Yield Line Approaches for Design of End Plate Tension Connections for Square and Rectangular HSS Members Using End Plate Tensile Strength

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ABSTRACT

End plates, which are sometimes called flange plates, are a common way to treat HSS members loaded in tension. In this application, prying action must be considered in the design of the plate and bolts. This paper demonstrates that the prying action model can use the end plate minimum tensile strength rather than the yield strength to achieve satisfactory designs. Only connections with bolts on all four sides of the HSS are considered here.

Keywords: HSS connections, end plate, prying action, yield line pattern.

INTRODUCTION

Calculations for prying action have, for many years, used the material minimum yield strength F_y in the calculations of the Struik and deBack (1969) model. This model has been the basis of the AISC *Steel Construction Manual* prying action analysis since the 8th Edition (1980). As early as 1965, Douty and McGuire suggested that the material minimum tensile strength F_u gives a better fit to the experimental results for tee stubs. Thornton (1992, 1996) showed that the use of F_u in place of F_y in the Struik-deBack model gave excellent predictions of the failure loads obtained by Kato and McGuire (1973). Because the experimental data of these two papers (Douty and McGuire; Kato and McGuire) were based on steels available in the 1960s, the AISC *Manual* Committee was reluctant to replace F_y with F_u in the *Manual* prying calculations. In 2002, Swanson showed that the Struik-deBack model with F_u in place of F_y gave excellent correlation for tee stub connections using modern materials. This is the reason that the *Manual* Committee adopted F_u for the prying calculations in the 13th Edition *Manual* (2005), and this continues in the 14th Edition (2011) and the soon-to-be-available 15th Edition *Manual*. Because the mode of failure of the plate (T-stub) material in the Swanson tests was ductile yield, not rupture, the resistance factor ϕ of 0.90 is used with the tensile strength F_u.

A recent AISC publication, *Design Guide 24—Hollow Structural Section Connections* (Packer et al., 2010), uses F_y rather than F_u for the prying action analysis of end plated HSS tension connections with bolts on all four HSS faces. This is a variation in the prying action formulation of the current 14th Edition *Manual*, which uses F_u in these calculations. This may cause confusion in the industry. For instance, when is it correct to use F_u and when should F_y be used? Many engineers will opt for the more conservative approach if there seems to be disagreement in AISC publications as to the correct approach.

The author notes that the method of Design Guide 24 is completely viable. It uses yield line patterns that are different than those proposed here, coupled with the use of F_y rather than F_u. The purpose of this paper is to show that with appropriate yield line patterns it is possible to use F_u in lieu of F_y. This is verified by the comparison of the predicted results with the available test results.

Using a yield line approach to the end plated HSS tension connection, which is similar to the method validated by Swanson (2002) for tee stub tension connections, this paper shows that, using the experimental data for end plated hollow structural steel (HSS) tension connections produced by Willibald, Packer and Puthli (2002, 2003); Kato and Mukai (1985); and Caravaggio (1988), a valid design method based on the tensile strength F_u can be justified.

DISCUSSION

Use of the Tensile Strength, F_u, in the Plate Flexure Model

A stress block with F_u at all points above and below the neutral axis is not likely to be achieved. The fibers near the neutral axis will not achieve F_u, but because of their proximity to the neutral axis, they are relatively unimportant in the

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overall capacity calculation. Because of this fact, the plastic stress block is currently used in many structural connection calculations even though it is theoretically impossible to achieve. It will be used in the method developed here.

Development of Yield Line Patterns and Bolt Tributary Length

There are many possible families of yield lines for the end plate HSS connections considered in this paper. Following the work of Willibald, Packer and Puthli (2002), three bolt arrangements are considered here. These are shown in Figures 1, 2 and 3 and are called patterns A, B and C, respectively. For these three bolt patterns, there are available a total of 55 physical tests; 26 for pattern A, 2 for pattern B, and 27 for pattern C. These are obtained from Willibald et al. (2002, 2003), Kato and Mukai (1985), and Caravaggio (1988).

Because of the availability of the physical test data for the A, B and C bolt arrangements, only these arrangements are considered here. As mentioned earlier, many possible yield line families are available for each of the three bolt patterns. For instance, circular yield lines at the HSS corners with radial fans are a possible family, as are straight line yield families. The author has reviewed a number of possibilities and determined by "trial and error" that the families chosen for this paper give the best correlation to the test data.

Note that the bolt holes are not explicitly removed in any of the three bolt patterns A, B and C. Bolt holes are removed through the use of the quantity δ in the prying action formulation presented in the "Proposed Analysis and Design Methods" section of the paper.

Yield Line Pattern A

This bolt pattern is applicable to both rectangular and square HSS members. Test data are available for both. The assumed yield line pattern is shown in Figure 1 for bolt pattern A. It is a combined curvilinear and straight line yield line pattern. It has an axial load capacity P_u given by

$$P_u = \frac{F_y t_p^2}{b}\left(w_i + h_i + \pi b\right) \tag{1}$$

where

F_y = end plate yield stress, ksi

t_p = end plate thickness, in.

and b, h_i and w_i are defined in Figure 1. Figure 1 also shows the end plate size as $w_p \times h_p$.

Equation 1 is derived by the usual upper-bound virtual work method of structural mechanics (see, e.g., Save and Massonnet, 1972), which satisfies equilibrium and compatibility but not necessarily the constitutive equations.

From Dowswell (2011), the strength of an equivalent pair of straight-line yield lines of length, l, is

$$P_u = \frac{F_y t_p^2}{2b} l \tag{2}$$

Setting Equation 1 equal to Equation 2, the effective straight-line yield line pattern that gives the same strength as the multiple yield line pattern will have a length, l, using a T-stub analogy, as given by Equation 3, as

$$l = 2(w_i + h_i + \pi b) \tag{3}$$

Thus, for yield line pattern A, the tributary yield line length per bolt, where n is the number of bolts, is

$$p_A = \frac{2(w_i + h_i + \pi b)}{n} \tag{4}$$

The tributary yield length per bolt is required for the prying action formulation that was mentioned earlier in this paper. This prying action formulation will be completely developed subsequently.

