# Analysis of Horizontally Curved Bridges

JAMES W. GILLESPIE

MANY YEARS AGO, highway bridges were located by determining the most convenient crossing site, with little regard to the general alignment of the roadway. After the bridge location was established, the highway designer or surveyor laid out the highway to meet the bridge.

During the last several decades, this situation has reversed and now bridges must fit the highway alignment that has been predetermined by many other considerations. The increasingly frequent occurrence of structures on curved alignment is presenting real challenges to engineers, especially in the design of urban freeways where multi-level interchanges must be built within tight geometric restrictions.

The present-day emphasis on good appearance is also an important factor. Welding has helped to produce structures with smooth surfaces, interrupted by a minimum amount of detail. Outside transverse stiffeners are no longer used on many highway girders. The use of curved supporting beams or girders in a structure on curved alignment is a natural outgrowth of this trend toward aesthetic design.

Along with aesthetic considerations, curved girders offer certain technical advantages where structures must be built to fit curved highway alignment, when compared with girders composed of straight segments. The roadway slab design and construction become much simpler because the stringer spacing and parapet overhang from the exterior stringer are constant over the entire length of structure. This provides for equally spaced slab reinforcement, a more uniform stress distribution, and panel forms which can be re-used as pouring of the deck slab progresses.

In addition, curved girders permit the designer to make use of continuous construction and its inherent advantages in situations which might otherwise be limited to simple spans. Continuous spans make more efficient use of materials and permit the elimination of many undesirable expansion details. A stiffer structure is obtained and in some cases more vertical clearance is available due to the use of shallower girders.

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Curved girders also permit the designer increased flexibility, where possible locations of substructures are often limited because of required clearances. The use of straight girders to span the same distance could mean a complicated framing system to support the deck. When high substructures are involved, the use of longer spans may also result in savings.

The major advantages of curved girders are structural efficiency, appearance and simplicity in certain phases of design, detailing, and construction.

### TYPES OF CURVED FRAMING

For convenience, curved girder framing may be categorized into two types: "closed framing" and "open framing."

In the closed framing type, curved girders are tied together by diaphragms or floorbeams and horizontal lateral bracing at the girder flange levels (see Fig. 1). Torsion is resisted by individual curved girders

