Analysis and Design of Cable-Stayed Steel Columns Using the Stiffness Probe Method

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ABSTRACT

The stiffness probe method (SPM) is a new numerical procedure that calculates buckling loads. SPM probes the local stiffness of a given structure at the point of application of a small transverse perturbation force as the applied load is increased. The local stiffness degrades from a maximum for an unloaded structure to zero at the buckling load. An artifice spring is added to the original structure that eventually absorbs the full perturbation force at a prescribed small deflection, thereby keeping structural deformations small as the buckling load is approached. As a result, using an indicator that approaches zero at buckling rather than having to rely on increasingly larger deflections at buckling as in conventional *P*-Δ methods, SPM ensures an accurate numerical result for the critical load. We use SPM herein to study the behavior of one and two cross-arm cable-stayed columns under applied load. A formula is given to calculate the minimum slenderness that justifies converting a tube into a cable-stayed column. Various factors such as cable prestrain, cable cross-sectional areas, and tiers of cross-arms affecting column strength are examined for a series of cable-stayed columns. We find that cable-stayed columns may buckle either in a one-lobe symmetrical mode or two-lobe anti-symmetrical mode, the latter case being contrary to conventional thinking. A design example for a given cable-stayed column using the AISC *Specification* is presented. The effect of optimum cable prestrain to enhance column buckling strengths is discussed. A strength enhancement ratio (SER) is defined that evaluates the additional column strength gained after transforming a given steel tube into a cable-stayed column.

Keywords: analysis, behavior, buckling modes, eigenvectors, cable (slackening, stays, optimum prestraining), columns, design (ASD, LRFD), elastic stability, failure mode, numerical methods, residual tension, cross-arms, load (applied, external), spring (augmented, parallel, series), steel, stiffness probe, strength (enhancement, nominal).

INTRODUCTION

Cable-stayed steel columns consist of a central steel tube, to which one or more sets of transverse cross-arms are welded at equal spacings along the length and to which prestressed steel cables are attached as required to enhance column strength. Review of the existing literature indicates a number of papers that have dealt with the subject in various attempts to understand and predict their behavior and strength. The following investigators have contributed to this field: R.J. Smith et al. (1975), Hafez et al. (1979), Hathout et al. (1979), Temple (1977), Temple et al. (1984), E.A. Smith (1985), and Saito and Wadee (2009). We present a new numerical approach to the subject that examines column behavior under axial load and calculates an accurate value of strength, followed by design using the 2010 AISC *Specification for Structural Steel Buildings* (AISC, 2010), hereafter referred to as the AISC *Specification*.

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For this purpose, we first provide a simple introduction to the stiffness probe method (SPM), which was conceived jointly by the senior author and the late A.R. Robinson at the University of Illinois. R.E. Miller also contributed initially. Fundamentally, SPM is based on the incontrovertible fact that the stiffness of an axially compressed structure to resist the effects of a perturbation force or moment becomes zero only when subjected to its buckling load (Bleich et al., 1952; Hoff, 1941, 1956). We proceed to explain SPM first in full detail and then use the method specifically to calculate the buckling load of cable-stayed columns.

We recognize that two sets of internal forces are generated in any cable-stayed column—namely, compression in the central tube and tension in the stay cables. We identify these forces and provide equations for columns with one and two sets of cross-arms, respectively. We emphasize two distinct loading stages, initially at cable prestraining and finally at buckling. We also discuss two possible ways of specifying cable prestrains for use in the field and provide a formula to relate them both.

Cable slackening takes place as the external load increases because the applied compression on the tube causes shortening of both it and the cable stays. Ominous consequences to column stability may occur if total cable slackening takes place under service conditions. This effect could occur only if small amounts of cable prestrain were specified, a situation that must be prevented. We provide a formula to predict the applied load at which total cable slackening occurs based on the initial amount of cable prestrain. For any case where the column load for total cable slackening is less than the column critical load, an enhancement of prestrain is required. We avoid this pitfall by specifying enough cable prestrain to make the load at which total cable slackening occurs to be greater than the column critical load by a reasonable margin.

We now calculate the critical load for a number of cable-stayed columns. We find that there are cases where the governing mode for a cable-stayed column may not be the conventional one-lobe symmetrical mode but, rather, the two-lobe anti-symmetrical mode. This counterintuitive situation calls for designers to always calculate and examine both modes and then use the smaller of the two as the critical value for design. We found that for a given stayed column, the critical mode depends on the initial cable pretension. There is a value of the latter that gives the same critical load, P_{cr}, for both modes. This value is usually but not always the optimum prestrain. Thus, we label as optimum only the actual prestrain value that gives the maximum P_{cr}.

The fact that an antisymmetric two-lobe mode could become the governing buckling mode, instead of the conventional symmetric one-lobe mode, was first pointed out for a given column using an analytical solution (Blumenthal, 1937) and later verified numerically (Newmark, 1943). Both papers found the buckling load of a double-hinged column subjected to two opposing compression loads not at the ends but at the column mid-third points instead. The governing buckling mode corresponding to the smallest buckling load was an anti-symmetrical, two-lobe configuration. Newmark remarked at that time that "It is of interest and importance that the critical load corresponding to the anti-symmetrical deflection is lower than that corresponding to the symmetrical configuration for the arrangement of loads chosen" and warned also that "this would not have been discovered if only symmetrical defection curves had been assumed." This caveat also applies to the calculation of the critical load for a cable-stayed column, a fact that has been verified by Smith et al. (1975) as well as by our own calculations using SPM. Further work using SPM showed that had the distance between the two compressive loads been $0.361L$ instead of $L/3$, both buckling modes would have provided the same critical load at $(\pi/0.361L)^2\,EI$.

For columns provided with cross-arms in a cruciform configuration, there are two principal axes about which buckling of the column may occur. One axis is defined by the tube diameter where two opposite cross-arms are located. The other axis is at 45° with respect to the cross-arms. We studied columns with one and two sets of cross-arms, including various areas of steel cables and prestrain levels, before concluding that the difference in buckling strength between the two axes is insignificant.

Designers shall not use the critical load, P_{cr}, to verify the strength of the supporting tube. Instead, they must use the actual compressive force, N_{cr}, in the tube, which is larger than P_{cr} by the amount of residual tension in the stay cables. We provide an example for this calculation using a given cable prestrain and find the AISC design strength for the given column for both LRFD and ASD values. For the sake of comparison, we now use the optimum cable prestrain and realize a major enhancement of the column design strength. Following this, we introduce the concept of a nominal strength enhancement ratio (*SER*) that effectively evaluates the additional strength gained by the unstayed-tube.

STIFFNESS PROBE METHOD

The stiffness probe method (SPM) is a new numerical procedure that can be used to evaluate the elastic stability of structures subjected to compressive forces. It is specifically used herein to calculate the buckling load of cable-stayed columns.

Consider a column made of a steel tube hinged at both ends and subjected to an axial load, *P*, (see Figure 1). The tube is reinforced with four steel cables, or rods, attached to each end and to the ends of horizontal cross-arms. The latter may be placed at either the mid-height of the column, at the mid-third points, or at equal spacing along the tube length as needed. Note that the cables are not continuous between the top and bottom ends of the tube but are segmental instead (see Figure 2). The schematic elevation and plan views of a one cross-arm column as shown in Figure 1 provide all necessary structural and geometric information to analyze the column for elastic stability. A one cross-arm column is used for the sake of simplicity. Both principal axes for flexural buckling—namely, *a-a* and *b-b*—are shown. Axis *a-a* is for bending about a cruciform configuration of the stay cables, and axis *b-b* is for bending about a two-paired configuration of stay cables. Two possible buckling modes—namely, a symmetric (one-lobe) and an anti-symmetric (two-lobe)—must always be calculated to ascertain the lowest critical load, as either mode may govern buckling. Calculations of higher modes using SPM is possible as shown herein, although modes with three or higher number of lobes will not govern column design.