Yield Line Pattern B

This bolt pattern is applicable to square and rectangular HSS, but test data are available only for the square case. The

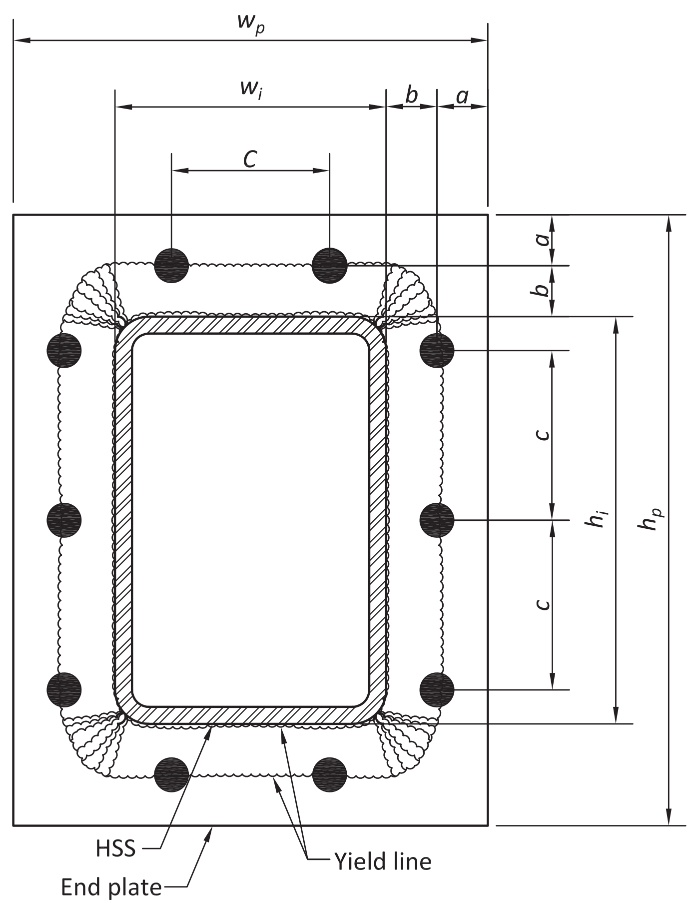

Fig. 1. Bolt pattern A and associated yield line.

bolt pattern and the assumed yield line pattern are shown in Figure 2. This yield line pattern was solved by Kapp (1974) to give the axial capacity P_u as

$$P_u = \frac{F_y t_p^2}{b}(w_i + h_i + 4b) \tag{5}$$

Using Dowswell's (2011) approach to determine the effective length of an equivalent straight-line yield line, using a T-stub analogy, gives

$$l_B = 2(w_i + h_i + 4b) \tag{6}$$

and the effective tributary length per bolt is

$$p_B = \frac{2(w_i + h_i + 4b)}{n} \tag{7}$$

Yield Line Pattern C

This yield line pattern has a single bolt on each HSS side. It could be applicable to both square and rectangular HSS, but test data are available in the referenced literature only for the square case. In order that the bolts on all sides be equally loaded, it is recommended that this pattern be used only for the square HSS case. Figure 3 shows the bolt pattern and the assumed yield line pattern. This yield line pattern is the same as that for bolt pattern A. Therefore

$$p_C = \frac{2(w_i + h_i + \pi b)}{n} \tag{8}$$

Limitation on Tributary Yield Line Length per Bolt Length

It is assumed that the yield line patterns of Figures 1, 2 and 3 will develop as shown. It is apparent that if the bolt spacing (tributary yield line length per bolt length) is too great, yield line patterns with less capacity than that calculated by Equation 1, 5 or 8, can develop. Dowswell (2011) has shown that if the tributary bolt length p is greater than

$$p = 4\sqrt{b'(a+b)} \tag{9}$$

where $b' = b - d/2$, a and b are defined in the figures, and d is the bolt diameter, independent yield line patterns can develop at each bolt, producing a capacity less than that determined by any of the patterns A, B or C. This is shown in Figure 4. The bolt spacing at which the localized pattern of Figure 4 can develop at each bolt is

$$p = 4\sqrt{b'(a+b)} \tag{10}$$

Therefore, to prevent this and to maintain the validity of the

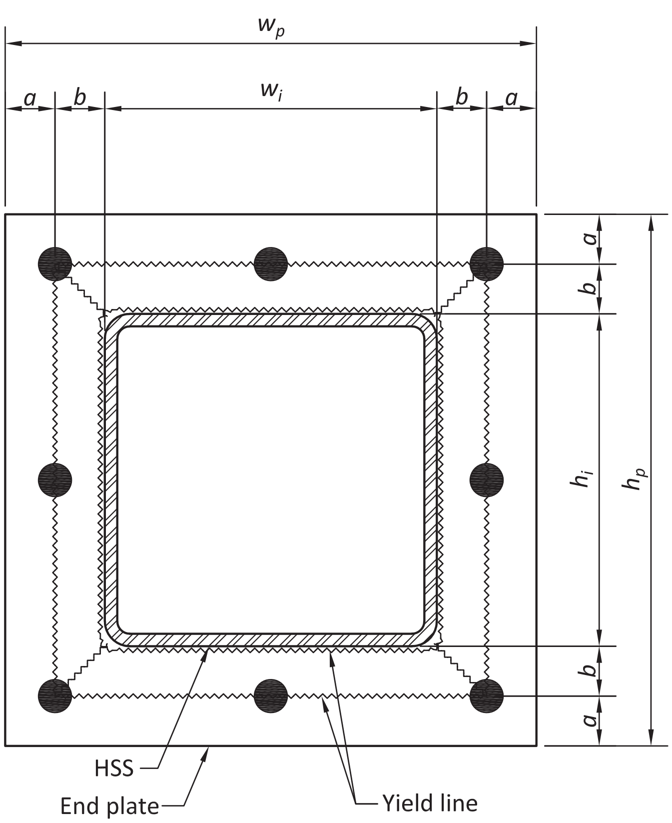

Fig. 2. Bolt pattern B and associated yield line.