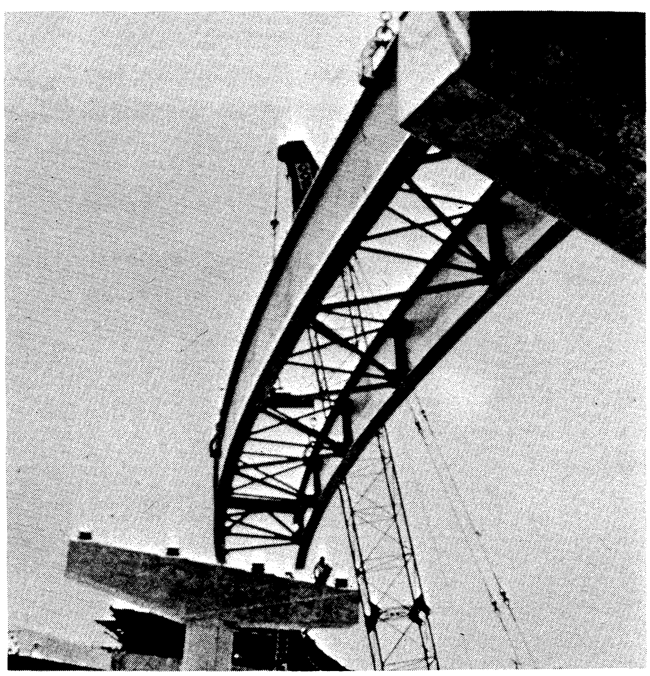

*Fig. 1. Curved girders tied together with diaphragms and lateral bracing*

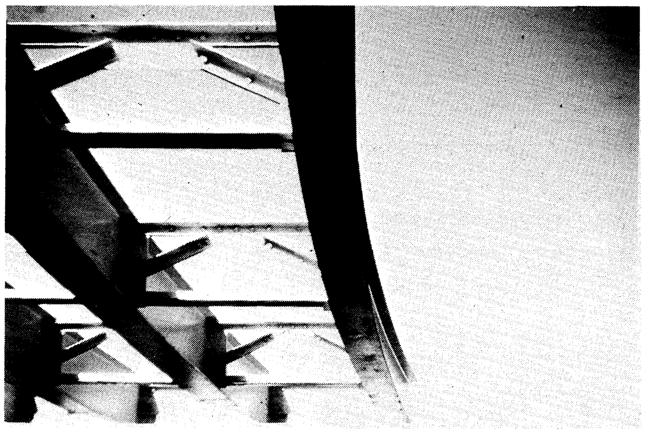

*Fig. 2. Curved girders tied together with diaphragms*

and interaction of these girders through diaphragms or floorbeams and lateral bracing.

In the open framing type, curved girders are tied together by diaphragms or floorbeams only, with no horizontal lateral bracing (see Fig. 2). In this case, torsion must be resisted by individual curved girders and interaction of curved girders through diaphragms or floorbeams.

A combination of closed and open framing can be used to form a third type of curved girder framing. For example, in Fig. 3 the exterior girders of a four-girder curved bridge are tied to the adjacent interior girders by diaphragms and horizontal lateral bracing, while the interior girders are tied to each other by diaphragms only.

### "EXACT" METHOD OF ANALYSIS

Curved girders may be of the plate-girder type or of the box-girder type. Although the analysis procedure discussed in this paper is specifically directed to the analysis of the plate-girder type, much of the procedure is also applicable to the analysis of box girders—except

*Fig. 3. Curved girders tied together with diaphragms and lateral bracing in alternate bays*

for the consideration of the torsional stiffness. The analysis procedure presented assumes that the plate girder has torsional stiffness due to warping only; stiffness due to pure (or St. Venant) torsion is neglected. For box girders, the contribution of St. Venant torsion should not be neglected.

To gain a feel for what is involved in analyzing a curved girder bridge, consider a two-girder open framing system (Fig. 4). This bridge is continuous over one intermediate support and the ends are simply supported. The complete structure is assumed to have eleven diaphragms, including the end diaphragms.

To formulate the problem by the stiffness method, the number of unknowns is equal to the number of independent displacements (degrees of freedom) possible (see Fig. 5). In general, assuming that certain basic conditions of symmetry of individual members, etc., are satisfied, three displacements are possible at each joint (point of intersecting members or external restraint). These displacements consist of one vertical translation and two rotations about axes in the "horizontal" plane of the structure.

However, certain joints are normally restrained against displacing in specified directions. These "restraints" can be subtracted from the total number of potential joint displacements. Thus,

$$n = 3j - r$$

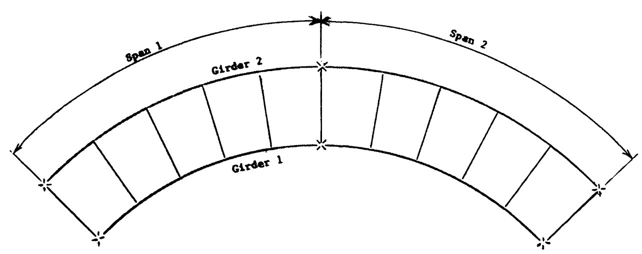


*Fig. 4. Two-girder two-span bridge*

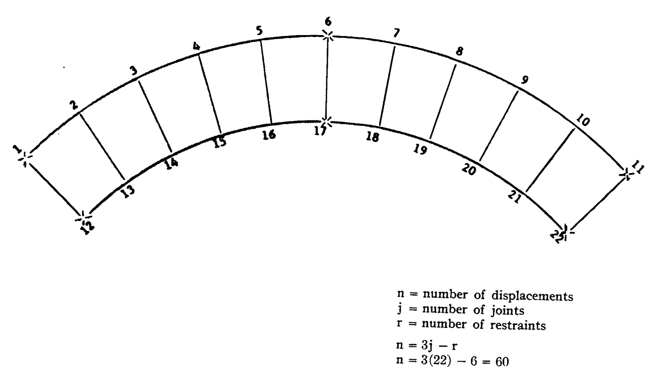


*Fig. 5. Stiffness formulation—two-girder two-span bridge*

where

$n$ = number of independent displacements
$j$ = number of joints
$r$ = number of restraints

