure 8. Hence, this nonconservatism increases as the member slenderness (L/r) and/or element slenderness (b/t) increases.

To address the nonconservatism in the strong-axis buckling results, the two alternative formulations proposed for the Q factor were investigated. In the companion paper, the adoption of all the 2010 AISC *Specification* buckling equations and the adoption of selected provisions addressing the built-up member slenderness (KL/r) modification equation were evaluated. In this study, the influence of the proposed changes to the Q-factor calculation in order to specifically address the nonconservatism associated with the low Q factors is evaluated. Using the proposed Q factors both with and without the AISC built-up member slenderness modification is also assessed.



(a)



(b)

Fig. 4. Relationship between unstiffened compression plate elements and predicted maximum compressive stress: (a) correlation between test data and predicted maximum stress (Yu, 2010); (b) comparison between idealizations that include and exclude post-bucking strength (AISI S100-2007-C).

PROPOSED MODIFICATIONS TO ELEMENT AND COMPONENT SLENDERNESS EFFECTS

Element Slenderness

The slender cross-section element buckling reduction factor alternatives Q_{A1} and Q_{A2} introduced in Figures 6 and 7 are based on the 1968 AISI assumptions regarding slender angle cross-sections and their inability to develop post-buckling strength due to twisting instability in the angle legs once the element elastic buckling load is reached. A preliminary comparison of both alternatives suggests the following characteristics:

1. Alternative Q_{A1} represents a more limited change from the 2010 provisions, while alternative Q_{A2} represents a more simplified relationship for design.
2. For elements in the elastic local buckling region ($Q < 0.8$), both alternatives result in lower Q factor values than

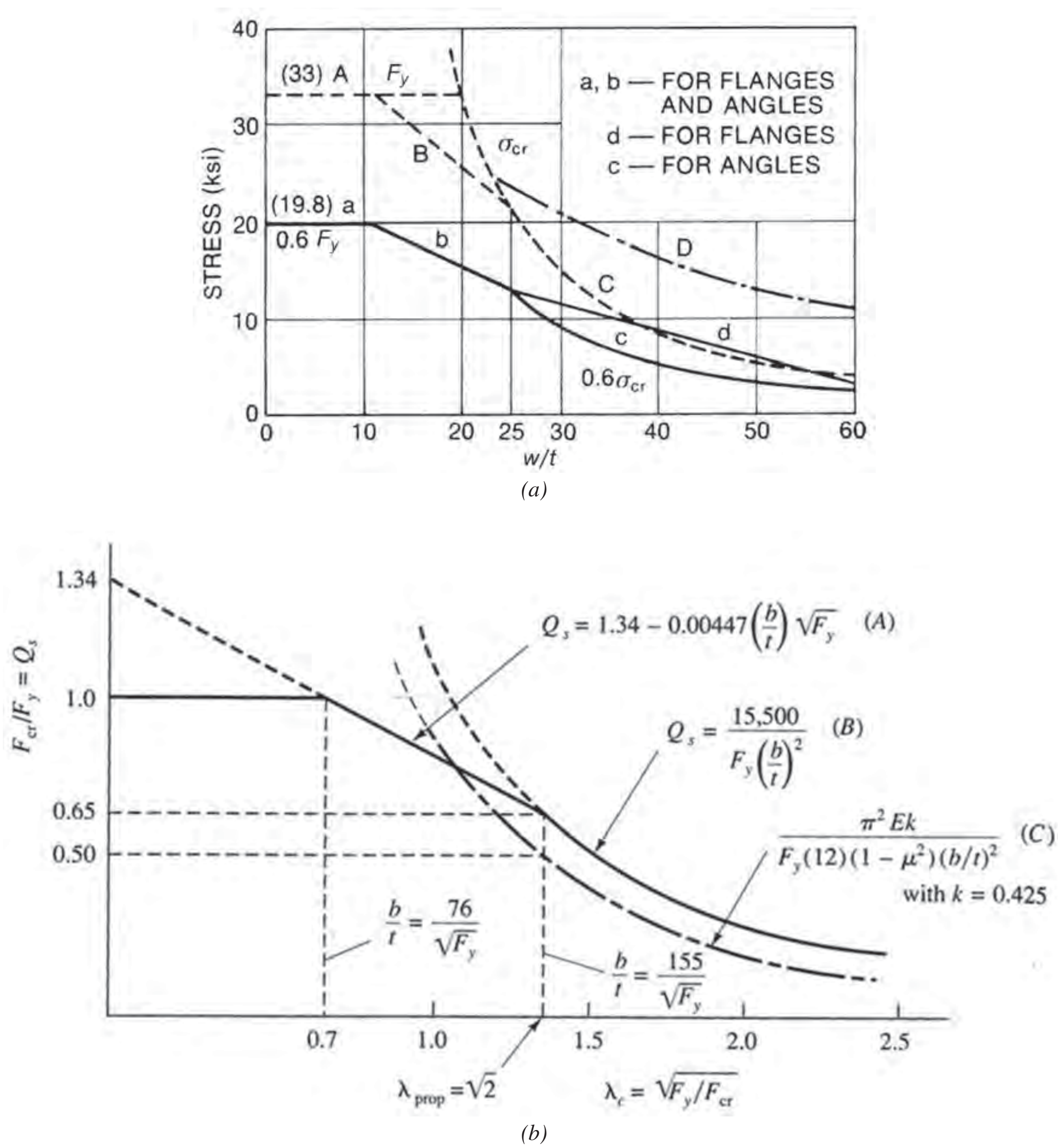
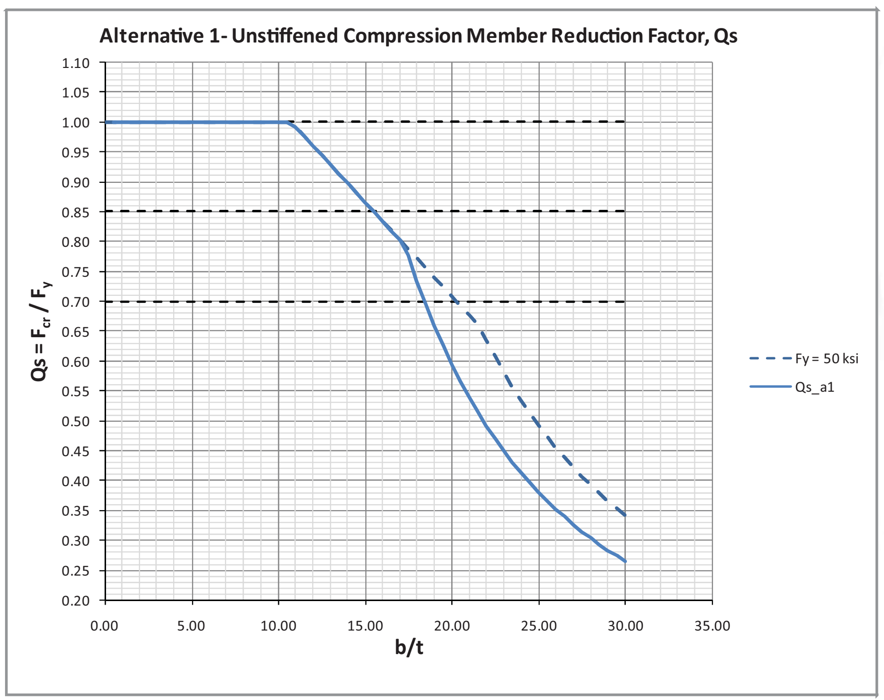