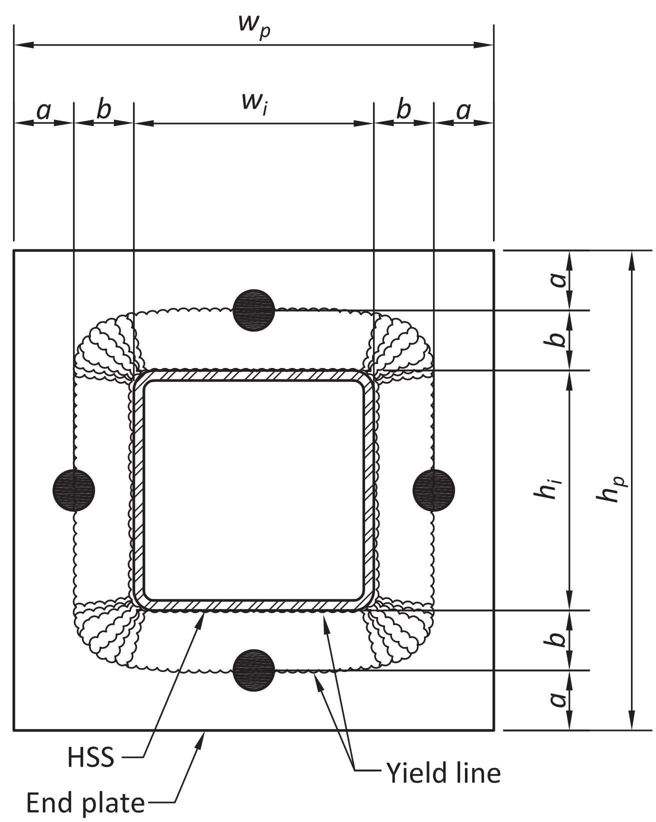

Fig. 3. Bolt pattern C and associated yield line.

assumed yield patterns of Figures 1, 2 and 3, the maximum tributary bolt lengths (p_A, p_B and p_C) are limited to

$$p_{i,max} = 4\sqrt{b'(a+b)} \tag{11}$$

Ultimately, whether or not the yield lines proposed for bolt patterns A, B and C are reasonable depends on how well they correlate to the physical test data. This is the subject of the "Physical Test Data" section of this paper. The next section of the paper will develop analysis and design methods proposed for the general analysis and design of this system. The analysis method was also used to develop the results given in Table 2. The design method is perhaps more convenient to use for routine design of this system. An example problem will show the use of both methods.

PROPOSED ANALYSIS AND DESIGN METHODS

Analysis Method

The proposed analysis method is that given in Part 9 of the AISC *Manual* (2011), with an additional requirement on the maximum value of α'. The method is as follows:

Given a, b, d, t_p, n, F_u, T and $B = \phi F_{nt} A_b$, find T_u and $N_u = nT_u$

A1. Check $a \le 1.25b$; if not, set $a = 1.25b$

A2. Calculate a', b', ρ, d':

$$a' = a + d/2 \tag{12}$$

$$b' = b - d/2 \tag{13}$$

$$\rho = b'/a \tag{14}$$

$$d' = d + 1/16 \text{ (for standard holes)} \tag{15}$$

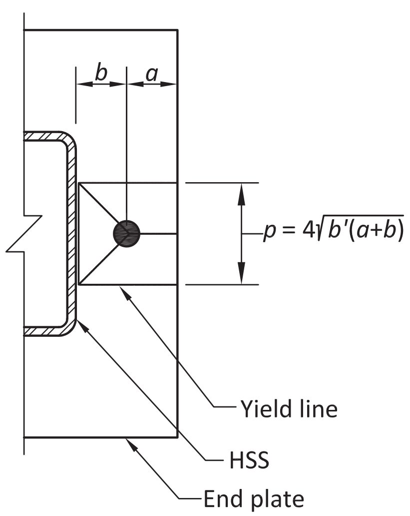

Fig. 4. Local yield line pattern when spacing is too large to allow patterns A, B or C to develop their associated p.

A3. Determine $p = p_i$, as appropriate for the bolt pattern; p_A, p_B or p_C:

$$p_A = \frac{2(w_i + h_i + \pi b)}{n} \tag{4}$$

$$p_B = \frac{2(w_i + h_i + 4b)}{n} \tag{7}$$

$$p_C = \frac{2(w_i + h_i + \pi b)}{n} \tag{8}$$

A4. Check that the determined p does not exceed $p_{i,max}$. If it does, use $p = p_{i,max}$.

$$p_{i,max} = 4\sqrt{b'(a+b)} \tag{11}$$

A5. Calculate δ:

$$\delta = 1 - d'/p \tag{16}$$

A6. Calculate t_c:

$$t_c = \sqrt{\frac{4Bb'}{\phi p F_u}} \tag{17}$$

Note that, as discussed in the "Introduction," $\phi = 0.90$ is used here.

A7. Calculate α':

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t_p}\right)^2 - 1\right] \tag{18}$$

$$T_u = B\left(\frac{t_p}{t_c}\right)^2 (1 + \delta\alpha') \tag{19}$$

If $\alpha' \le 0$, set $\alpha' = 0$, $T_u = B \rightarrow$ bolts control

If $0 < \alpha' < 1$, $T_u = B\left(\frac{t_p}{t_c}\right)^2 (1+\delta\alpha')$

$\rightarrow$ bolts and plate control

If $\alpha' \ge 1$, set $\alpha' = 1$, $T_u = B\left(\frac{t_p}{t_c}\right)^2 (1+\delta)$

$\rightarrow$ plate controls

If $\alpha' > 1.5$, choose a larger t_p and repeat until $\alpha' \le 1.5$.

A8. Calculate N_u:

$$N_u = nT_u \tag{20}$$

A9. If $N_u \ge nT$, where T is the required strength per bolt, the design is satisfactory. In Table 2, N_u as calculated in Equation 20, is compared with the experimental value, N_{ux}.

Design Method

The proposed design method is also that given in Part 9 of the AISC *Manual* (2011), with an additional requirement on the maximum value of α', and is as follows:

Given T, a, b, p, F_u and $B = \phi F_{nt} A_b$, find t_p.

D1. Check $T \le B$. If true, proceed; otherwise increase the bolt number or bolt strength.

D2. Calculate β:

$$\beta = \frac{1}{\rho}\left(\frac{B}{T} - 1\right) \tag{21}$$

D3. If $\beta \ge 1$, set $\alpha^* = 1$

If $0 \le \beta < 1$, set $\alpha^* = \min\left\{\frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right), 1\right\}$ (22)

D4. With the determined value of α^* and t_c from Equation 17, calculate t_p:

$$t_p = t_c\sqrt{\frac{T}{B}\left(\frac{1}{1+\delta\alpha^*}\right)} \tag{23}$$

D5. Calculate α':

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t_p}\right)^2 - 1\right] \tag{18}$$

D6. Check that $\alpha' \le 1.5$. If so, t_p is the required end plate thickness. If not, increase t_p until $\alpha' \le 1.5$. Note that α' and α^* are not the same physical quantity.

For the development of these methods, see Thornton (1985). Note that the methods are "reciprocal" to each other. That is, when the analysis method is run with a specified plate thickness t_p, a design strength T_u results. If the design method is then run with T_u as the required tension, the plate thickness t_p will result.