For this case, there are 22 joints and vertical displacement is restrained at each point of support (6 restraints); thus,

$$n = 3(22) - 6 = 60$$

From this general discussion, it is concluded that 60 simultaneous equations are necessary to solve this problem by the stiffness method. This does not imply that the structure is statically indeterminate to the 60th degree.

A flexibility solution for this problem would be formulated with the member forces expressed as unknowns. See Fig. 6. For each member, six member forces are possible (three at each end). Also a number of external reactive forces may exist; this must be added to the number of member forces. From this, the number of releases (discontinuities of force) is subtracted. Thus,

$$p = 6m + R - s$$

where

$p$ = number of unknown forces
$m$ = number of members
$R$ = number of external reactive forces
$s$ = number of releases

For this case, there are 31 members, 6 external reactions, and zero releases; thus,

$$p = 6(31) + 6 - 0 = 192$$

Of these 192 unknown forces, only 33 are redundant forces (159 statical equations are available). If it is assumed that the individual members have no torsional stiffness, the number of redundant forces is reduced from 33 to 22.

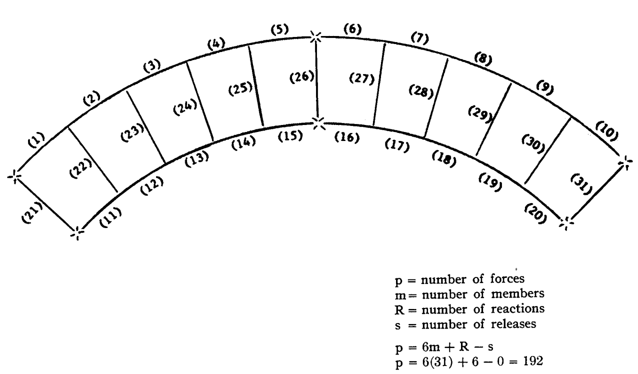


*Fig. 6. Flexibility formulation—two-girder two-span bridge*

**APPROXIMATE METHOD OF ANALYSIS**

Because of the difficulty of obtaining theoretically "exact" solutions, especially if manual methods are employed, there is considerable incentive to develop a simplified method of analysis. Such a method has been developed; following are the steps of the procedure for analysis:

1. Isolate each curved girder under consideration and straighten it to its full developed length. The external load is then applied to the girder, considering it supported at its developed span lengths, and the moment diagram is constructed as for straight girders. This diagram is referred to as the primary moment diagram.

2. Evaluate the moments at the ends of the diaphragms, from which the end shears are determined. These end shears are applied to the girders in their assumed straight configuration and additional bending moments calculated.

3. Evaluate the lateral bending moment in the girder flanges. This is accomplished by assuming an equivalent lateral load applied to the flanges and suitable support conditions.

4. Evaluate the stresses resulting from the forces calculated in steps 1, 2 and 3. These stresses are superimposed and checked against the allowable design stress.

5. Review the design, revise portions of it, if necessary, and repeat the appropriate steps of the analysis procedure.

Figure 7 shows a portion of a curved girder in plan. The arc length is assumed to be $d$, the spacing between diaphragms. A bending moment, $M$, is shown at each end of the girder segment. The bending moment is producing tension on the bottom surface. The bending moment vectors are resolved into vertical and horizontal components. The vertical components are in equilibrium; however, the horizontal components are additive.