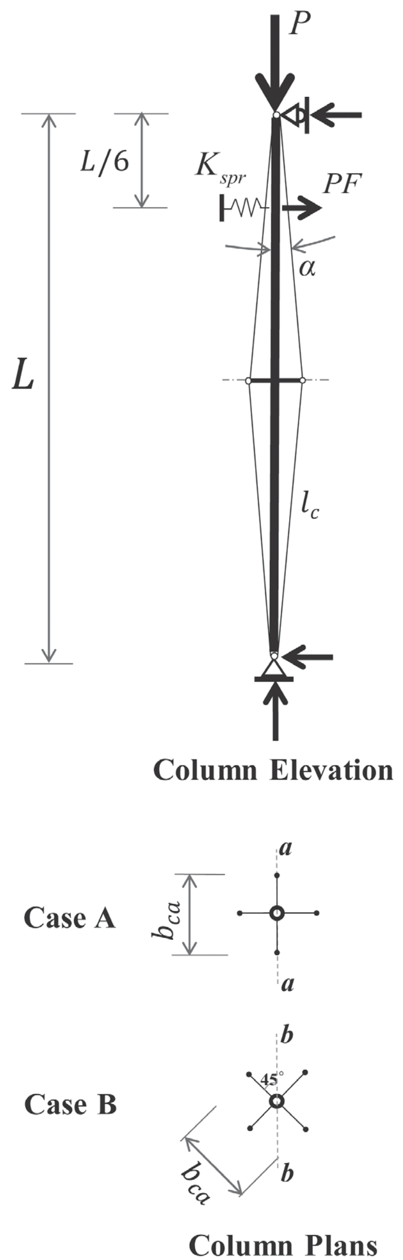
Characteristic to SPM are a perturbation force, *PF*, and an elastic artifice spring of stiffness K_{spr}. After both are attached to a given column, the latter is transformed into an augmented structure (Figure 1). For any given value of load *P*, the force *PF* triggers a transverse displacement $\delta(P)$ at its location on the column. For any $P < P_{cr}$, the column achieves an equilibrium configuration. At $P = P_{cr}$, the column buckles while the augmented structure still remains stable because of the enhanced stiffness K_{spr} provided by the artifice spring. We note that all of the above would apply as well if a perturbation moment *PM* and an artifice rotational spring of stiffness K_{rot} were used instead of PF and K_{spr}, respectively.

As PF is applied to the augmented structure in the presence of the external load P, it is resisted by both the column and the artifice spring K_{spr}. We recognize these two components of PF as PF_{col} and PF_{spr}, where $PF = PF_{col} + PF_{spr}$. Because the column and the artifice spring actually behave as a set of two parallel springs of stiffness $K_{col}(P)$ and K_{spr}, respectively, we find that for any given value of the transverse displacement $\delta(P)$, the two components PF_{col} and PF_{spr} can be calculated as follows:

$$PF_{col} = PF \frac{K_{col}(P)}{\left[K_{col}(P) + K_{spr}\right]} \tag{1}$$

and

$$PF_{spr} = PF \frac{K_{spr}}{\left[K_{col}(P) + K_{spr}\right]} \tag{2}$$



Column Data

Core: 12-in. outer dia × 0.5-in. thk. steel tube

$L = 930.7$ in.
$E = 29{,}000$ ksi
$I = 299.2$ in.^{4}
$A = 18.06$ in.^{2}
$r = 4.07$ in.
$F_y = 42$ ksi
$N_E = \pi^2 EI/L^2 = 98.86$ kips
$N_y = Af_y = 758.5$ kips

Cross-arm: 4-in. outer dia × 0.33-in. thk. tube

$E_{ca} = 29{,}000$ ksi
$A_{ca} = 3.81$ in.^{2}
$b_{ca} = 96$ in.

Cable-stays

$E_s = 24{,}000$ ksi
$A_s = 1$ in.^{2} for each cable
$F_y = 243$ ksi
$F_u = 20$ ksi
$l_c = 468$ in.
$\alpha = \tan^{-1}\frac{b_{ca}}{L} = 5.8891°$
$\varepsilon_{s_{spec}} = 0.002$
$\varepsilon_{s_{ini}} = 0.00169$

Stiffness-Probe Parameters

Perturbation Force, $PF = 0.1$ kip
Artifice spring stiffness, $K_{spr} = 0.1$ kip/in.

Fig. 1. Cable-stayed column showing applied load P *and SPM parameters* PF *and* K_{spr}.

Note that K_{spr} is always constant, while $K_{col}(P)$ decreases as P increases. At $P = 0$, the column takes a substantial portion of PF, while the artifice spring takes little. As P continues to increase, PF_{col} decreases while PF_{spr} increases. At $P = P_{cr}$, the column is devoid of any stiffness and cannot support any portion of PF, resulting in $PF_{col} = 0$ and $PF_{spr} = PF$. Thus, at buckling, the column is subjected exclusively to P and not at all to PF. Regardless of the value of PF used to trigger the column away from its initial vertical configuration, the resulting P_{cr} is not affected by PF. This is because at $P = P_{cr}$, the column is no longer subjected to PF; the latter is fully resisted by the elastic spring alone. Similarly, the value of K_{spr} does not affect the resulting value of P_{cr}. It only decides the limiting value of the transverse deflection for an equilibrium position of the augmented structure at $P = P_{cr}$. Therefore, any reasonable set of values for PF and K_{spr} (say, $PF = 0.1$ kips and $K_{spr} = 0.1$ kip/in., or $PF = 0.5$ kips and $K_{spr} = 0.2$ kip/in.) should render the same result for P_{cr}. We have verified that the equilibrium configurations at P_{cr} from the preceding two sets of PF and K_{spr} have the same shape, except that the target displacement is 0.1 kip/0.1 kip/in. = 1 in. for the former, and 0.5 kip/0.2 kip/in. = 2.5 in. for the latter set.

The purpose of using an artifice spring is thus clear. To wit, K_{spr} keeps the augmented structure stable even as the applied load approaches the column critical load. As a result, there is always computational control using SPM, and at the location of PF, the transverse deflection is always small and limited to the target value. An accurate calculation of P_{cr} is thus possible, with as many significant digits as desired. In the absence of an artifice spring, transverse deflections at P close to P_{cr} may increase nonsensically toward infinity, making an accurate calculation of P_{cr} difficult. We note again that at $P = P_{cr}$, all of the PF has been absorbed by the artifice spring, and none of it is acting on the column. As such, SPM differs substantially from the conventional way of triggering P-δ effects for which the initial perturbation force remains in the column and displacements grow uncontrollably as P approaches P_{cr}.