The additional requirement for the application of these methods to end plated HSS tension members is based on the results discussed in the section of the paper entitled, "Further Discussion and Observations." It is that α' should be less than or equal to 1.5.

Physical Test Data and Development of Tables 1, 2 and 3

Table 1 lists all of the physical properties of 55 specimens from four sources: (1) Willibald et al. (2002), (2) Kato and Mukai (1985), (3) Caravaggio (1988), and (4) Willibald et al. (2003). These test results are used to validate the yield line patterns proposed for the bolt patterns A, B and C. Except for some nominal HSS and plate dimensions, all of the data are measured rather than nominal.

Table 1 gives the physical specimen data, and Table 2 gives all of the calculations necessary to validate the proposed yield line patterns. The analysis method to provide these calculations is the same as the proposed analysis method given in the "Proposed Analysis and Design Methods" section of the paper, except that ϕ is taken to be 1.0 and actual measured values of B_u and F_u from Table 1 are used in the formula for t_c. Thus, the experimental value of t_c can be denoted as

$$t_{cx} = \sqrt{\frac{4B_u b'}{pF_u}} \tag{19}$$

Other than the t_{cx} value using $\phi = 1.0$ and the actual measured bolt and plate material tensile strengths, as shown in Equation 12, the method used to generate the connection capacities of Table 2 is exactly that of the "Proposed Analysis Method" section of this paper, without the $\alpha' \le 1.5$ requirement, which results from the physical data of Table 2.

The value of p, from Equation 4, 7 or 8, or from Equation 10, as appropriate, is included in Table 2. It is noted that Kato and Mukai (1985) include two specimens that exceeded the capacity of their testing system and, therefore, yielded no useful failure information. These are included in Tables 1 and 2 for completeness.

Discussion of Results

General

Table 2 shows excellent correlation between the experimental capacities N_{ux} and the predicted capacities N_u. The ratio N_{ux}/N_u shown in the last column of Table 2 should be approximately 1.0 or slightly bigger. Except for three outliers, this is the case, for types A and C. A discussion for types A and C follows. Type B will be discussed separately.

Types A and C

There are 26 type A specimens. Two of these exceeded the capacity of the testing apparatus and yielded no useful information, so there are 24 specimens to consider. Of these, two can be considered to be outliers. These are specimens 16 and 17. These had very thin end plates, approximately ¼ in. and 5⁄16 in., respectively. The prying ratios, β_{ux}, for these specimens were 233.4% and 77.5%, respectively. These very large prying ratios indicate that these connections are not acting as the theory assumes. The prying ratio is usually in the range of 0 to 30% for most connections in the author's experience. Willibald et al. (2003) reported that no visible separation of the plates occurred at a prying ratio of 41.7%. Ratios greater than about 50% indicate excessive deformation is occurring and that the load is very likely being carried by catenary action or membrane action, rather than flexure as assumed

Table 1. Specimen Physical Data														
				HSS	Plates	n								
		Specimen		$h_i \times w_i \times t$	$h_p \times w_p$	# of	a	b	c	t_p	d	d'	F_u	B_u
No.	Source	ID	Type	(in.)	(in.)	Bolts	(in.)	(in.)	(in.)	(in.)	(in.)	(in.)	(ksi)	(kips)
1	Willibald et al. (2002)	1-1	A	6.02×6.02×0.370	11.5×11.5	8	1.40	1.36	5.50	0.638	0.618	0.681	72.3	36.0
2	Willibald et al. (2002)	1-2	A	6.02×6.02×0.370	11.5×11.5	8	1.41	1.35	5.50	0.791	0.618	0.681	75.6	34.2
3	Willibald et al. (2002)	1-3	A	6.02×6.02×0.370	11.5×11.5	8	1.40	1.36	2.74	0.642	0.618	0.681	72.3	36.0
4	Willibald et al. (2002)	1-4	A	6.02×6.02×0.370	11.5×11.5	8	1.43	1.36	2.74	0.791	0.618	0.681	75.6	34.2
5	Willibald et al. (2002)	2-1	A	6.02×6.02×0.370	12.25×12.25	8	1.57	1.57	5.50	0.496	0.618	0.681	74.8	31.0
6	Willibald et al. (2002)	2-2	A	6.02×6.02×0.370	12.25×12.25	8	1.58	1.56	2.74	0.496	0.618	0.681	74.8	31.0
7	Willibald et al. (2002)	2-3	A	6.02×6.02×0.370	12.25×12.25	8	1.36	1.75	5.50	0.504	0.618	0.681	74.8	31.0
8	Willibald et al. (2002)	2-4	A	6.02×6.02×0.370	12.25×12.25	8	1.37	1.76	5.50	0.626	0.618	0.681	74.3	30.2
9	Willibald et al. (2002)	2-5	A	6.02×6.02×0.370	12.25×12.25	8	1.32	1.76	2.74	0.492	0.618	0.681	74.8	31.0
10	Willibald et al. (2002)	2-6	A	6.02×6.02×0.370	12.25×12.25	8	1.35	1.76	2.75	0.626	0.618	0.681	74.3	30.2
11	Kato and Mukai (1985)	26	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.449	0.630	0.692	63.5	37.1
12	Kato and Mukai (1985)	27	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.602	0.630	0.692	67.4	36.8
13	Kato and Mukai (1985)	28	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.728	0.630	0.692	66.3	38.4
14	Kato and Mukai (1985)	29	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.882	0.630	0.692	62.8	36.5
15	Kato and Mukai (1985)	30	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.992	0.630	0.692	62.9	37.7
16	Kato and Mukai (1985)	31	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.246	0.787	0.850	70.2	59.6
17	Kato and Mukai (1985)	32	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.350	0.787	0.850	71.2	59.6
18	Kato and Mukai (1985)	33	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.602	0.787	0.850	67.4	59.9
19	Kato and Mukai (1985)	34	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.728	0.787	0.850	66.3	59.9
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21	Kato and Mukai (1985)	36	A	7.87×7.87×0.303	12.6×12.6	8	1.18	1.18	3.94	0.992	0.787	0.850	62.9	58.9
22	Caravaggio (1988)	1	A	5×5×0.177	11.2×11.2	8	1.72	1.38	3.94	0.630	0.626	0.688	73.5	32.8
23	Willibald et al. (2003)	R1	A	9.98×5.98×.287	16.36×12.33	10	1.41	1.77	4.31	0.492	0.626	0.697	74.8	32.8
24	Willibald et al. (2003)	R2	A	9.98×5.98×.287	16.42×12.30	10	1.42	1.77	4.31	0.624	0.626	0.695	74.3	31.7
25	Willibald et al. (2003)	R3	A	9.98×5.98×.287	16.43×12.29	10	1.61	1.56	4.31	0.492	0.626	0.691	74.8	32.8
26	Willibald et al. (2003)	R4	A	9.98×5.98×.287	16.33×12.36	10	1.60	1.57	4.32	0.623	0.626	0.694	74.3	31.7
27	Willibald et al. (2002)	1-5	B	6.02×6.02×0.370	11.5×11.5	8	1.41	1.36	—	0.642	0.618	0.681	72.3	36.0
28	Willibald et al. (2002)	1-6	B	6.02×6.02×0.370	11.5×11.5	8	1.41	1.36	—	0.787	0.618	0.681	75.6	34.1
29	Kato and Mukai (1985)	1	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.355	0.630	0.693	59.0	37.5
30	Kato and Mukai (1985)	2	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.449	0.630	0.693	63.5	37.1
31	Kato and Mukai (1985)	3	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.618	0.630	0.693	66.1	39.6
32	Kato and Mukai (1985)	4	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.728	0.630	0.693	66.3	38.4
33	Kato and Mukai (1985)	5	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.842	0.630	0.693	64.1	36.4
34	Kato and Mukai (1985)	8	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.350	0.787	0.850	71.2	59.6
35	Kato and Mukai (1985)	9	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.449	0.787	0.850	63.5	58.0
36	Kato and Mukai (1985)	10	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.602	0.787	0.850	67.4	59.8
37	Kato and Mukai (1985)	11	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.728	0.787	0.850	66.3	59.8
38	Kato and Mukai (1985)	12	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.882	0.787	0.850	62.8	58.2
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40	Kato and Mukai (1985)	14	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.449	0.787	0.850	63.5	58.0
41	Kato and Mukai (1985)	15	C	5.91×5.91×0.236	10.63×10.63	4	1.18	1.18	—	0.992	0.787	0.850	62.9	58.9
42	Kato and Mukai (1985)	16	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.449	0.787	0.850	63.5	58.0
43	Kato and Mukai (1985)	17	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.606	0.787	0.850	65.5	59.8
44	Kato and Mukai (1985)	18	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.729	0.787	0.850	66.3	59.8
45	Kato and Mukai (1985)	19	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.882	0.787	0.850	62.8	58.2
46	Kato and Mukai (1985)	20	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.941	0.787	0.850	68.3	58.9
47	Kato and Mukai (1985)	21	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.606	0.945	1.01	65.5	84.5
48	Kato and Mukai (1985)	22	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.728	0.945	1.01	66.3	88.6
49	Kato and Mukai (1985)	23	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.882	0.945	1.01	62.8	85.4
50	Kato and Mukai (1985)	24	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	0.940	0.945	1.01	68.3	88.1
51	Kato and Mukai (1985)	25	C	7.87×7.87×0.315	12.6×12.6	4	1.18	1.18	—	1.08	0.945	1.01	61.5	84.1
52	Willibald et al. (2002)	2-7	C	6.02×6.02×0.370	12.25×12.25	4	1.59	1.52	—	0.496	0.744	0.807	74.8	57.6
53	Willibald et al. (2002)	2-8	C	6.02×6.02×0.370	12.25×12.25	4	1.57	1.53	—	0.622	0.744	0.807	74.3	55.6
54	Willibald et al. (2002)	2-9	C	6.02×6.02×0.370	12.25×12.25	4	1.38	1.72	—	0.496	0.744	0.807	74.8	57.6
55	Willibald et al. (2002)	2-10	C	6.02×6.02×0.370	12.25×12.25	4	1.38	1.73	—	0.622	0.744	0.807	74.3	55.7