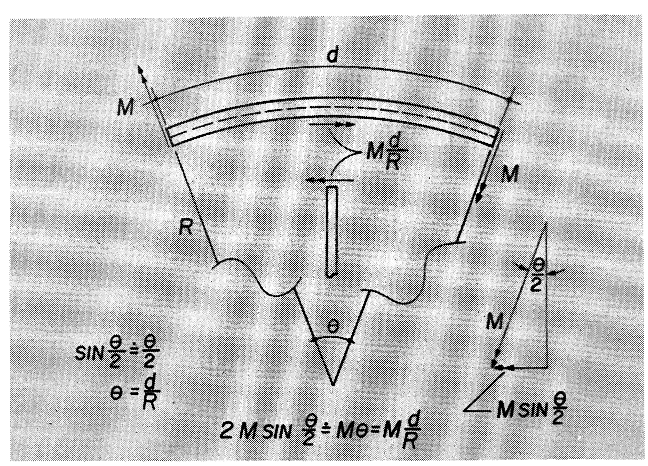


*Fig. 7. Segment of a curved girder*

The resultant horizontal component is

$$m = 2M \sin \frac{\theta}{2}$$

but, because the included angle, $\theta$, is small

$$\sin \frac{\theta}{2} = \frac{\theta}{2}$$

and by definition

$$\theta = d/R$$

thus

$$m = M\ (d/R)$$

This is the total torque in the curved girder developed between diaphragms that must be balanced (Fig. 8). The only mechanism available to resist this torque is the diaphragm. Therefore, the diaphragms must be attached to the girders and capable of developing the end moment

$$m = M\ (d/R)$$

The existence of these end moments requires that the diaphragms also transfer shear between the girders (Fig. 9). These shears must be applied to the girders and additional bending moments calculated.

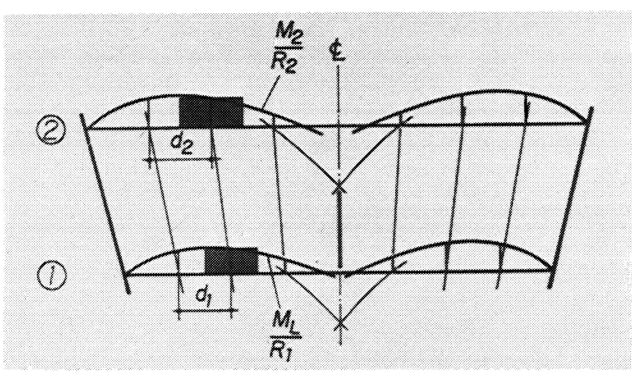

*Fig. 8. M/R diagram for a curved girder*

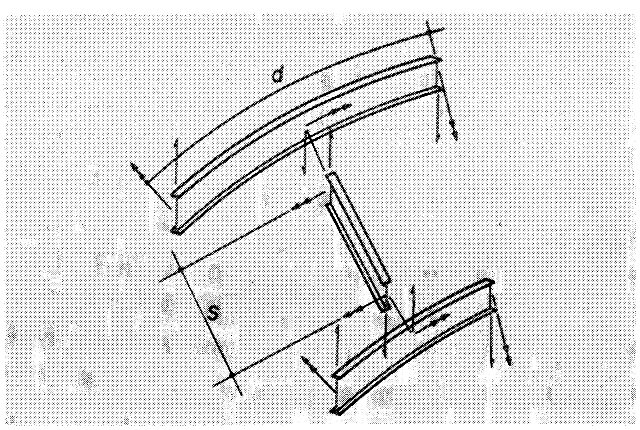


*Fig. 9. Segment of a curved bridge showing interaction of forces between curved girders and diaphragms*

If the bridge consists of more than two girders, it is necessary to make an additional assumption as to how the shear is distributed between the girders. It is assumed that the total shear transferred to each girder is proportional to the distance of the girder from the centerline of the bridge (Fig. 10). With this assumption, it is possible to calculate the end moments in the diaphragms and the shears to be applied to the girders (Fig. 11). For a four-girder structure the shear load to be applied to the exterior girder (Fig. 12) is

$$V = \frac{\Sigma M}{(10/9)(RD/d)}$$

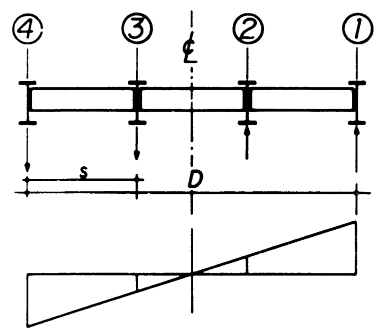


*Fig. 10. Cross-section of a four-girder bridge and assumed distribution of diaphragm shears to girders*