Fig. 5. Q factor according to 1968 AISI and 2010 AISC: (a) permissible design stress for unstiffened compression elements for 33-ksi steel in the 1968 AISI Specification Commentary (Winter, 1970); (b) 2010 AISC Specification *definition (dashed line labeled (C) shows plate elastic buckling).*

Table 1. Global Comparison of FE Analysis to SJI Nominal Strength Ratios							
Global Average of P_{fe}/P_n per SJI Specification							
		L/1500			**L/500**		
Axis	**Variable**	**50 ksi**	**65 ksi**	**80 ksi**	**50 ksi**	**65 ksi**	**80 ksi**
	***Q* Factor**						
Strong	0.7	0.855	0.890	0.868	0.782	0.808	0.787
	0.85	1.033	1.034	1.021	0.900	0.916	0.917
	1	1.019	1.011	1.004	0.928	0.930	0.929
Weak	0.7	1.045	1.104	1.095	0.945	1.016	1.021
	0.85	1.186	1.172	1.160	1.066	1.083	1.080
	1	1.136	1.126	1.116	1.062	1.060	1.055
	Number of Spacers						
Strong	1	0.973	0.973	0.958	0.872	0.878	0.873
	2	1.005	1.015	1.005	0.899	0.918	0.915
Weak	1	1.124	1.127	1.116	1.030	1.050	1.043
	2	1.148	1.151	1.142	1.047	1.069	1.072
	Mean: Strong	0.989	0.994	0.982	0.886	0.898	0.894
	Mean: Weak	1.136	1.139	1.129	1.038	1.060	1.058
	Mean: Weak and strong	1.063	1.067	1.055	0.962	0.979	0.976
	Grand total	1.061			0.972		



(i) $b/t \le 0.45\sqrt{E/F_y}$

$$Q_s = 1.0$$

(ii) $0.45\sqrt{E/F_y} < b/t \le 0.71\sqrt{E/F_y}$

$$Q_s = 1.34 - 0.76\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}$$

(iii) $b/t > 0.71\sqrt{E/F_y}$

$$Q_s = \frac{0.41E}{F_y\left(\frac{b}{t}\right)^2}$$

Fig. 6. Proposed Q_{A1} *factor* Q_s *equation versus angle leg-to-thickness ratio for 50-ksi steel (dashed curve shows* Q *factor per AISC, 2010).*

calculated by the 2010 AISC provisions. This can offset the nonconservatism observed in Table 1 for $Q = 0.7$ cases.

3. For elements in the transition region ($1 > Q > 0.8$), Q_{A1} follows the 2010 AISC provisions, while Q_{A2} results in higher Q factor values. This can reduce the consistent slight conservatism in $Q = 0.85$ cases.
4. For elements in the inelastic local buckling region ($Q = 1$), Q_{A1} and Q_{A2} follow the 2010 AISC provisions.

Member Slenderness

The use of the slender cross-section element buckling reduction factor alternatives was investigated with and without the built-up member slenderness modification per the 2010 AISC provisions according to Equations 1 and 2. The built-up member slenderness modification only affects the strong-axis buckling strength. It primarily affects the buckling strength of slender members in the elastic buckling region by increasing their effective slenderness and, therefore, reducing their nominal buckling strengths. (Meanwhile, the modified element slenderness—i.e., modified Q factor—reduces the nominal buckling strength of members with slender elements in the inelastic buckling range.) The companion paper (Webster et al., 2017) demonstrated that the adoption of AISC slenderness modification adequately addresses the observed nonconservatism in analytical-to-nominal buckling strength ratios with increasing member slenderness.

ASSESSMENT OF PROPOSED MODIFICATIONS

Modification of Element Slenderness Q Factor

Table 2 summarizes the ratios of the analytically determined to the calculated buckling strengths using the SJI Specification and modified using the proposed Q factor alternative Q_{A1}. Comparison with Table 1 reveals the following observations:

- The proposed Q factor modifications only affects the results for $Q = 0.7$ cases.
- For $Q = 0.7$, strong-axis buckling, the average ratios for all steel grades is between 0.99 and 1.04 (for code-basis L/1500 imperfection magnitude). This reflects a consistent margin of safety when compared to components with higher Q-factor values (the corresponding margins of safety for $Q = 0.85$ and $Q = 1.0$ cases in Table 2 range between 1.00 and 1.03). The overall mean ratios for 50-, 65- and 80-ksi steels grades are 1.03, 1.02 and 1.01, respectively.
- For weak-axis buckling, the average ratios for all steel grades and Q factor combinations are between 1.13 and 1.17, reflecting a consistent margin of safety when compared to angles of higher Q-factor values. The overall mean ratios for all three steel grades are between 1.14 and 1.16.

Table 3 summarizes the ratios of the analytically determined to the calculated buckling strengths using the