Table 2. Validation of the Proposed Analysis Method													
No.	a' (in.)	b' (in.)	ρ	$*P_i$ (in.)	$*P_{i,max}$ (in.)	δ	t_{cx} (in.)	α'	T_u (kip)	N_u (kip)	N_{ux} (kip)	β_{ux} (%)	N_{ux}/N_u
1	1.71	1.05	0.615	4.08	6.81	0.833	0.716	0.194	33.2	265	249	15.7	0.939
2	1.72	1.04	0.606	4.07	6.78	0.833	0.680	–0.195	34.2	274	261	4.8	0.954
3	1.71	1.05	0.615	4.08	6.81	0.833	0.716	0.182	33.3	266	279	3.2	1.05
4	1.74	1.05	0.604	4.08	6.85	0.833	0.683	–0.191	34.2	274	268	2.1	0.980
5	1.88	1.26	0.671	4.24	7.96	0.840	0.702	0.715	24.8	198	203	22.2	1.02
6	1.89	1.25	0.662	4.24	7.93	0.839	0.700	0.710	24.9	199	213	16.4	1.07
7	1.67	1.26	0.756	4.24	7.69	0.840	0.702	0.637	24.5	196	190	30.5	0.968
8	1.68	1.45	0.864	4.39	8.52	0.845	0.733	0.235	26.4	211	213	13.4	1.01
9	1.63	1.45	0.891	4.39	8.46	0.845	0.740	0.790	22.9	183	198	25.3	1.08
10	1.66	1.45	0.875	4.39	8.50	0.845	0.733	0.234	26.4	211	229	5.5	1.08
11	1.50	0.865	0.579	4.86	5.72	0.858	0.645	0.640	30.8	246	234	26.8	0.95
12	1.50	0.865	0.579	4.86	5.72	0.858	0.623	0.053	35.9	287	264	11.5	0.919
13	1.50	0.865	0.579	4.86	5.72	0.858	0.642	–0.164	38.4	307	299	2.7	0.973
14	1.50	0.865	0.579	4.86	5.72	0.858	0.643	–0.331	36.5	292	286	2.1	0.979
15	1.50	0.865	0.579	4.86	5.72	0.858	0.653	–0.418	37.7	302	302	–0.1	1.00
16	1.57	0.787	0.500	4.86	5.45	0.825	0.741	6.409	12.2	97	143	233.4	1.47
17	1.57	0.787	0.500	4.86	5.45	0.825	0.738	3.488	20.6	165	270	77.5	1.64
18	1.57	0.787	0.500	4.86	5.45	0.825	0.758	0.474	52.5	420	367	30.6	0.874
19	1.57	0.787	0.500	4.86	5.45	0.825	0.765	0.083	58.0	464	416	15.2	0.896
20	1.57	0.787	0.500	4.86	5.45	0.825	0.775	–0.184	58.3	466	—	—	—
21	1.57	0.787	0.500	4.86	5.45	0.825	0.778	–0.310	58.9	471	—	—	—
22	2.03	1.067	0.525	3.58	7.27	0.808	0.729	0.275	29.9	240	209	25.6	0.873
23	1.72	1.458	0.846	4.30	8.61	0.840	0.771	0.937	23.9	239	232	41.4	0.971
24	1.73	1.458	0.841	4.30	8.63	0.840	0.760	0.313	27.0	270	259	22.4	0.960
25	1.92	1.248	0.649	4.17	7.95	0.835	0.724	0.847	25.8	258	249	31.7	0.963
26	1.91	1.258	0.658	4.18	7.99	0.835	0.717	0.233	28.6	286	279	13.6	0.975
27	1.72	1.051	0.611	4.37	6.82	0.844	0.692	0.119	34.1	273	236	22.0	0.865
28	1.72	1.051	0.611	4.37	6.82	0.844	0.659	–0.220	34.1	273	256	6.6	0.938
29	1.50	0.865	0.579	7.76	5.72	0.879	0.620	1.480	23.1	92	97.6	53.7	1.06
30	1.50	0.865	0.579	7.76	5.72	0.879	0.595	0.544	31.3	125	126	17.8	1.01
31	1.50	0.865	0.579	7.76	5.72	0.879	0.602	–0.036	39.6	158	155	202	0.98
32	1.50	0.865	0.579	7.76	5.72	0.879	0.592	–0.244	38.4	154	154	–0.3	1.00
33	1.57	0.787	0.500	7.76	5.45	0.844	0.573	–0.409	36.4	146	149	–2.3	1.02
34	1.57	0.787	0.500	7.76	5.45	0.844	0.695	2.326	27.9	111	144	65.6	1.29
35	1.57	0.787	0.500	7.76	5.45	0.844	0.726	1.276	40.9	164	167	38.9	1.021
36	1.57	0.787	0.500	7.76	5.45	0.844	0.716	0.326	54.0	216	214	11.8	0.991
37	1.57	0.787	0.500	7.76	5.45	0.844	0.722	–0.014	59.8	239	236	1.4	0.987
38	1.57	0.787	0.500	7.76	5.45	0.844	0.731	–0.247	58.2	233	235	–0.9	1.009
39	1.57	0.787	0.500	7.76	5.45	0.844	0.735	–0.356	58.9	236	235	0.3	0.997
40	1.57	0.787	0.500	7.76	5.45	0.844	0.726	1.276	40.9	164	164	41.5	1.003
41	1.57	0.787	0.500	7.76	5.45	0.844	0.735	–0.356	58.9	236	233	1.1	0.989
42	1.57	0.787	0.500	9.72	5.45	0.844	0.726	1.276	40.9	164	172	34.9	1.05
43	1.57	0.787	0.500	9.72	5.45	0.844	0.726	0.344	53.8	215	209	14.4	0.97
44	1.57	0.787	0.500	9.72	5.45	0.844	0.722	–0.016	59.8	239	238	0.5	0.995
45	1.57	0.787	0.500	9.72	5.45	0.844	0.738	–0.237	59.2	237	237	–0.1	1.001
46	1.57	0.787	0.500	9.72	5.45	0.844	0.706	–0.346	58.9	236	234	0.7	0.993
47	1.65	0.708	0.428	9.72	5.17	0.805	0.840	0.803	72.3	289	262	29.0	0.905
48	1.65	0.708	0.428	9.72	5.17	0.805	0.855	0.331	81.3	325	298	18.9	0.917
49	1.65	0.708	0.428	9.72	5.17	0.805	0.863	–0.037	85.4	342	333	2.6	0.975
50	1.65	0.708	0.428	9.72	5.17	0.805	0.840	–0.175	88.1	352	348	1.3	0.988
51	1.65	0.708	0.428	9.72	5.17	0.805	0.865	–0.311	84.1	336	332	1.3	0.987
52	1.96	1.148	0.585	8.41	7.56	0.893	0.706	0.725	46.8	187	191	20.6	1.02
53	1.94	1.158	0.596	8.42	7.58	0.894	0.678	0.132	52.6	210	215	4.0	1.02
54	1.75	1.348	0.769	8.72	8.18	0.901	0.713	0.667	44.7	179	178	29.4	0.996
55	1.75	1.358	0.775	8.74	8.22	0.902	0.704	0.175	50.4	201	205	8.7	1.02