The artifice spring must be placed so that it acts together with the column as two springs in parallel, and not in series (Figure 2). For the configuration shown in Figure 2a, the column is between the PF and the artifice spring; both the column and the artifice spring are subjected to the same displacement, and both share PF according to their respective stiffness, $K_{col}(P)$ and K_{spr}. This is an acceptable configuration that we have used for the column shown in Figure 1. One could also achieve the same result by placing the PF between the column and the artifice spring (Figure 2b). What one cannot do is place the artifice spring between the column and the PF (Figure 2c). Such a location would make the artifice spring and the column act as two springs in series, rather than in parallel. As a result, the PF would not be shared because both the artifice spring and the column

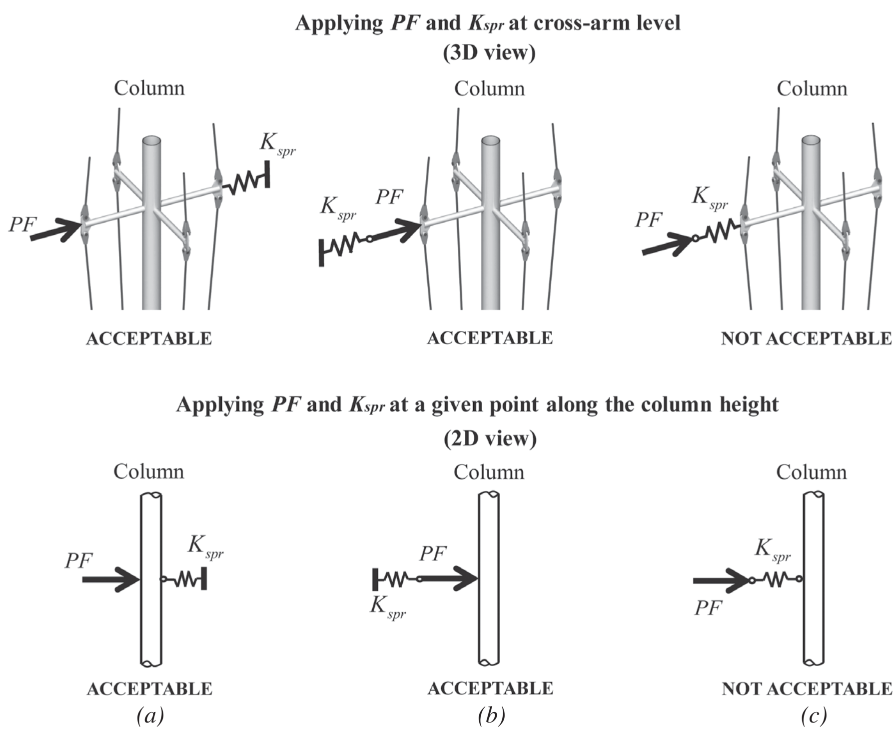


Fig. 2. Three possible ways for placing the PF *and* K_{spr} *with respect to the column.*

would take the whole PF, except at different displacements. To summarize, never place the artifice spring between the PF and the column.

For any given value of the load, P, we define the stiffness of the augmented structure, $K_{aug}(P)$, as the sum of the column stiffness, $K_{col}(P)$, and artifice spring stiffness, K_{spr}—that is, $K_{aug}(P) = K_{col}(P) + K_{spr}$. We calculate $K_{aug}(P)$ as the ratio $PF/\delta(P)$, where $\delta(P)$ is the transverse displacement of the column (in the presence of P) as measured at the location of PF. We may use any commercial program with a P-δ analysis option such as SAP 2000, to calculate $\delta(P)$. The quantities $K_{col}(P)$ and $K_{aug}(P)$ are functions of P, while K_{spr} is a constant. Once the quantity $K_{aug}(P)$ is calculated as before, we may find $K_{col}(P)$ as follows:

$$K_{col}(P) = K_{aug}(P) - K_{spr} \tag{3}$$

For an unloaded structure—that is, at $P = 0$—$K_{aug}(0)$ and $K_{col}(0)$ are maximum values. As P increases, both $K_{aug}(P)$ and $K_{col}(P)$ diminish in value. We probe the stiffness $K_{col}(P)$ as P increases. Instability ensues at $P = P_{cr}$ when the stiffness of the column $K_{col}(P_{cr})$ becomes zero. This leads to $K_{aug}(P_{cr}) - K_{spr} = 0$, or $K_{aug}(P_{cr}) = K_{spr}$. Note that although the column has buckled, the augmented structure remains stable at $P = P_{cr}$ because its remnant stiffness $K_{aug}(P_{cr}) = K_{spr}$ is greater than zero. Users of SPM may, alternatively, probe the transverse displacement $\delta(P)$, instead of $K_{col}(P)$, as an indicator to check the onset of instability. In which case, $\delta < \delta_{max}$ denotes $P < P_{cr}$, while $\delta = \delta_{max} = PF/K_{spr}$ is true only at $P = P_{cr}$. In addition, either the value of $PF_{col}(P)$ or $PF_{spr}(P)$ may also be probed as other possible indicators of instability. As such, instability may ensue when $PF_{col}(P) = 0$ or $PF_{spr}(P) = PF$.

The correctness and accuracy of SPM results have been verified using cases with closed-form solutions from Timoshenko and Gere (1961), Gurfinkel and Robinson (1965), Column Research Committee of Japan (1971), and others. SPM may be used to calculate structural instability caused not just by static compression loads, as discussed in the paper, but also by vibrations at their natural frequencies. The concept is the same whether instability ensues by either the application of a static load or by structural excitation to a natural frequency. They both would cause zero stiffness in any given structure. As such, the P-Δ effects caused by vibrating masses in reducing the natural frequency of a given structure may be readily calculated using SPM. That said, SPM is applicable to any problem for which the stiffness of a structure will go to zero, thereby leading to instability. Some other examples include lean-on columns, polygonal frames, arches, and domes. SPM has also been used in practice by the senior author in 2007 for stability calculations of a huge, steel-framed, paper storage structure that collapsed in Wisconsin. The method was used to calculate the critical load of cable-stayed struts on cable-dome structures by the co-author as part of his doctoral dissertation at the Department of Civil Engineering at the University of Illinois at Urbana-Champaign (Krishnan, 2015). SPM has been taught in class to graduate students by the senior author since 2010.

Numerical Example

We apply SPM to find the buckling modes for the stayed column shown in Figure 1 that are within the possible load range, $0 \le P \le P_{sl}$. Note that P_{sl} is the applied load that would be large enough to cause the stay cables to fully slacken. When this happens at $T(P) = 0$, the cables become ineffective thereby rendering the column into an unstayed tube. Let the steel cables be prestrained to $\varepsilon_{s_{spec}} = 0.002$, which results in an initial prestrain value $\varepsilon_{s_{ini}} = 0.00169$ after losses caused by compression of the tube. Also, let $PF = 0.1$ kips and $K_{spr} = 0.1$ kip/in.

Starting at $P = 0$, we increase the load, P, gradually for the sake of probing the variation of K_{col} with P in order to ascertain P_{cr}. The results are given in Figures 3 and 4.

For $P = 0$:

$$\delta(0) = 0.0289 \text{ in.}$$

$$\begin{aligned} K_{avg}(0) &= PF/\delta(0) \\ &= 0.100 \text{ kip}/0.0289 \text{ in.} \\ &= 3.46 \text{ kip/in.} \end{aligned}$$

$$\begin{aligned} K_{col}(0) &= K_{avg}(0) - K_{spr} \\ &= 3.46 \text{ kip/in.} - 0.100 \text{ kip/in.} \\ &= 3.36 \text{ kip/in.} \end{aligned}$$

Further increasing P results in $P_{cr} = 343$ kips at $K_{col} = 0$. At this point, we also find:

$$\begin{aligned} \delta &= \delta_{max} \\ &= PF/K_{spr} \\ &= 0.100 \text{ kip}/0.100 \text{ kip/in.} \\ &= 1.00 \text{ in.} \end{aligned}$$

Note that we have chosen not to plot $K_{aug}(P)$ because it is irrelevant for design and also for the sake of clarity. Had we done so, the plot of $K_{aug}(P)$ would have been exactly like $K_{col}(P)$ except separated above it by a vertical distance $K_{spr} = 0.1$ kip/in. As such, the value of P that would cause buckling of the entire augmented structure would always be larger than P_{cr} and lie to the right of it in Figure 3.

Finding $P_{cr} = 343$ kips is all a designer needs, and no further work would be necessary, as far as instability is concerned, because this is the load at which the given column would fail. Nevertheless, if we look at the column eigenvector (Figure 4b), we might be surprised to recognize it as a two-lobe, anti-symmetric configuration that is conventionally associated by designers with the second mode of buckling and not the first. Further loading of the column gives the actual second buckling mode as a one-lobe configuration at $P_{cr2} = 422$ kips $> P_{cr1} = 343$ kips (Figures 3 and 4d).