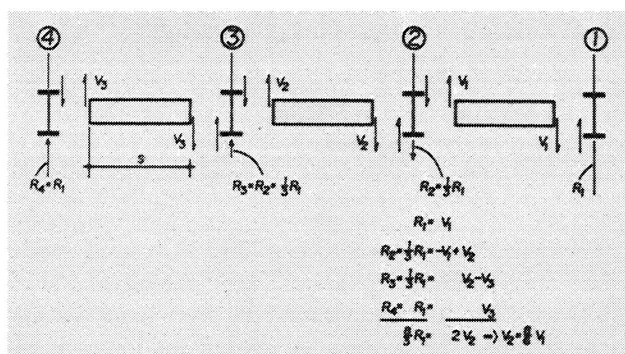

*Fig. 11. Cross-section of a four-girder bridge—shear equilibrium*

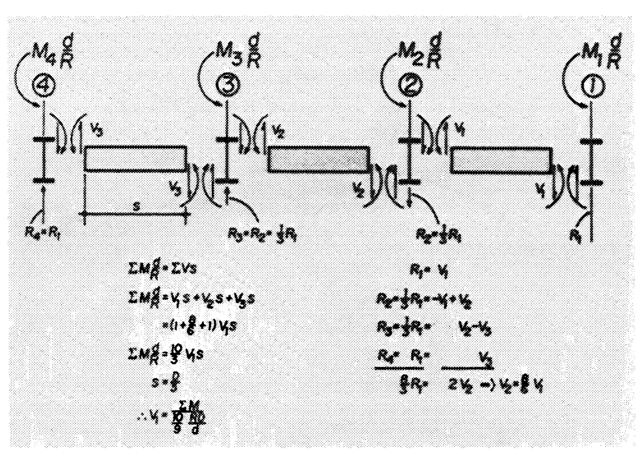

*Fig. 12. Cross-section of a four-girder bridge—moment equilibrium*

Figures 13 through 16 illustrate the percent increase in primary bending moment due to the shear loads for three, four, five and eight-girder bridges, respectively. These data are for simple spans or for lengths between inflection points of continuous girders. For a three-girder system consider a radius of 1,000 ft and a span of 100 ft; a percent increase of 18 is noted (Fig. 13). For a four-girder system and the same criteria, the percent increase is approximately 14 (Fig. 14). The percent increase is approximately 12 for a five-girder system (Fig. 15) and approximately 6 for an eight-girder system (Fig. 16). Note that the increase in bending moment due to the shear loads decreases as the number of girders increase. This percent increase is for the outside girder. Because the structure is in static equilibrium, there must be corresponding decreases in some of the girders to offset the increases in the other girders.

After the primary bending moment and the additional bending moment due to the shear loads are calculated, the analyst must check the lateral bending moment in the top and bottom flanges. This is evaluated by loading the flanges with a transverse load of

$$q = \frac{M}{Rh}$$

and assuming support at the diaphragm locations (see Fig. 17). An average negative moment of

$$M_L = \frac{1}{10} qd^2 = \frac{1}{10} \frac{Md^2}{Rh}$$

is considered for design purposes. The stress caused by this lateral bending moment is