*i = A, B or C based on bolt pattern type; see Column Heading “Type” in Table 1

Shaded Indicates outlier not included in statistical analysis

Table 3 Statistical Analysis				
Type	Mean (μ)	$\Sigma(\mu - x_i)^2$	n	COV (%)
A	0.977	0.075	22	5.99
B	0.902	0.001	2	2.87
C	1.00	0.010	26	1.96

in the design method. This is indicated by the large N_{ux}/N_u for these specimens, 1.47 and 1.64, respectively. In scanning Tables 1 and 2 and comparing t_p to d, it can be seen that reasonable prying ratios β_{ux} occur when $t_p/d > ½$. In addition to the two type A specimens that do not satisfy this criterion, there is one type C specimen that does not. This is specimen 34, which has a prying ratio of 65.6% and a N_{ux}/N_u ratio of 1.29. The author considers these three specimens, numbers—16, 17 and 34—to be outliers that skew the mean of N_{ux}/N_u in a desirable direction—that is, $\mu > 1$—but this skewing is not correct. Thus, they are discounted in the Table 3 statistical analysis. Table 3 shows the mean, μ, the sum of the squares of the deviation from the mean, $\Sigma_i(\mu - x_i)^2$, and the covariance as a percentage, COV(%), for each of the bolt group types, A, B and C, with the outliers excluded. The means (μ) for types A and C are 0.977 and 1.00, respectively, with covariances of 5.99% and 1.96%, respectively. These values indicate excellent agreement of theory and test results and that designs performed with the proposed type A and C yield lines can be used with confidence.

Type B

The sample size for this type of connection is too small to provide confidence in its use. The mean is 0.902, which, being less than 1.0, indicates that this type will yield unconservative results. As such, its use is not recommended. This same observation is made by Willibald et al. (2002). They recognize that the corner bolts do not share the load equally with the side bolts. The side bolts fail first and reduce the connection capacity below what it would be if all bolts were equally loaded. The low mean value of 0.902 is suggestive of this observation.

The statistical data for type B is included in Table 3 only for completeness. The sample is too small for these data to be meaningful.