Table 1. Single Cross-Arm Column as Shown in Figure 1. Summary of SPM Results for the First Three Buckling Modes (results in bold characters are for the governing two-lobe buckling mode)

Buckling Lobes	Applied Load	Tension in Cables	Force in Tube	Transverse Display at *PF* location	Stiffness		Perturbation Force Components	
					Augmented Structure	Column	Artifice Spring	Column
No.	P kips	$\sum T\cos\alpha$ kips	N kips	$\delta(P)$[1] in.	$K_{aug}(P)$[2] kip/in.	$K_{col}(P)$[3] kip/in.	$PF_{spr}(P)$[4] kips	$PF_{col}(P)$[5] kips
—	0	162	162	0.0289	3.46	3.36	0.0029	0.0971
2	**343**	**109**	**452**	**1**	**0.1**	**0**	**0.1**	**0**
1	422	97	519	1	0.1	0	0.1	0
3	956	15	971	1	0.1	0	0.1	0

Notes

[1] Maximum value of δ at $P = P_{cr}$ is $\delta_{max} = PF/K_{spr}$. For the given case, $\delta_{max} = 1$ in.

[2] $K_{aug}(P) = PF/\delta(P)$; where $PF = 0.1$ kip.

[3] $K_{col}(P) = K_{aug}(P) - K_{spr}$; where $K_{spr} = 0.1$ kip/in.

[4] $PF_{spr}(P) = K_{spr}\,\delta(P)$.

[5] $PF_{col}(P) = PF - PF_{spr}(P)$.

This sequence of buckling configurations is definitively counterintuitive to conventional thinking. Finally, we find a third buckling mode (a three-lobe configuration) at $P_{cr3} = 956$ kips, which is still less than $P_{sl} = 1{,}048$ kips, although otherwise irrelevant (Figure 3). A summary of the results is shown in Table 1.

We emphasize in both Figures 3 and 4 that stable equilibrium for the given column under applied compression load, even ideally, would be only possible for $P < 343$ kips, and any applied load larger than that would not be realistic, but only a mathematical curiosity. In Figure 4, note that for all three buckling modes, the column is only subjected to the

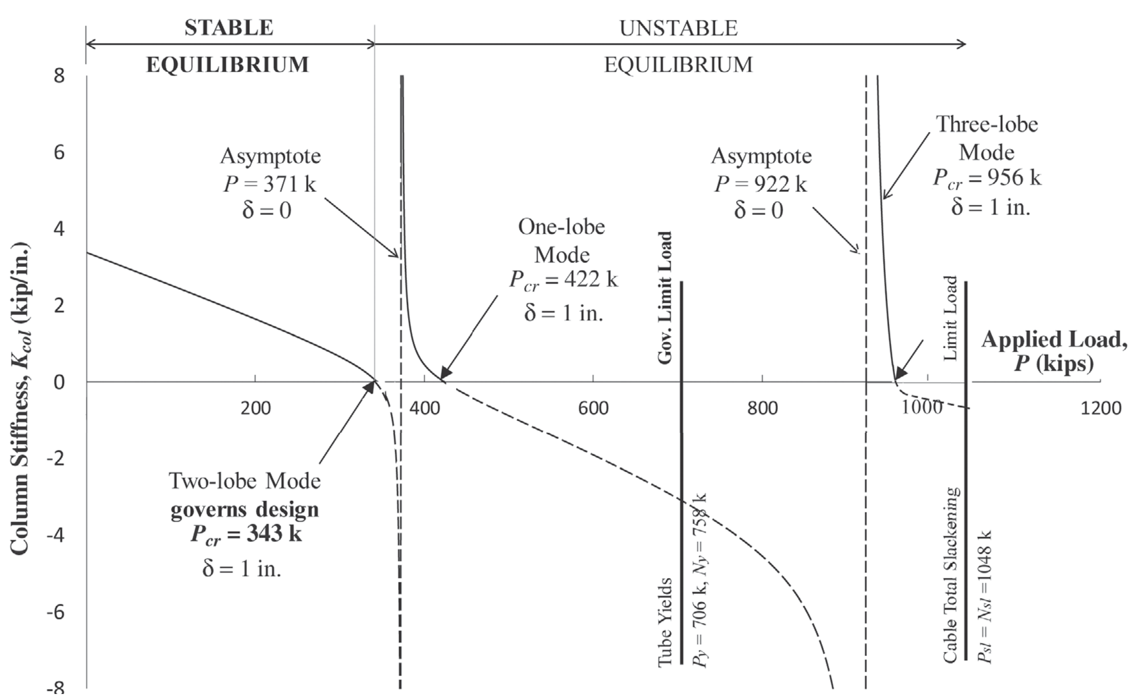


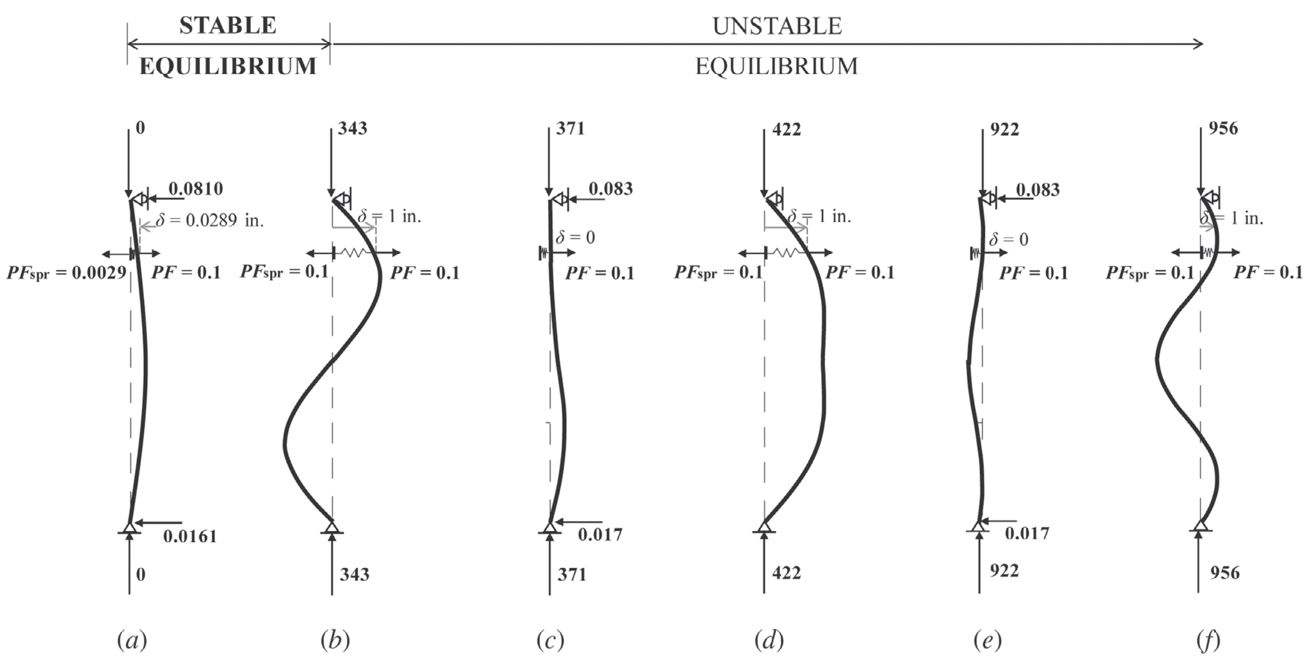
Fig. 3. Variation of column stiffness K_{col} *with applied load* P*, indicating* P_{cr} *for the first three modes of buckling and* P_{sl} *at the end of range. Note that the two-lobe buckling mode governs.*

corresponding P_{cr} and not to any transverse force or horizontal reactions at the column supports. This is true because using SPM guarantees that, at any $P = P_{cr}$, all of the *PF* is absorbed by the artifice spring—that is, $PF_{spr} = PF$. As such, no part of *PF* remains on the structure—that is, $PF_{col} = 0$.