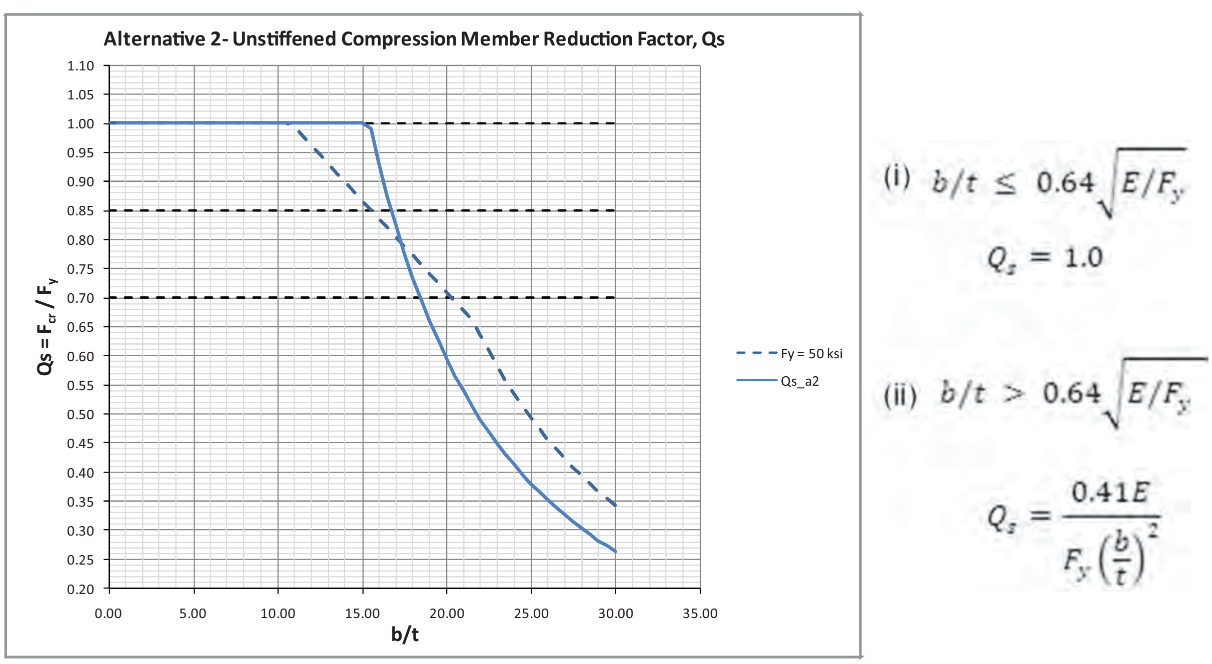


Fig. 7. Proposed Q_{A2} factor, Q_s equation versus angle leg-to-thickness ratio for 50-ksi steel (dashed curve shows Q factor per AISC, 2010).

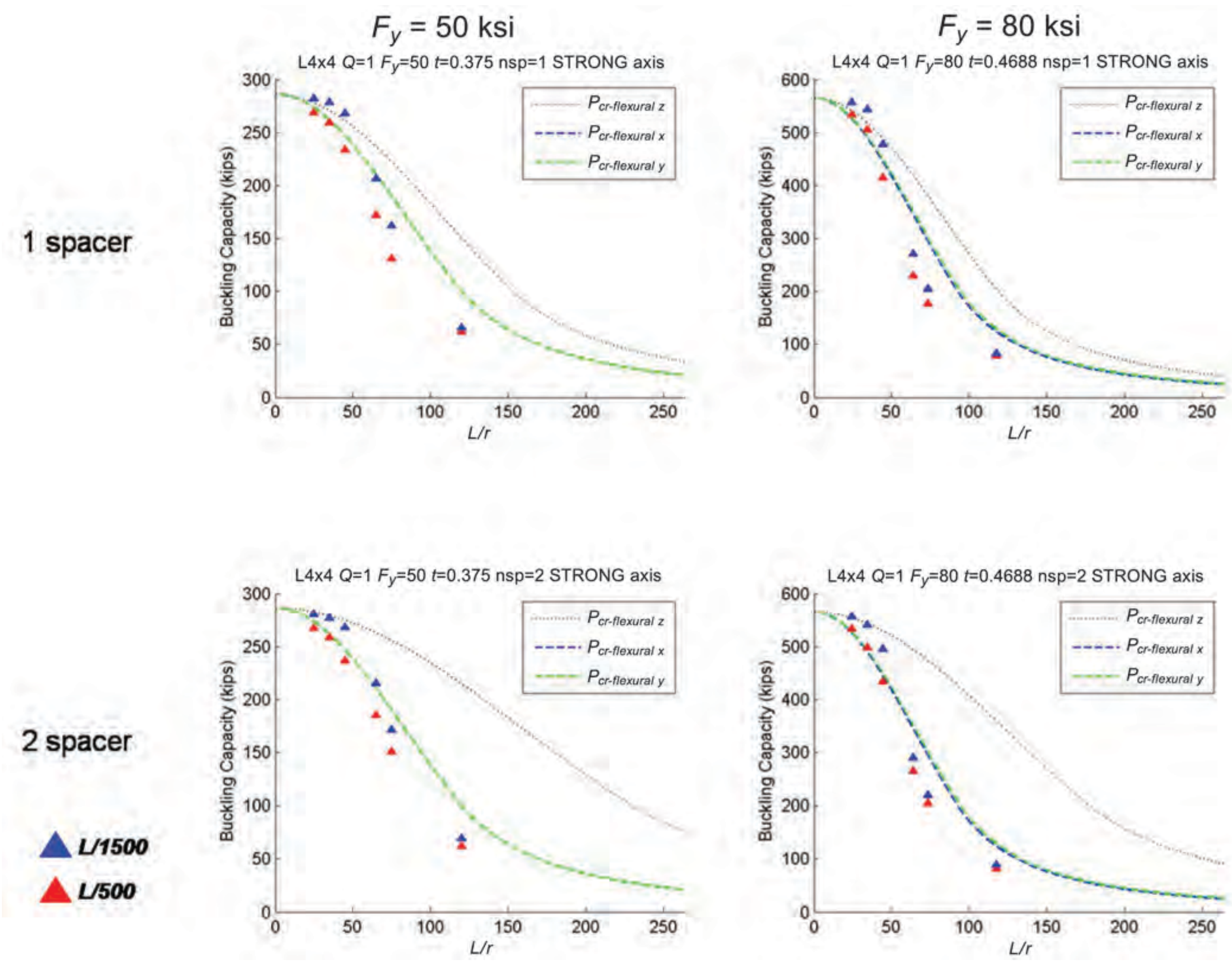


Fig. 8. LL4×4 analytical strong-axis (pinned) buckling strengths compared to the SJI nominal strengths; L/500 and L/1500 imperfection magnitudes; F_y *= 50-ksi and 80-ksi materials;* Q = *1.0.*

Table 2. Global Comparison of FE Analysis to Q_{A1} Modified SJI Strength Ratios (compare to Table 1)								
Global Average of $(F_{fe}/F_y)/(F_{cr}/F_{y\_}Q_{A1})$								
		L/1500				L/500		
Axis	Variable	50 ksi	65 ksi	80 ksi		50 ksi	65 ksi	80 ksi
	Q Factor							
Strong	0.7	1.044	0.986	0.988		0.958	0.897	0.898
	0.85	1.032	1.034	1.022		0.899	0.916	0.917
	1	1.018	1.012	1.003		0.927	0.930	0.929
Weak	0.7	1.171	1.162	1.164		1.061	1.070	1.088
	0.85	1.185	1.172	1.161		1.065	1.083	1.081
	1	1.135	1.126	1.115		1.062	1.061	1.054
	Number of Spacers							
Strong	1	1.012	0.993	0.983		0.908	0.897	0.896
	2	1.046	1.037	1.032		0.937	0.938	0.939
Weak	1	1.149	1.139	1.130		1.054	1.062	1.057
	2	1.176	1.165	1.157		1.071	1.081	1.086