$$f_2 = \frac{M_L}{S_L}$$

where $$S_L = \frac{tb^2}{6}$$

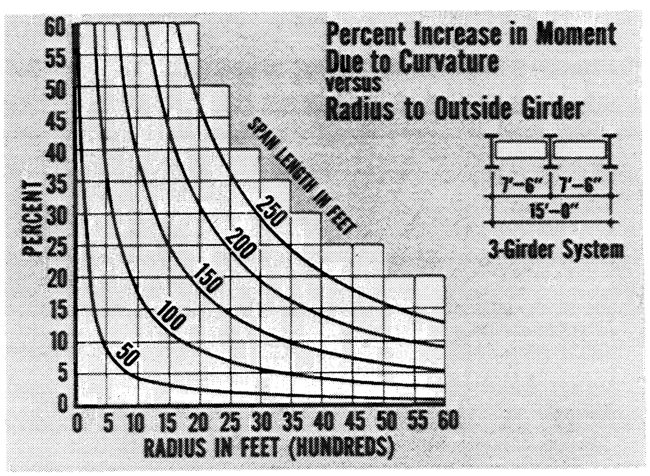


*Fig. 13. Percent increase in primary bending moment due to shear loads—three girder system*

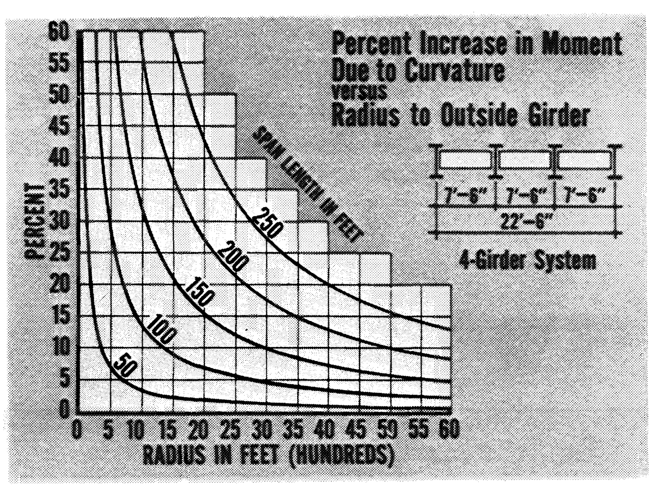


*Fig. 14. Percent increase in primary bending moment due to shear loads—four girder system*

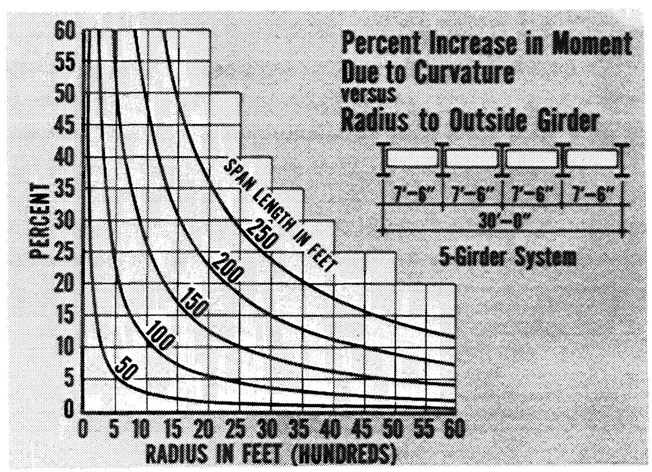


*Fig. 15. Percent increase in primary bending moment due to shear loads—five girder system*

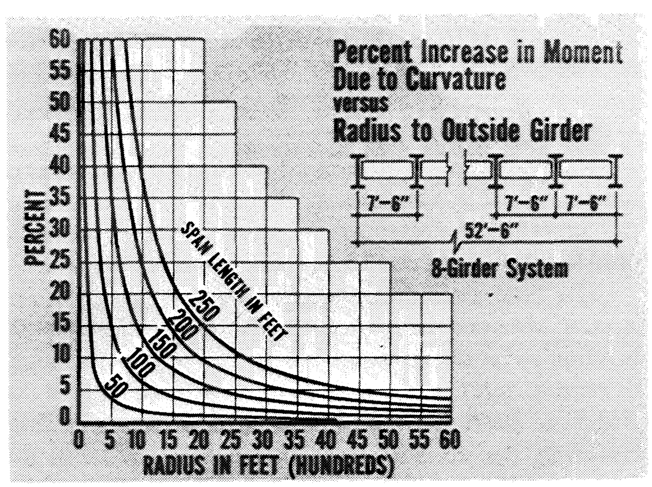


*Fig. 16. Percent increase in primary bending moment due to shear loads—eight girder system*