Simplified SPM Models for Calculating P_{cr}

Let us try the following two simple models to ascertain which buckling mode governs. To find the one-lobe buckling mode, use the column in Figure 1 with the same perturbation set $PF = 0.1$ kips and $K_{spr} = 0.1$ kip/in., except located now at mid-height. Using SPM, again we obtain $P_{cr} = 422$ kips, at $\delta_{max} = 1$ in. as before. Note that the two-lobe mode would not be captured by this model because it requires zero displacement at column mid-height, which is an obvious impossibility when the *PF* is located therein. To find the two-lobe buckling mode, use the column in Figure 1 again, except we apply instead a perturbation moment $PM = 1$ kip-in. and rotational artifice spring $K_{rot} = 100$ kip-in./rad at mid-height, which results in $\theta_{max} = 1/100 = 0.01$ rad, at P_{cr}. Use SPM as before to obtain the two-lobe mode at $P_{cr} = 343$ kips as before. Neither the one-lobe nor the three-lobe modes may be captured using this model because they both require $\theta = 0$ at column mid-height, which is again an obvious impossibility when *PM* is located therein. Comparing the results from the two models, we find that the governing $P_{cr} = 343$ kips corresponds to the two-lobe configuration.

The calculation process for the preceding simplified models need not start at $P = 0$. We may use any reasonable value of $P_1 > 0$ for this purpose. If $K_{col}(P_1)$ were found to be



LOADING STAGES

Variables	Prestressed state	Two-lobe Mode	Asymptote	One-lobe Mode	Asymptote	Three-lobe Mode
P (kips)	0	**343**	371	422	922	956
δ (in.)	0.0289	**1**	0	1	0	1
PF_{spr} (kips)	0.0029	**0.1**	0	0.1	0	0.1
PF_{col} (kips)	0.0971	**0**	0.1	0	0.1	0
K_{col} (kip/in.)	3.36	**0**	—	0	—	0

Fig. 4. Column eigenvectors at various loading stages, including a tabulated list of variables; cable stays and cross-arms are not shown for clarity.

positive, this would mean $P_1 < P_{cr}$. We then try $P_2 > P_1$ and calculate $K_{col}(P_2)$. A negative value for $K_{col}(P_2)$ would mean $P_2 > P_{cr}$. Linear interpolation between $K_{col}(P_1)$ and $K_{col}(P_2)$ should provide an educated guess for the next trial $P = P_3$. The process should converge quickly to a value of $K_{col}(P_i)$ as close to zero as possible or to a value small enough to satisfy the designer's criterion for accuracy.

INTERNAL FORCES GENERATED IN CABLE-STAYED COLUMNS

Cable-stayed columns are usually hinged at both ends and braced there against lateral displacements. Actual bracing may take place either by direct attachment or through diaphragm action of the floor or roof against shear walls or X-braced frames available elsewhere in the structure. For design purposes, we emphasize two distinct stages of loading: at initial cable stressing and at ultimate cable stressing. Vertical equilibrium calls for the following equation:

$$N = P + \sum T(P)\cos\alpha \tag{4}$$

to be true at all times, where $\sum T(P)$ is the sum of the tension of all four individual cables at any given load P and α is the angle between each cable and the axis of the tube. We also consider the small variation of the angle α between the initial stage and any given value of P that is caused by column shortening. Although practically this variation is negligible, we have accounted for it by automatically updating the angle α as the load increases. We recognize that $N(P) > P$ at all times. The difference $N(P) - P$ is largest at cable prestressing ($P = 0$), and smallest at buckling ($P = P_{cr}$) when the cables are subjected to a residual tension, $T_{res} > 0$. Only for the unacceptable case when cables slacken totally as P increases (usually because of small initial prestressing) would $\sum T = 0$ and $N(P) = P$. The quantity T_{res} depends mostly on the amount of initial cable prestressing T_o, which is gradually reduced as the column shortens by a strain $\Delta\varepsilon$, while inclined cables shorten simultaneously, except by a smaller strain equal to $\Delta\varepsilon\cos^2\alpha$.

Consider now a stayed column with two tiers of cross-arms. Force equilibrium at the two end segments of this column is also given by Equation 4. For the central segment, the same equation holds true, except for $\alpha = 0$. Our previous discussion on the difference between forces N and P is applicable here as well. However, we note that the change in strain imposed on the central segment by a change in applied load, is identical to that in the corresponding steel cables. This is because being parallel to each other makes the cables and the central segment of the tube of the same length. Also for $\alpha = 0$, $\Delta\varepsilon\cos^2\alpha = \Delta\varepsilon$, which results in the tube internal force N being always somewhat larger in the central rather than in the end segments. Regardless of the number of tiers of cross-arms, it is the maximum force in the tube $N(P_{cr})$, and not the applied P_{cr}, that governs the design of the tube.

PRESTRAINING THE STAYED CABLES

The amount of cable prestraining influences the behavior of a stayed column under axial load. For any given cable-stayed column, the amount of initial straining that is imposed on the stays determines the axial strength of the column. Consider the stayed column shown in Figure 1. We may specify the amount of cable prestrain by requiring that either the cables be stretched by a certain amount (as measured by cable elongation or recovery) or, at the end of the stretching operation, they be prestrained to a given level (as measured by a strain gauge or transverse vibration device). Let us specify a prestrain $\varepsilon_{s_{spec}} = 0.002$ for the inclined cable segment measuring 468 in. We calculate the cable elongation at 0.002(468 in.) = 0.936 in. but recognize that, at the end of the cable-stretching operation, the actual prestrain in the cable would be less than 0.002 because of tube shortening from cable-induced compression. We may calculate the prestrain in the cable using the following approximate formula:

$$\varepsilon_{s_{ini}} = \frac{\varepsilon_{s_{spec}}}{\left[1 + \left(\frac{\sum A_s}{A}\right)\left(\frac{E_s\cos^2\alpha}{E}\right)\right]} \tag{5}$$

where

$\varepsilon_{s_{ini}}$ = actual initial strain in the stays at $P = 0$
$\varepsilon_{s_{spec}}$ = specified prestrain
$\sum A_s$ = total area of the cables, in.^{2}
A = area of the tube, in.^{2}
E_s and E = respective moduli of elasticity of the cable-stays and the tube, ksi
α = initial value of the angle between the cables and the axis of the tube

Substituting $\varepsilon_{s_{spec}} = 0.002$ and the values of other variables shown in Figure 1 into Equation 5, we obtain $\varepsilon_{s_{ini}} = 0.001693 < 0.002$, as expected. Compare this value to $\varepsilon_{s_{ini}} = 0.001695$ obtained using SAP 2000 or an equivalent program, which accounts not just for the shortening of the column as in Equation 5, but also for the resulting small increase in angle α, which Equation 5 would not do. The two results are practically identical.

The second option considers the specified prestrain as that required in the cable at the end of the prestretching operation—say, $\varepsilon_{s_{ini}} = 0.002$. This is the strain to be read in a strain gauge attached to the stay when we stop stretching the cable. Using Equation 5, we calculate an equivalent $\varepsilon_{s_{spec}} = 0.002363$ and a corresponding cable elongation of 1.105 in. As expected, both values for the second option are larger than those corresponding to the first option. For computational purposes, we recommend $\varepsilon_{s_{spec}}$, while for implementation in the field, the anticipated value of $\varepsilon_{s_{ini}}$ would be easier to verify for consistency. Whichever option is used must be clearly specified by the designer for proper implementation in the field. As far as our study is concerned, we shall show both corresponding values in our results. Further, we expect that a designer would specify low-relaxation cables. As such, any consideration of time-induced prestrain loss in the cables can be safely ignored.