Mean: Strong	1.029	1.015	1.007		0.923	0.918	0.918
Mean: Weak	1.163	1.152	1.144		1.063	1.072	1.072
Mean: Weak and strong	1.096	1.084	1.075		0.993	0.995	0.995
Grand total	1.085				0.994		

SJI Specification and modified using the proposed Q factor alternative Q_{A2}. Comparison with Tables 1 and 2 reveals the following observations:

- The effect of the QA2 proposal on the $Q = 0.7$ cases is identical to that of the QA1 proposal.
- For $Q = 0.85$, the Q_{A2} proposal results in a reduction in the average ratios of analytical-to-predicted buckling strengths for all steel grades. For strong-axis buckling, this reduction results in nonconservative predictions of the buckling load. For weak-axis buckling, this reduction results in a minor decrease in the margin of safety.
- For strong-axis buckling, the overall mean ratios for all three steel grades are between 0.98 and 1.0, which reflects a nonconservative bias.

Figures 9 through 13 show graphical comparisons of the analytical results to the predictions of the SJI Specification using the 2010 AISC nominal buckling strengths and the buckling strengths modified using the two proposed Q-factor definitions.

Figures 9 and 10 show the results for LL8×8 LL6×6 angles, respectively. These two angle sizes have Q-factor values not lower than 0.8. For angles with relatively low member slenderness parameters ($\lambda_{cr} < 1$), it is clear that Q_{A2} returns a better fit on average, while Q_{A1} and Q_{SJI} can be seen as a lower bound for design, which maintains a safety margin. For angles with higher member slenderness parameter λ_{cr}, there are cases where the SJI Specification slightly overpredicts the strong-axis buckling strength for all three Q-factor formulations.

Figures 11 and 12 show the results for LL4×4 and LL2×2 angles, respectively. These two angle sizes have Q-factor values as low as 0.67. For specimens with $Q = 0.8$ or higher, the comparison yields similar results to that of LL8×8 and LL6×6 angles (Figures 9 and 10). For angles with lower Q-factor values and relatively low member slenderness parameter ($\lambda_{cr} < 1$), both proposed Q-factor formulations result in predicted buckling strengths that constitute a lower bound of nearly all the analytical results, while the 2010 AISC Q-factor formulation overpredicts the strong-axis analytical strength in all but four of 72 cases. For angles with higher member slenderness parameters λ_{cr}, there are cases where the SJI Specification slightly overpredicts the strong-axis buckling strength for all three Q-factor formulations.

Figure 13 shows that the inclusion of residual stresses in the analytical model to determine the buckling strength produces limited reduction in the analytical strength for

Table 3. Global Comparison of FE Analysis to Q_{A2} Modified SJI Strength Ratios (compare to Table 1)								
Global Average of $(F_{fe}/F_y)/(F_{cr}/F_{y\_}Q_{a2})$								
		L/1500				L/500		
Axis	**Variable**	**50 ksi**	**65 ksi**	**80 ksi**		**50 ksi**	**65 ksi**	**80 ksi**
	***Q* Factor**							
Strong	0.7	1.044	0.986	0.988		0.958	0.897	0.898
	0.85	0.953	0.956	0.951		0.829	0.846	0.853
	1	1.018	1.012	1.003		0.927	0.930	0.929
Weak	0.7	1.171	1.162	1.164		1.061	1.070	1.088
	0.85	1.126	1.118	1.114		1.012	1.033	1.037
	1	1.135	1.126	1.115		1.062	1.061	1.054
	Number of Spacers							
Strong	1	0.981	0.963	0.955		0.882	0.870	0.871
	2	1.015	1.006	1.004		0.909	0.910	0.913
Weak	1	1.127	1.118	1.112		1.034	1.043	1.040
	2	1.152	1.143	1.138		1.051	1.061	1.069
	Mean: Strong	0.998	0.984	0.979		0.895	0.890	0.892
	Mean: Weak	1.139	1.131	1.125		1.042	1.052	1.054
	Mean: Weak and strong	**1.069**	**1.057**	**1.052**		**0.969**	**0.971**	**0.973**
	Grand total	**1.059**				**0.971**		