Further Discussion and Observations

Note that type C has p_{max} controlling in all specimens. This indicates that this type should be used only for small HSS, say, 3×3 and 4×4. For HSS5×5 and larger, there is no reason not to use a type A configuration.

From the data of Table 2, a correlation between α' and β_{ux} can be observed. When $\alpha' < 0$, the bolts theoretically control the design, which requires that β_{ux} be zero or small. That this is the case can be seen in Table 2. When $0 < \alpha' < 1$, the largest value of β_{ux} is 41.4%. In this range of α', both the plate and the bolts are active in controlling the design capacity. The β_{ux} value of 41.4% indicates that prying is taking place but that no separation or excessive deformations are occurring. When $\alpha' > 1$, the plate controls the design capacity. If the plate is too thin relative to bolt size, say, $t_p/d < ½$, large values of α' and β_{ux} occur. Table 2 shows that if $\alpha' \leq 1.5$, reasonable values of prying ratio, less than approximately 50%, occur. (Specimen 29, with $\alpha' = 1.48$ and $\beta_{ux} = 53.7\%$, is slightly over this 50% limit, but it is accepted here as a viable design.) Note that the outliers all have large values of α'.

The correlation of α' to β_{ux} is important. The ideal parameter to test a design is β_{ux}, but this is an experimental parameter that is not available to the designer. The parameter α', which mirrors the β_{ux} experimental values, is available to the designer. The author recommends, based on the Table 2 results, that the designer control α' to be ≤ 1.5. If $\alpha' > 1.5$, a thicker plate should be used.

When rectangular HSS are used, the bolt spacing should be kept uniform around the perimeter of the HSS. This will ensure that the bolts and HSS wall are uniformly loaded, as is assumed in the design of the HSS member. The parameter c shown in Figure 1 for pattern A is used to guarantee this.

CONCLUSIONS

Design and analysis procedures for end plate connected HSS tension members have been developed. The methods use yield lines matched to the end plate bolt pattern. The methods use the end plate tensile strength F_u rather than the end plate yield strength F_y, thereby bringing them into conformance with the AISC *Manual* (2011) procedure for prying action. The method recommends that, for end plated HSS tension members, the calculation parameter α' be limited to 1.5.

EXAMPLE PROBLEM—DESIGN

Given:

Determine the end plate thickness and the weld size, and the bolts required to resist forces of 16 kips dead load and 50 kips live load on an ASTM A500 Grade B HSS4×4×¼ section. The end plate is ASTM A36. Use E70 electrodes.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are:

HSS strut
$F_y = 46$ ksi
$F_u = 58$ ksi

End plate
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS4×4×¼
$t = 0.233$ in.
$A = 3.37$ in.^{2}

Find the required end plate thickness, t_p.

From Chapter 2 of ASCE/SEI 7 (ASCE, 2016), the required tensile strength is:

$P_u = 1.2(16.0 \text{ kips}) + 1.6(50.0 \text{ kips})$
$= 99.2$ kips

From Figure 5,

$a = 1.5$ in
$b = 1.5$ in.

A1. Check $a \le 1.25b$:

$a = 1.50$ in.
$1.25b = 1.25(1.50 \text{ in.})$
$= 1.88$ in.
$a \le 1.25b$ **o.k.**

A2. Calculate a', b', ρ, d':

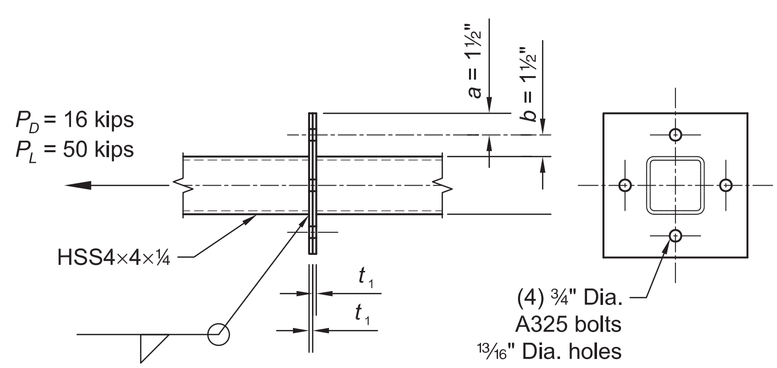

Fig. 5. Data for example problem.

$$a' = a + \frac{d}{2} \tag{12}$$

$$= 1.50 \text{ in.} + \frac{(0.75 \text{ in.})}{2}$$

$$= 1.88 \text{ in.}$$

$$b' = b - \frac{d}{2} \tag{13}$$

$$= 1.50 \text{ in.} - \frac{(0.75 \text{ in.})}{2}$$

$$= 1.13 \text{ in.}$$

$$\rho = \frac{b'}{a'} \tag{14}$$

$$= \frac{1.13 \text{ in.}}{1.88 \text{ in.}}$$

$$= 0.600$$

$$d' = d + \tfrac{1}{16} \text{ in.} \tag{15}$$

$$= \tfrac{3}{4} \text{ in.} + \tfrac{1}{16} \text{ in.}$$

$$= \tfrac{13}{16} \text{ in.}$$

A3. The tributary length per bolt for pattern C is:

$$p_c = \frac{2(w_i + h_i + \pi b)}{n} \tag{8}$$

$$= \frac{2[4.00 \text{ in.} + 4.00 \text{ in.} + \pi(1.50 \text{ in.})]}{4}$$

$$= 6.36 \text{ in.}$$

A4. Check p_{max}:

$$p_{max} = 4\sqrt{b'(a+b)} \tag{11}$$

$$= 4\sqrt{1.13 \text{ in.} (1.50 + 1.50 \text{ in.})}$$

$$= 7.35 \text{ in.}$$

Because $p_c < p_{max}$, the pattern C can be developed. Use $p = p_c = 6.36$ in.

A5. Calculate δ:

$$\delta = 1 - \frac{d'}{p} \tag{16}$$

$$= 1 - \frac{\tfrac{13}{16} \text{ in.}}{6.36 \text{ in.}}$$

$$= 0.872$$

A6. Calculate t_c:

From AISC *Manual* Table 7-2, the design strength per bolt is:

$B = 29.8$ kips.

$$t_c = \sqrt{\frac{4Bb'}{\phi p F_u}} \tag{17}$$

$$= \sqrt{\frac{(4)(29.8 \text{ kip/bolt})(1.13 \text{ in.})}{(0.90)(6.36 \text{ in.})(58 \text{ ksi})}}$$

$$= 0.636 \text{ in.}$$

D1. Check $T \le B$:

The required tension per bolt, T, is:

$$T = \frac{P_u}{n}$$

$$= \frac{99.2 \text{ kips}}{4 \text{ bolts}}$$

$$= 24.8 \text{ kips}$$

Because $T < B$, the procedure can continue.