LIMIT LOADS

We now discuss the following two limit loads, namely, 1. Load at which stay cables fully slacken; and 2. Load at which the steel tube yields. We calculate these loads to verify if any of the two might be lower than the column critical load.

1. Load at Which Stay Cables Fully Slacken

Design of a cable-stayed column is not adequate unless the stays remain taut as the load varies from zero to ultimate. We must guarantee that $P_{sl} > P_{cr}$ is satisfied by a reasonable margin, where P_{sl} is the load at which full slackening may occur and P_{cr} is the column buckling strength. Let us calculate P_{sl} now.

We first recognize that the initial tensile strain in the stay cables, $\varepsilon_{s_{ini}}$, is reduced by the load P. Also, the actual strain in the cables, $\varepsilon_s(P)$, is given as the difference between $\varepsilon_{s_{ini}}$ and the strain caused by the shortening of the tube under the load, P. Using elastic analysis, we find:

$$\varepsilon_s(P) = \varepsilon_{s_{ini}} - \frac{(P/AE)}{\left[1+\left(\frac{\sum A_s}{A}\right)\left(\frac{E_s \cos^2 \alpha}{E}\right)\right]} \tag{6}$$

where all terms have been previously identified. Consider the column example in Figure 1. Let us calculate if the stay cables are still taut for $P = P_{cr} = 343$ kips (Figure 3). We have already found $\varepsilon_{s_{ini}} = 0.001693$ after specifying $\varepsilon_{s_{spec}} = 0.002$. Substituting $P = 343$ kips and the values of all other variables in Equation 6, we find $\varepsilon_s(P) = 0.001693 - 0.000554 = 0.00114 > 0$. Thus, at $P = 343$ kips, there is residual tension in the cables at $T_r = 0.00114$ (24,000 ksi)(1 in.^{2}) = 27.36 kip/cable. As such, full cable slackening will not occur at a load smaller than the critical load. To calculate the actual value of the load $P = P_{sl}$ at which full slackening occurs, we substitute $\varepsilon_s(P_{sl}) = 0$ into Equation 6 and obtain:

$$P_{sl} = \varepsilon_{s_{ini}} EA\left[1+\left(\frac{\sum A_s}{A}\right)\left(\frac{E_s \cos^2 \alpha}{E}\right)\right] \tag{7}$$

Using Equation 7 or SAP 2000, we find $P_{sl} = 1{,}048$ kips after substituting all previous values from Figure 1 for the given column. Note this limit load is indicated by a thick vertical line in Figure 3. Because $P_{cr} = 343$ kips $< P_{sl}$, the stay cables will not fully slacken before failure. For the unacceptable case where $P_{sl} < P_{cr}$, the designer must increase the specified cable prestrain to a value large enough to guarantee compliance.

Note that for the case of a column with two cross-arms, where cable slackening may first occur at the central segment of the tube, use $\alpha = 0$ in Equations 6 and 7.

2. Load at Which Steel Tube Yields

We now consider the possibility that the steel tube for the column in Figure 1 may yield under a force, N_y, caused by a load $P_y < P_{cr}$. If such were the case, total slackening of the stay cables might occur abruptly following inelastic deformations in the steel tube. Failure would result from the column reverting to an unstayed-tube condition.

Hand calculation of P_y is possible using the following linearized relation between P_y and P_{sl}:

$$P_y = P_{sl}(N_y - N_o)/(N_{sl} - N_o) \tag{8}$$

which assumes that the cable inclination angle α remains constant. Substituting data already available to us in Equation 8, such as $N_o = 162$ kips from Table 1, $P_{sl} = N_{sl} = 1{,}048$ kips, and $N_y = 758$ kips from Figure 3, we obtain $P_y = 706$ kips. This value is also indicated by a thick vertical line in Figure 3. Because $P_y > P_{cr} = 343$ kips, the latter value remains the governing critical load.

DECIDING COLUMN CRITICAL AXIS

The core of cable-stayed columns is conventionally made of a steel tube, although we note that a solid wood core has been used in Germany (Keil, 2000). Whether hollow or solid, the core is axisymmetric and may buckle about any transverse plane that contains its axis. However, once four stay cables are attached in cruciform fashion to the core, two distinct transverse planes are created about which the column may buckle. These we have labeled *a-a* (case A) and *b-b* (case B) in Figure 1. Plane *a-a* passes through the center of the core and two of the stays. Plane *b-b* makes a 45° angle with plane *a-a*, and does not contain any of the cable stays. Both planes create corresponding axes of bending about the column cross-section.

We calculate P_{cr} for both such axes of the one cross-arm column shown in Figure 1 using SPM. Various cable prestrains are considered. The relative difference between any pair of results for cases A and B for any given prestrain is small and well within 2% of each other. Similar results are found for a stayed column with two cross-arms. For practical design purposes, the buckling strength of a stayed column may be considered the same about either of its principal axes. These findings stem from our selected numerical examples and are not the result of a parametric study.

Tacitly, however, the transverse axis about which the column section buckles is within the prerogative of the column designers, who ultimately decide the position of both pin ends and their corresponding axis of rotation. The latter are conventionally contained in the same vertical plane, one at the top and one at the bottom of the column. As such, column buckling may take place only in a vertical plane normal to the axis of rotation of both hinges. Usually, designers provide hinges made of opposing vertical steel plates that transfer the load through a steel pin placed across them. This hinge is not perfect in that it may provide a small rotational restraint, mostly generated by friction of the transverse pin against its supporting plates. This may cause the actual ultimate load to exceed P_{cr} by a margin. Thus, calculating P_{cr} as if the column were ideally double hinged is conservative.

The axis of buckling, whether it is case A or B, is determined by the designer's selection of the position of the end hinges. As a result, only P_{cr} about the actual axis of buckling need be calculated. Buckling about the other axis would call for larger values of P_{cr} because of the resulting rotational restraint at the ends. For the same reason, if the two end hinges were placed in planes normal, rather than parallel, to each other, one could expect higher values of P_{cr}. In the extreme case where fixed ends were provided, using either bolted or welded end-plate connections instead of hinges will result in even higher values of P_{cr}. Using fixed-ends to enhance P_{cr} may become more cost-effective for large slenderness of the tube. The subject of cable-stayed columns on end supports other than hinges is outside the scope of this paper.

CRITICAL LOAD

Previously we showed that cable-stayed columns may fail by cable total slackening at an axial load that causes stayed-cable action to cease and the column to revert to a slender unstayed tube. However, the most prevalent cause for failure of these naturally slender columns is at an applied load equal to P_{cr}. Even after P_{cr} is calculated, we must design the tube to resist the force $N(P_{cr})$. This, because $N_{cr} > P_{cr}$ due to the additional compression imposed by the residual tension in the cables (see Equation 4). Once $N(P_{cr})$ is calculated, the design procedure for a cable-stayed column need not be different from that provided by the AISC *Specification* (2010) for a conventional steel column.

For any given column, P_{cr} is calculated using SPM to find the lowest value, which is given by either the one-lobe symmetric or the two-lobe anti-symmetric modes of buckling. As shown in Figures 3 and 4, this example confirmed that the anti-symmetric mode gives the lowest P_{cr} and may thus prevail as the governing condition. In the following section, we check the design validity of the tube selected for Figure 1.