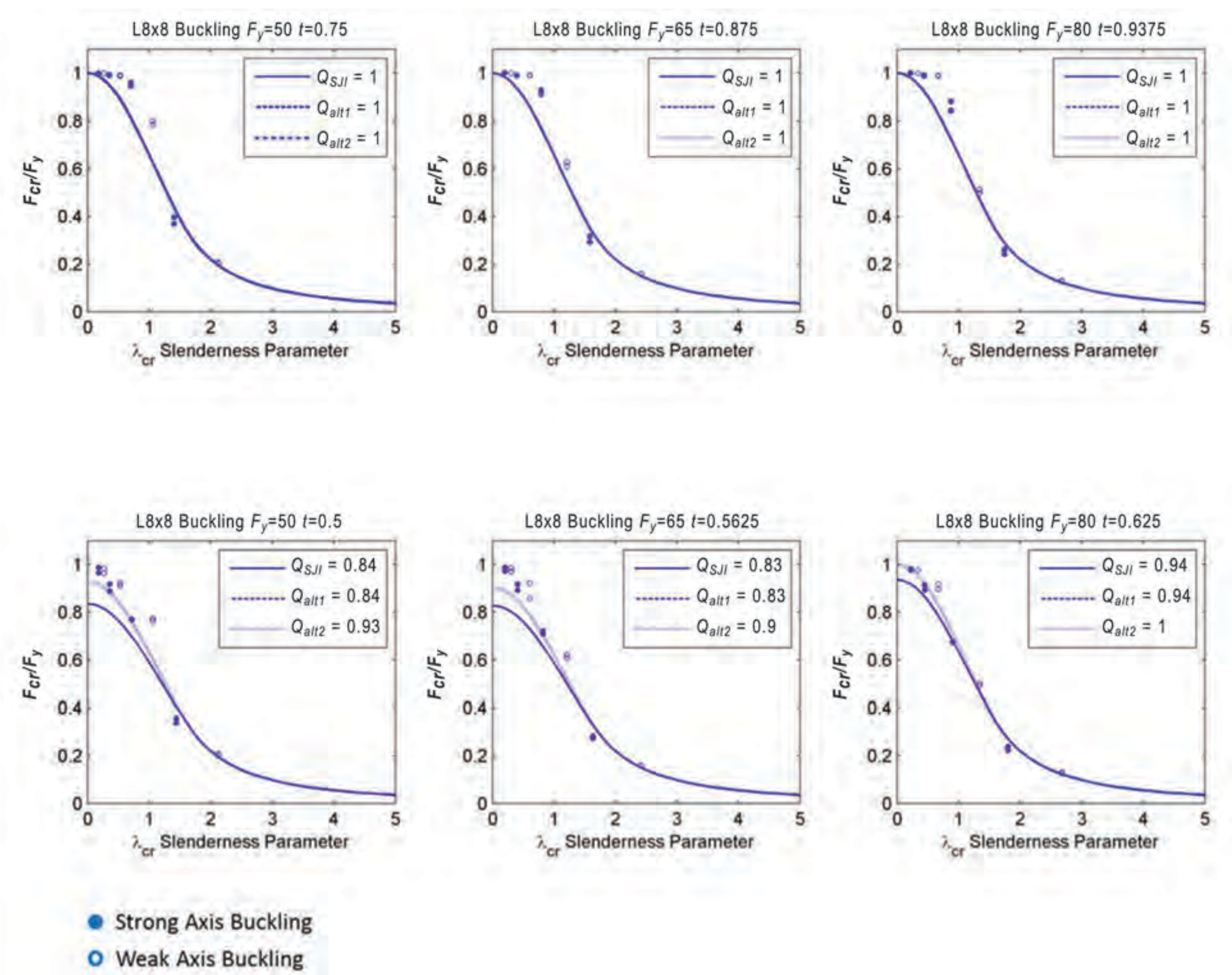


Fig. 9. LL8×8 normalized analytical buckling strength (L/1500 imperfection) compared to normalized flexural buckling equations using Q*-factor definitions per 2010 AISC* Specification, Q_{A1}, *and* Q_{A2}.

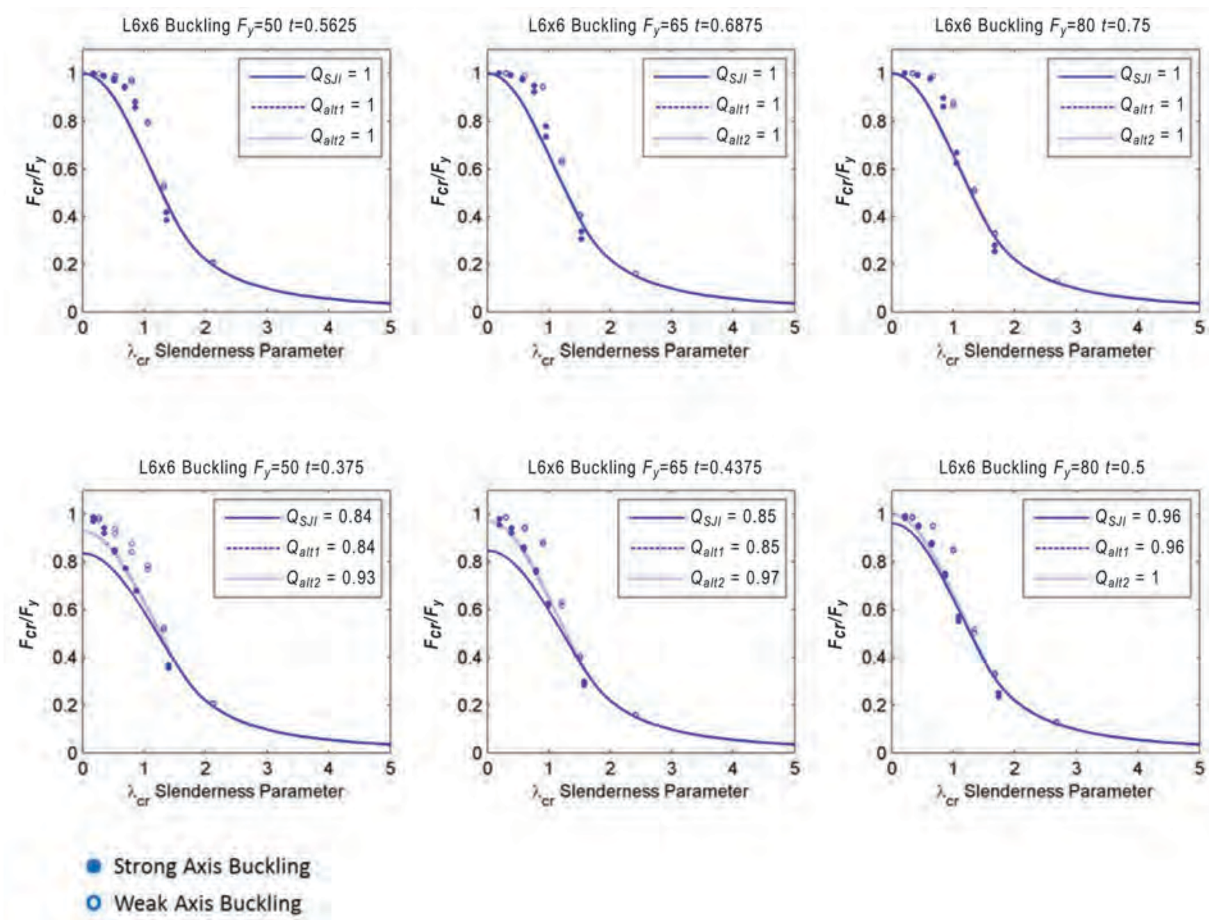


Fig. 10. LL6×6 normalized analytical buckling data (L/1500 imperfection) compared to normalized flexural buckling equations using Q*-factor definitions per 2010 AISC* Specification, Q_{A1}, *and* Q_{A2}.