D2. Calculate β:

$$\beta = \frac{1}{\rho}\left(\frac{B}{T} - 1\right) \tag{21}$$

$$= \frac{1}{0.600}\left(\frac{29.8 \text{ kips}}{24.8 \text{ kips}} - 1\right)$$

$$= 0.336$$

D3. Because $0 < \beta < 1$,

$$\alpha^* = \min\left[\frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right), 1\right] \tag{22}$$

$$= \frac{1}{0.872}\left(\frac{0.336}{1-0.336}\right)$$

$$= 0.580$$

D4. Calculate t_p:

$$t_p = t_c\sqrt{\frac{T}{B}\left(\frac{1}{1+\delta\alpha^*}\right)} \tag{23}$$

$$= 0.636 \text{ in.}\sqrt{\frac{24.8 \text{ kips}}{29.8 \text{ kips}}\left(\frac{1}{1+0.872(0.580)}\right)}$$

$$= 0.473 \text{ in.}$$

Use a ½-in. A36 end plate.

D5. Calculate α′:

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t_p}\right)^2 - 1\right] \tag{18}$$

$$= \frac{1}{0.872\,(1+0.600)}\left[\left(\frac{0.636 \text{ in.}}{0.500 \text{ in.}}\right)^2 - 1\right]$$

$$= 0.443$$

D6. Check α':

$\alpha' = 0.443 < 1.5$ **o.k.**

Therefore, the ½-in. A36 end plate is adequate.

Design the weld of the end plate to the HSS

From AISC *Manual* Table 1-12, the surface area of the HSS 4×4×¼ = 1.27 ft^2/ft

Length of weld = l_w = (1.27 ft^2/ft)(12 in./ft)
= 15.2 in.

From the AISC *Specification* Section J2.4 (AISC, 2016):

$$F_{nw} = 0.60F_{EXX}(1.0 + 0.50\sin^{1.5}\theta) \quad \text{(\textit{Spec}. Eq. J2-5)}$$

With $\theta = 90°$:

$$1.0 + 0.50\sin^{1.5} 90 = 1.5$$

The weld size required, D, is then:

$$D = \frac{P_u}{1.392(1.5)l_w} \quad \text{(from \textit{Manual} Eq. 8-2a)}$$

$$= \frac{99.2 \text{ kips}}{1.392(1.5)(15.2 \text{ in.})}$$

$$= 3.12 \text{ sixteenths}$$

Use ¼-in. fillet weld.

EXAMPLE PROBLEM—ANALYSIS

Find the design strength of the previous example when a ½-in. A36 end plate is specified.

From the calculations of the previous problem:
t_c = 0.636 in.
α' = 0.443

A7. Because $0 < \alpha' < 1$, bolts and plate control

$$T_u = B\left(\frac{t_p}{t_c}\right)^2(1+\delta\alpha') \quad (19)$$

$$= (29.8 \text{ kip/bolt})\left(\frac{0.500 \text{ in.}}{0.636 \text{ in.}}\right)^2\left[1+(0.872)(0.443)\right]$$

$$= 25.5 \text{ kip/bolt}$$

A8. Calculate N_u:

$$N_u = nT_u \quad (20)$$
$$= (4 \text{ bolts})(25.5 \text{ kip/bolt})$$
$$= 102 \text{ kips}$$

A9. Because N_u = 102 kips > P_u = 99.2 kips, the design is satisfactory.

SYMBOLS

B — Design bolt tensile strength, $\phi F_{nt} A_b$, used for routine analysis design calculations, kips [The notation used here is that of the AISC *Specification*, ANSI/AISC 360-10; see the AISC *Manual* (2011), Part 16, Table J3.2.]

B_u — Measured bolt tensile strength, Table 1, kips

F_u — End plate tensile strength, measured values for Table 2 calculations, specified minimum values for routine analysis and design calculations, ksi

F_y — End plate yield strength, ksi

N_u — Predicted connection axial capacity, nT_u, kips

N_{ux} — Experimental connection capacity, kips

P_u — Required axial tension strength for the design problems and the yield line nominal tensile strengths for patterns A, B and C, kips

T — Required tensile strength per bolt, kips

T_u — Predicted tensile capacity per bolt, or design strength, kips

a — Plate dimension from bolt center to edge of end plate, in.

a' — $a + d/2$, in.

b — Plate dimension from bolt center to edge of HSS member, in.

b' — $b - d/2$, in.

c — Reported bolt spacing in type A bolt pattern specimens, in. (Not explicitly used in this study, but see the discussion in the "Further Discussion and Observations" section.)

d — Bolt diameter, in.

d' — Hole size (bolt diameter plus 1⁄16 in., except measured values for specimens 23–26), in.

h_i — Dimension of HSS, in.

h_p — Dimension of end plate, in.

l — Length of equivalent straight line yield line, in.

n — Number of bolts in bolt pattern or number of specimens in statistical sample

p — Tributary length of end plate per bolt, in.

p_i — Tributary length of end plate per bolt for the *i*th pattern, A, B or C, in.

$p_{i,max}$ — Maximum bolt spacing for the *i*th pattern, in.

t_c — End plate thickness that will develop the design bolt strength B, in.

t_{cx} — End plate thickness that will develop the experimental bolt strength B_u, in.

t_p — End plate thickness, in.

w_i — Dimension of HSS, in.

w_p — Dimension of end plate, in.

x_i — Specific datum value, $(N_{ux}/N_u)_i$, for the *i*th datum value

α^* — Calculation parameter for the proposed design method

α' — Calculation parameter for the proposed analysis method, representing the theoretical point at which the bolt strength and the plate strength are equal

β — Calculated prying ratio, $\dfrac{1}{\rho}\left(\dfrac{B}{T}-1\right)$, Eq. 9-25 of the AISC *Manual* (2011)

β_{ux} — Experimental prying ratio, $\dfrac{nB_u - N_{ux}}{N_{ux}}(100)$, % [similar to AISC *Manual* (2011), Eq. 9-25]

δ — Factor in analysis and design methods that removes the bolt holes, $1 - d'/p$

ϕ — Resistance factor from AISC *Manual* (2011) Eq. 9-30a

μ — Mean value of a data set

ρ — Parameter in analysis and design methods, b'/a'

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