First, however, we study the effects of cable size and number of cross-arms on the axial strength of cable-stayed columns. We run a series of numerical evaluations for the governing buckling load for two types of columns, one with a single mid-height cross-arm and the other with two cross-arms at mid-third points. For either case, we consider four cable stays, placed orthogonally to each other, and use three specific areas of steel cable, namely, A_s = 0.5 in.^{2}, $2A_s$ = 1.0 in.^{2} and $4A_s$ = 2.0 in.^{2} per cable. Various levels of cable prestrain ranging between zero and 0.004 are used.

The results are shown in two sets of three plots each (see Figure 5). All plots show the two respective variations of P_{cr} (but N_{cr} only for governing P_{cr}) with cable prestrains $\varepsilon_{s_{spec}}$ and $\varepsilon_{s_{ini}}$. Specifically shown is the following information: (1) column buckling load, P_{cr}, for the two buckling modes, including the governing P_{cr} shown in bold; (2) compression force in the tube, N_c, for the governing mode only; and (3) two indicators of tube compressive strength—namely, Euler buckling load of the unstayed tube, $N_E = (\pi/L)^2\ EI$ = 98.86 kips as the lower limit, and yield load, $N_y = AF_y$ = 758 kips of a robust short tube as the upper limit. Each point indicates the results for either the one or two cross-arm column at a given cable prestrain. It is always true that (1) $N_{cr} > P_{cr}$ and (2) the quantity $T_{res} = N_{cr} - P_{cr}$ measures the residual tension in the stay cables. Theoretically, for a given solution to be acceptable, N_{cr} must not exceed the upper limit, N_y. However, to account for potential nonlinear behavior and uncertainties predicting column strength, we conservatively recommend that this limit be taken instead as $0.85N_y$. If the latter value were exceeded by N_{cr}, a designer should consider specifying a higher yield strength for the steel.

Engineers interested in drawing the most value from their designs may wish to use the optimum cable prestrain for that purpose. We define this prestrain as that which generates the largest P_{cr} of the governing set; see the bold variations in each plot of Figure 5. For the four plots in Figures 5b and 5c, the optimum prestrain is found at the intersection of the P_{cr} (symmetric) and the P_{cr} (anti-symmetric) buckling modes. As such, it is the prestrain at which both modes give the same value of P_{cr}. For the two plots in Figure 5a, the variations of P_{cr} do not intersect; the optimum prestrain for both plots is given by the largest value of the P_{cr} (symmetric) variations. Clearly, for the latter case, the anti-symmetric buckling mode never governs. This makes the resulting stayed columns behave as conventional columns if ever loaded to buckling. Optimum prestrain points are identified by large solid circles in Figure 5.

We recognize both quantities N_y and N_E as approximate bounds to the corresponding actual values for the lower and upper strengths, respectively, of the cable-stayed column. As such, it may not be unreasonable to take the vertical distance between N_y and N_E in Figure 5 as an approximate indicator of the maximum amount of strength enhancement caused by the transformation of a steel tube into a cable-stayed column. Justification for this transformation increases with the quantity $N_y - N_E$, as in the case of slender columns. From a strength point of view, tubes need not be transformed when

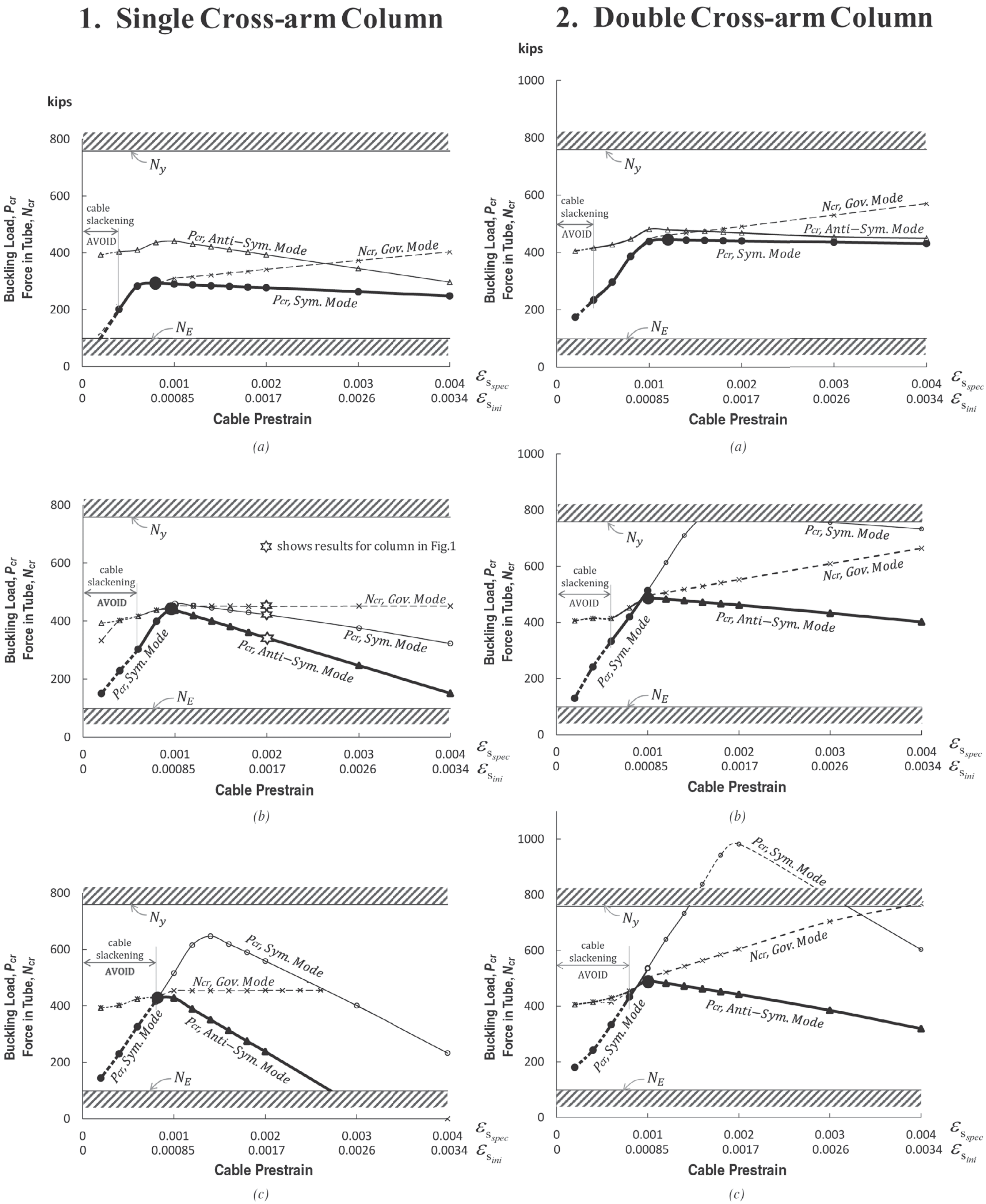


Fig. 5. Variation of P_{cr} *and* N_{cr} *with* $\varepsilon_{s_{spec}}$ *and* $\varepsilon_{s_{ini}}$ *for single and double cross-arm columns with four cables each at (a) 0.5 in.*^{2}*, (b) 1.0 in.*^{2} *and (c) 2.0 in.*^{2}*.* P_{cr} *is given for both symmetrical and anti-symmetrical modes. The governing* P_{cr} *variation is shown in bold.* N_{cr} *is only shown for the governing* P_{cr}*. Optimum cable prestrain for maximum* P_{cr} *is shown by large solid circles.*

the quantity $N_E \geq N_y$ as in the case of robust tubes. On the other hand, if $N_y > N_E$ were true for a given tube, a simple indicator (of the minimum value of slenderness for a double-hinged tube) that might justify transforming it into a cable-stayed column would be obtained as follows:

$$(L/r)_{min} = \pi\sqrt{E/F_y} \tag{9}$$

Note, however, that the additional costs in materials and labor that are associated with cable-stayed columns may make the preceding limit on slenderness substantially smaller than the actual practical value.