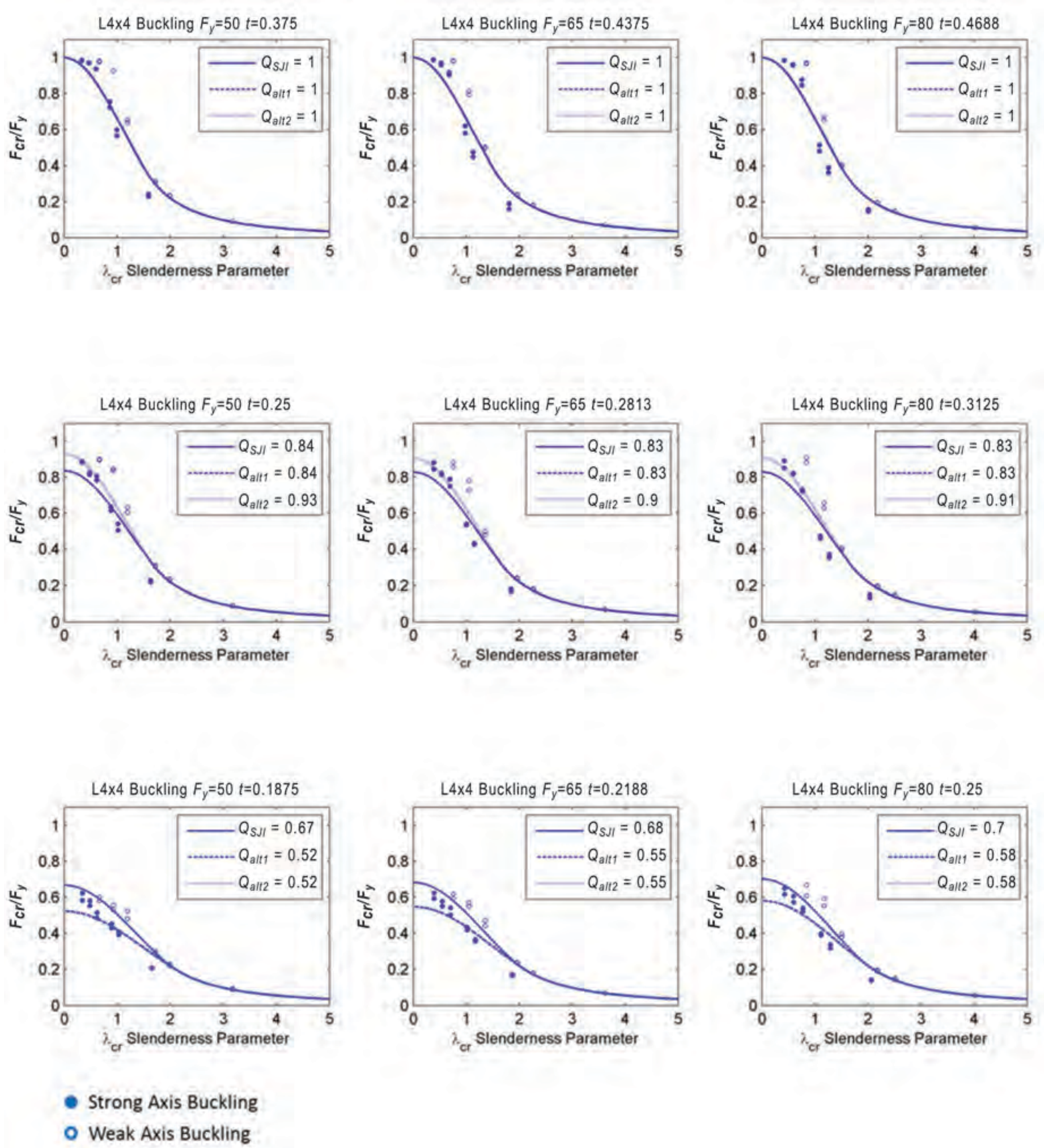


Fig. 11. LL4×4 normalized analytical buckling data (L/1500 imperfection) compared to normalized flexural buckling equations using Q*-factor definitions per 2010 AISC* Specification*,* Q_{A1}*, and* Q_{A2}.

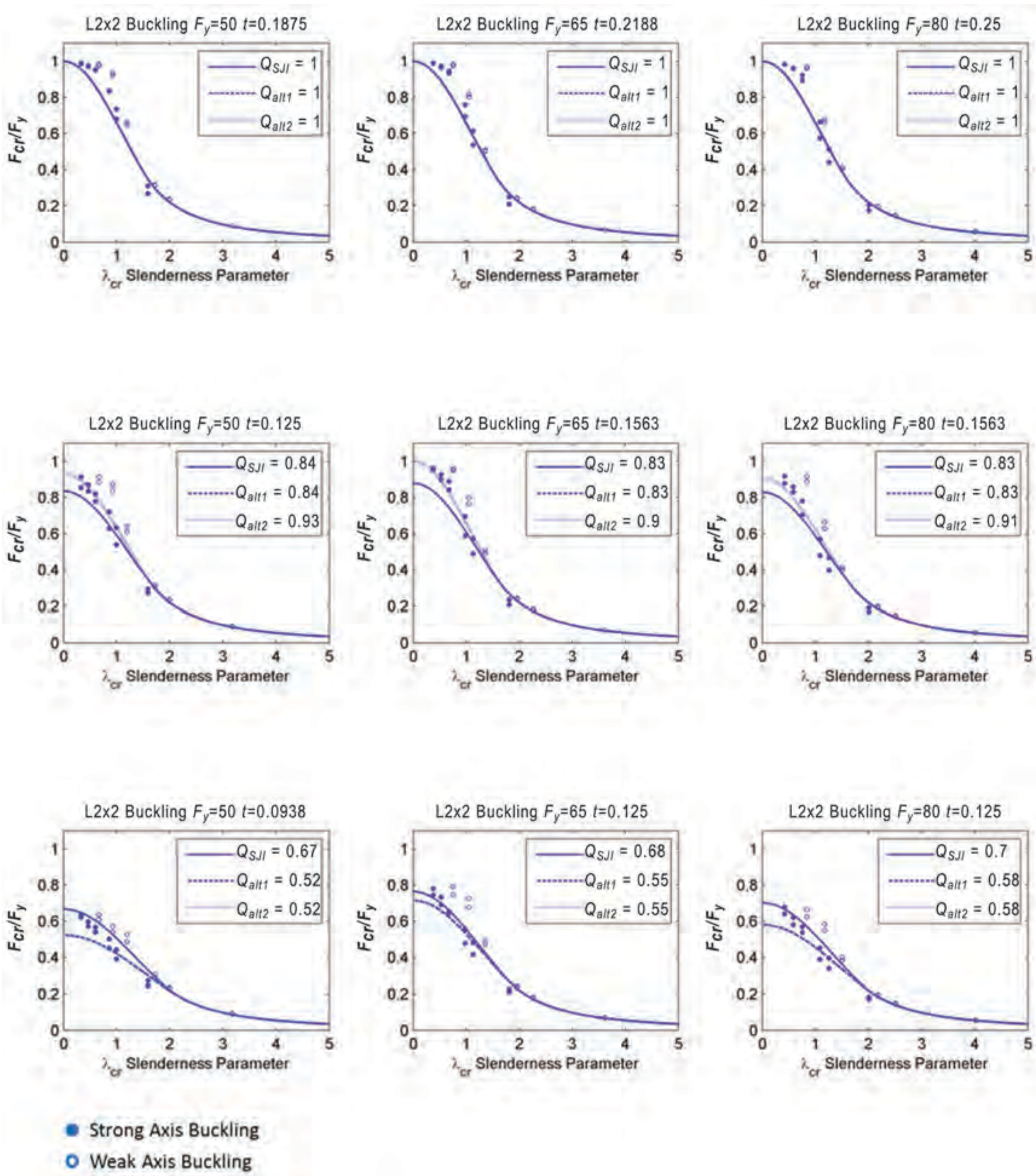


Fig. 12. LL2×2 normalized analytical buckling data (L/1500 imperfection) compared to normalized flexural buckling equations using Q*-factor definitions per 2010 AISC* Specification, Q_{A1}, *and* Q_{A2}.