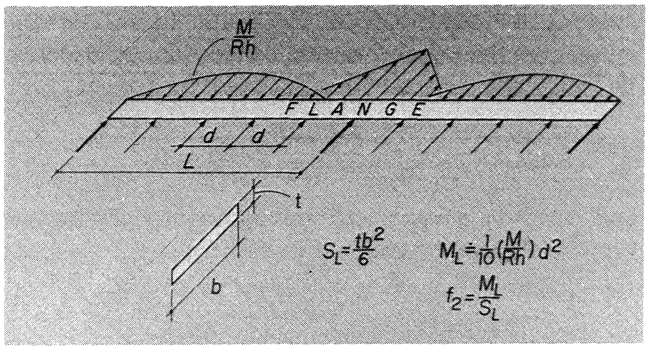


*Fig. 17. Girder flange with transverse load*

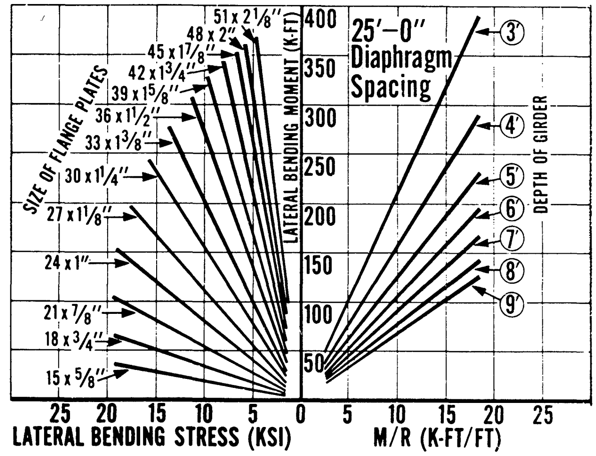


*Fig. 18. Lateral bending moment and bending stress for a diaphragm spacing of 25 ft*

Note that the lateral bending stress is inversely proportional to the section modulus of the flange and that the section modulus is proportional to the flange thickness and to the flange width squared; thus, the wider the flange, the lower the stress. However, consideration must be given to other factors such as fabrication and erection, to arrive at the most desirable flange dimensions.

Note also that the lateral bending stress is directly dependent on the lateral bending moment which is related to the diaphragm spacing to the second power. Thus, if this component of stress becomes objectionably large, it can be lowered by reducing the diaphragm spacing. One must weigh the benefits to be gained from more closely spaced diaphragms against an expected increase in fabrication cost.

The solution to the equations

$$M_L = \frac{1}{10}\frac{Md^2}{Rh}$$

$$f_2 = M_L/S_L$$

is shown in Fig. 18 for a diaphragm spacing of 25 ft. Consider a $M/R$ of 10 kip-ft/ft, a girder depth of 4 ft, and a flange plate 30 x $1\frac{1}{4}$-in. A lateral bending moment of 156 kip-ft and a flange stress of 10 ksi are read from the plots.

Figure 19 shows similar relationships for a diaphragm spacing of 15 ft. Considering the same conditions as in the preceding paragraph, a lateral bending moment of 56 kip-ft and a stress of 3.6 ksi are observed.

The final stress in the girder is the resultant of the stresses obtained from steps 1 and 2 and the lateral bending stress obtained in step 3. The stresses obtained in steps 1 and 2 vary linearly through the depth of the girder as in the design of straight girders (Fig. 20). However, the stress obtained in step 3 varies across the flanges with a neutral axis coincident with the girder web, assuming that the girder is symmetrical about a vertical axis (Fig. 21). This stress due to lateral bend-