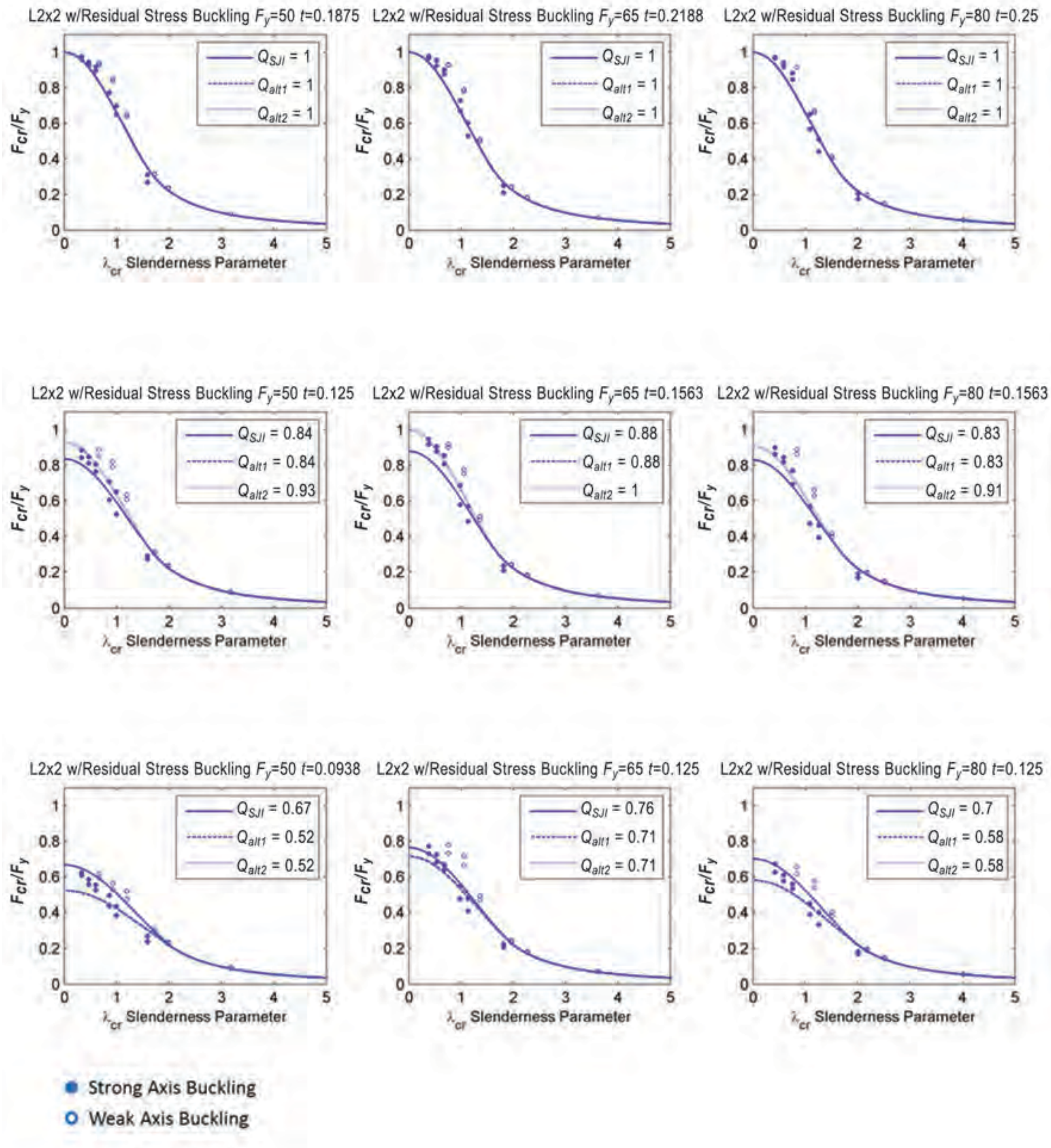


Fig. 13. LL2×2 (with residual stress) normalized analytical buckling data (L/1500 imperfection) compared to normalized flexural buckling equations using Q*-factor definitions per 2010 AISC* Specification, Q_{A1}, *and* Q_{A2}.

Table 4. Global Comparison of Ratios of FE Analysis to SJI Strength Modified Q_{A1} and AISC Built-Up Member Slenderness Modification (compare to Tables 1 and 2)								
Global Average of $(F_{fe}/F_y)/(F_{cr}/F_{y}$_Modified_$Q_{A1})$								
		L/1500				L/500		
Axis	Variable	50 ksi	65 ksi	80 ksi		50 ksi	65 ksi	80 ksi
	Q Factor							
Strong	0.7	1.092	1.053	1.064		0.999	0.956	0.965
	0.85	1.090	1.104	1.100		0.949	0.977	0.986
	1	1.087	1.090	1.090		0.989	0.999	1.008
Weak	0.7	1.171	1.162	1.164		1.061	1.070	1.088
	0.85	1.185	1.172	1.161		1.065	1.083	1.081
	1	1.135	1.126	1.115		1.062	1.061	1.054
	Number of Spacers							
Strong	1	1.102	1.101	1.103		0.987	0.992	1.004
	2	1.077	1.074	1.073		0.964	0.971	0.976
Weak	1	1.149	1.139	1.130		1.054	1.062	1.057
	2	1.176	1.165	1.157		1.071	1.081	1.086
	Mean: Strong	1.089	1.088	1.088		0.975	0.981	0.990
	Mean: Weak	1.163	1.152	1.144		1.063	1.072	1.072
	Mean: Weak and strong	**1.126**	**1.120**	**1.116**		**1.019**	**1.026**	**1.031**
	Grand total	**1.121**				**1.025**		

members with low slenderness parameters λ_{cr}, yet it does not have a significant effect on the relative impact of switching from the 2010 AISC provisions to the proposed Q-factor modifications when compared to Figure 12.

It is clear from the results that the use of the proposed Q-factor modifications eliminates the nonconservatism in the average analytical-to-predicted buckling strength ratios for members with low Q factors ($Q = 0.7$). Alternative Q_{A1} produces consistent average margins of safety for all steel grades and Q-factor values. Alternative Q_{A2} produces nonconservative average ratios for members with intermediate Q factors ($Q = 0.85$). For all three Q-factor formulations, there are a few individual cases with high member slenderness parameters ($\lambda_{cr} \geq 1$) where the predicted buckling strengths are nonconservative.

Inclusion of AISC Built-Up Member Slenderness Modification Provision

Table 4 summarizes the ratios of the analytically determined to the calculated buckling strengths using the SJI Specification with the AISC built-up slenderness modification and the proposed Q-factor alternative Q_{A1}. Comparison with Table 2 and Table 3 reveals the following observations:

- The proposed Q-factor and built-up slender modifications only affect the results for strong-axis buckling.
- For all Q-factor values and steel grade combinations, the average ratio for strong-axis buckling is between 1.06 and 1.10 (for code-basis L/1500 imperfection magnitude), which reflects a consistent average margin of safety above unity.
- The average ratio for weak-axis buckling is always greater than one. The average margins of safety are nearly consistent between 1.12 and 1.19.
- The overall mean ratios for strong-axis buckling in all three steel grades are about 1.09. The overall mean ratios for weak-axis buckling in all three steel grades are between 1.14 and 1.16. Hence, the design equation leads to similar levels of conservatism for both sets of boundary conditions and corresponding failure modes.
- The individual result plots (not shown) demonstrate a better performance for the calculated buckling strengths at relatively high member slenderness parameters λ_{cr} than

Table 5. Global Comparison of Ratios of FE Analysis to SJI Strength Modified by Q_{A2} and AISC Built-Up Member Slenderness Modification (compare to Tables 1, 3 and 4)								
Global Average of $(F_{fe}/F_y)/(F_{cr}/F_{y}$_Modified_$Q_{A2})$								
		L/1500				L/500		
Axis	Variable	50 ksi	65 ksi	80 ksi		50 ksi	65 ksi	80 ksi
	Q Factor							
Strong	0.7	1.092	1.053	1.064		0.999	0.956	0.965
	0.85	1.013	1.027	1.031		0.881	0.909	0.924
	1	1.087	1.090	1.090		0.989	0.999	1.008
Weak	0.7	1.171	1.162	1.164		1.061	1.070	1.088
	0.85	1.126	1.118	1.114		1.012	1.033	1.037
	1	1.135	1.126	1.115		1.062	1.061	1.054
	Number of Spacers							
Strong	1	1.072	1.072	1.077		0.961	0.966	0.980
	2	1.046	1.043	1.046		0.936	0.942	0.951
Weak	1	1.127	1.118	1.112		1.034	1.043	1.040
	2	1.152	1.143	1.138		1.051	1.061	1.069
	Mean: Strong	1.059	1.057	1.061		0.949	0.955	0.966
	Mean: Weak	1.139	1.131	1.125		1.042	1.052	1.054
	Mean: Weak and strong	**1.099**	**1.094**	**1.093**		**0.995**	**1.003**	**1.010**
	Grand total	**1.095**				**1.003**		

observed in Figures 9 through 13. Fewer cases underpredict the analytically determined buckling strength than shown in Figures 9 through 13.

Table 5 summarizes the ratios of the analytically determined to the calculated buckling strengths using the SJI Specification with the AISC built-up slenderness modification and the proposed Q-factor alternative Q_{A2}. Comparison with Tables 1, 3 and 4 reveals the following observations:

- The effect of Q_{A2} proposal on $Q = 0.7$ cases is identical to that of Q_{A1} proposal. Its effect on cases with relatively high member slenderness parameters, λ_{cr}, is also similar to using alternative Q_{A1}.
- For all combinations of Q-factor values and steel grades, the average ratio for strong-axis buckling is between 1.01 and 1.09 (for code-basis L/1500 imperfection magnitude), which reflects an average margin of safety slightly above 1. However, the average margin of safety for $Q =$ 0.85 cases are consistently lower than other cases.
- The overall mean ratios for strong-axis buckling in all three steel grades are about 1.06. The overall mean ratios for weak-axis buckling in all three steel grades are between 1.13 and 1.14.

The inclusion of the 2010 AISC slenderness modification produces better predictions for members with high slenderness parameter values. At relatively low member slenderness ratios, it introduces moderate conservatism in the strong-axis buckling prediction (within 10% margin on average). For Q_{A1}, the extra conservatism is consistent on average for each steel grade and range of Q factors. The margins of safety for strong- and weak-axis buckling are more aligned than they are using Q_{A2}. For Q_{A2}, the extra conservatism is not consistent, having lower values for $Q = 0.85$ cases than $Q = 0.70$ and $Q = 1.0$ cases for each steel grade.

In summary, the use of the modified Q-factor definitions eliminates the nonconservatism observed with the use of the SJI equations for strong-axis bending in members with low Q-factor values. However, alternative Q_{A1} produces consistent average margins of safety of about unity for all examined combinations of Q-factors and steel grades, while Q_{A2} produces less conservative average predictions for $Q = 0.85$ cases. To achieve a consistent factor of safety across the full range of Q factors, the authors recommend the use of the cross-section element slenderness factor alternative Q_{A1}. The effect of using the proposed Q-factor modifications only affects cases with low member slenderness parameters.

CONCLUSIONS

In a separate computational parametric study (Webster et al., 2017), the authors showed that the SJI buckling equations are nonconservative in the strong-axis direction for many double-angle configurations, independent of material strength, and that this nonconservatism increases as the Q factor decreases from 1.0 to 0.7 and as the member (global) slenderness ratio increases. The authors demonstrated that the modified slenderness ratio for built-up sections in the 2010 AISC *Specification* significantly improves the accuracy of the SJI nominal buckling strength predictions for strong-axis buckling of members with high slenderness ratios in steel grades ranging from 50 to 80 ksi, yet it does not completely eliminate the nonconservatism for members with low Q-factor values. The authors also found that the 2010 AISC equations for flexural-torsional buckling produced overly conservative results for members with low slenderness ratios.

The present paper investigated the adoption of modified Q-factor definitions for element slenderness, based on the 1968 AISI Specification, to resolve the observed lack of conservatism in strong-axis buckling predictions for members with low Q factors. Two alternative element slenderness definitions, Q_{A1} and Q_{A2}, were proposed.

For both conventional and HSLA-V steels, the use of the modified Q-factor definitions in conjunction with the AISC built-up member slenderness modification eliminated the nonconservatism observed with the use of the SJI equations for strong-axis bending for members with low Q factors. Alternative Q-factor definition Q_{A1} produced consistent average margins of safety of about 1.1 for all examined combinations of Q factors and steel grades, while alternative Q-factor definition Q_{A2} produced lower average margins of safety for $Q = 0.85$ cases compared to $Q = 0.7$ and $Q = 1.0$ cases.

The authors recommend modifying both the AISC and the SJI Specifications to uniformly address double-angle compression members as follows:

- Eliminate the flexural-torsional buckling provisions for double-angle compression members from the AISC *Specification*.
- Add the 2010 AISC modified slenderness ratio provisions for built-up members to the SJI Specifications.
- Replace the existing Q-factor definition with the proposed Q_{A1} definition in both the AISC Specification and the SJI Specification.

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