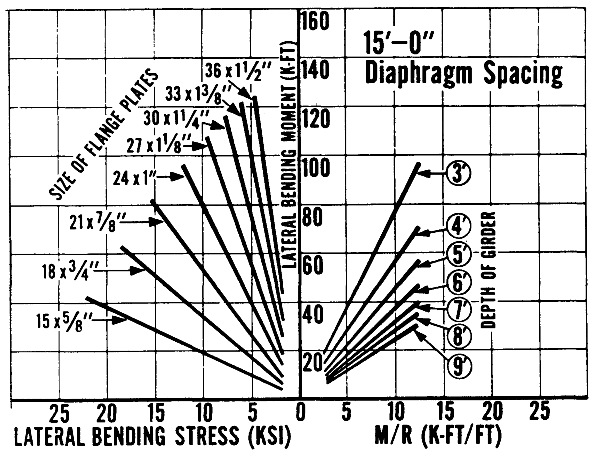


*Fig. 19. Lateral bending moment and bending stress for a diaphragm spacing of 15 ft*

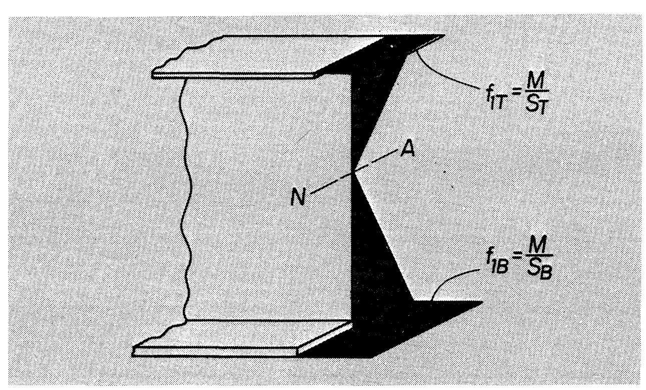


*Fig. 20. Distribution of stress due to bending moments obtained in steps 1 and 2*

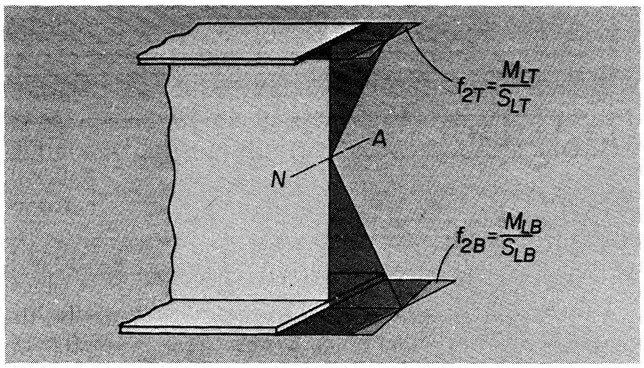


*Fig. 21. Distribution of stress due to lateral bending moments obtained in step 3*

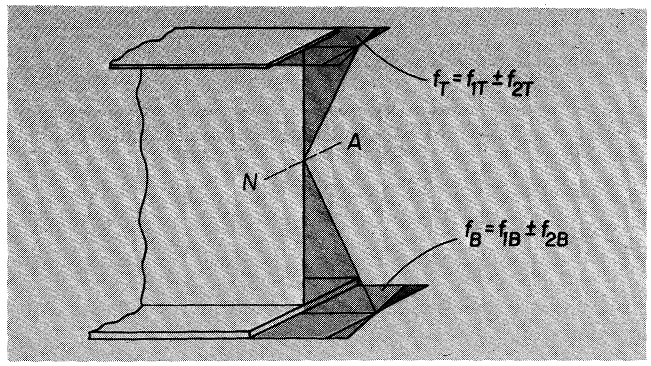


*Fig. 22. Final distribution of stress in the girder*

ing does not alter the average stress in the flanges but it does affect the distribution of stress—increasing it on one edge and decreasing it on the opposite edge (Fig. 22).

The final state of stress on the girder cross section indicates that the section does not remain plane, but is warped. It is, in fact, warped; however each stress calculation is concerned with a section or portion of a section that remains plane under that state of stress. Therefore, the elementary stress formulas are applicable.

As in any other design situation, if the final stresses are not satisfactory, it is necessary to revise the design and repeat the appropriate phases of the analysis.

**ACKNOWLEDGMENTS**

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