For the steel tube used in this study (Figure 1), we substitute E = 29,000 ksi, F_y = 42 ksi, in Equation 9 and obtain $(L/r)_{min}$ = 82.55 and L_{min} = 82.55(4.07 in.) = 336 in. Compare this to the actual L/r of the tube at 930.71 in./4.07 in. = 229 > 82.55. This difference may be enough to justify converting the given steel tube into a cable-stayed column to seek additional compressive strength. An alternative solution for using instead a larger and heavier conventional steel tube may be less attractive.

All six plots in Figure 5 show two distinct variations for P_{cr} as cable prestrain increases. There is first a steep increase to a maximum value of P_{cr} at a certain optimum cable prestrain. This is followed by a gradual decline for P_{cr} as cable prestrain increases beyond the optimum. For the initial segment, we find the quantity $T_{res} = N_{cr} - P_{cr}$ is rather small. It is only past the optimum value for P_{cr} that T_{res} increases gradually as cable prestrain increases. We have no structural concern with the latter effect unless it causes the compression force in the tube, N, to reach an unacceptable stress level under service conditions that would require using a heavier tube. There is no good reason to seek a larger prestrain for the stay cables than that necessary to obtain the optimum value for P_{cr} (Figure 5).

At the low end of cable prestrains, total slackening may result from the loss of cable tension as the applied axial load increases, thereby negating cable-stayed action for the column. Thus, all six plots in Figure 5 indicate an AVOID range for prestrain that leads to total cable slackening under the applied load. Designers should heed this advice by making sure that enough prestraining is specified to prevent premature failure.

We now examine Figure 5 again to compare the strength of stayed-columns with either one or two cross-arms for the same given cable areas and prestrain. All columns with one cross-arm provide a smaller buckling strength, case by case, than corresponding columns equipped with two cross-arms. We conclude that cable-stayed columns with two cross-arms are more efficient than those with one cross-arm because they require a smaller amount of cable area to achieve the same strength. Whether they are also more cost efficient would require a comparison accounting for the cost of additional cross-arms, connections and labor.

DESIGN AXIAL LOAD

Calculation of the critical load of a stayed column, P_{cr}, is followed by determination of its nominal compressive strength, $P_n = F_{cr}A_g$, where F_{cr} is the nominal critical stress and A_g is the gross area of the tube. We use the provisions of AISC *Specification* Chapter E, Section E3 (AISC, 2010) to calculate P_n for the one cross-arm stayed-column shown in Figure 1. Our previous calculations using SPM (see Figure 3 and Table 1) indicate that the two-lobe rather than the one-lobe buckling configuration governs the strength of the column. We obtain P_{cr} = 343 kips from Table 1. This corresponds to a compression force in the tube, N_{cr} = 452 kips, for which the compressive stress, given by N_{cr}/A = 452 kips/18.06 in.^{2} = 25 ksi. This is well within the elastic range for steel specified at 42 ksi yield strength and our recommended 0.85(42 ksi) = 37.5 ksi maximum compressive stress. There is also a residual vertical tension force in the stay cables, $\sum T\cos\alpha$ = 109 kips. These results verify $N_{cr} = P_{cr} + \sum T\cos\alpha$. We recognize that it is the force in the tube, N_{cr}, and not the applied load, P_{cr}, that controls the nominal compressive strength of the stayed column. As such, the value of AISC's design strength, F_{cr}, depends exclusively on the tube.

We may find the equivalent slenderness ratio of the cable-stayed tube, kL/r, using:

$$N_{cr} = \frac{\pi^2 EA}{\left(\frac{kL}{r}\right)^2}$$

Solving for kL/r:

$$\begin{aligned}\frac{kL}{r} &= \sqrt{\frac{\pi^2 EA}{N_{cr}}}\\ &= \sqrt{\frac{\pi^2(29{,}000\text{ ksi})(18.06\text{ in.}^2)}{452\text{ kips}}}\\ &= 107 < 200\end{aligned}$$

Because AISC's control quantity $\lambda = 4.71\sqrt{E/F_y}$ = $4.71\sqrt{29{,}000\text{ ksi}/42\text{ ksi}}$ = 123.8 > 107, we calculate F_{cr} using AISC *Specification* Equation E3-2:

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad (\textit{Spec.}\text{ Eq. E3-2})$$

where

$$F_e = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} \qquad (\textit{Spec.}\text{ Eq. E3-4})$$

$$\begin{aligned}F_e &= \frac{\pi^2(29{,}000\text{ ksi})}{(107)^2}\\ &= 25\text{ ksi}\end{aligned}$$

and then,

$$F_{cr} = \left[0.658^{\frac{42\text{ ksi}}{25\text{ ksi}}} \right](42\text{ ksi})$$
$$= 20.8\text{ ksi}$$

With this value of F_{cr}, the nominal compressive strength of the tube is:

$$N_a = F_{cr}A_g$$
$$= (20.8\text{ ksi})(18.06\text{ in.}^2)$$
$$= 375\text{ kips}$$

Let us now calculate the value of P_n on the column that would correspond to the preceding value of N_n. Hand calculation is possible using the following linearized relation between P_n and N_n, which is similar to Equation 8, except adapted to AISC design notations:

$$P_n = P_{cr}(N_n - N_o)/(N_{cr} - N_o) \qquad (10)$$

Equation 10 also assumes that the cable inclination angle α remains constant. Using Table 1, we find N_o = 162 kips, N_{cr} = 452 kips, and P_{cr} = 343 kips. Substituting these values and the preceding N_n = 375 kips into Equation 10, we find P_n = 252 kips. This value is slightly larger than the more accurate value 251.6 kips provided by computer analysis that accounts for the slight increase in the angle α due to tube shortening under load.

Designers using LRFD or ASD may now take the design compressive strength at ϕP_n = 0.90(252 kips) = 226 kips or the allowable compressive strength at P_n/Ω = 252 kips/1.67 = 151 kips, respectively. Let us compare P_n = 252 kips for the given cable-stayed column to that of the tube alone. We find kL/r = 1.0(931 in.)/(4.07 in.) = 229 > 200. While this exceeds the recommended limit of 200 for non-stayed columns as mentioned in AISC *Specification* Section E2 (AISC, 2010), it is viewed as acceptable for the purpose of strength comparison. We now calculate P_n:

$$P_n = AF_{cr}$$

Because λ = 123.8 < kL/r = 229, use AISC *Specification* Equation E3-3:

$$F_{cr} = 0.877F_e \qquad (\textit{Spec.}\text{ Eq. E3-3})$$

where,

$$F_e = \frac{\pi^2 E}{\left(\frac{kL}{r}\right)^2} \qquad (\textit{Spec.}\text{ Eq. E3-4})$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(229)^2}$$
$$= 5.46\text{ ksi}$$

and then,

$$F_{cr} = 0.877(5.46\text{ ksi})$$
$$= 4.79\text{ ksi}$$

Using this value of F_{cr}, $P_{n,tube}$ is calculated as:

$$P_{n,tube} = (18.06\text{ in.}^2)(4.79\text{ ksi})$$
$$= 86.5\text{ kips} \ll 252\text{ kips}$$

The nominal strength enhancement of the cable-stayed column over that of the tube alone is measured by the difference of the corresponding nominal strengths, which is substantial:

$$P_n - P_{n,tube} = 252\text{ kips} - 86.5\text{ kips}$$
$$= 165.5\text{kips}$$

The nominal strength enhancement ratio (*SER*) may be defined as the ratio $(P_n - P_{n,tube})/P_{n,tube}$. For the given column, we find *SER* = 165.5 kips/86.5 kips = 1.91. In other words, the relative nominal strength of the cable-stayed tube will be 2.91 times as large as that of the unstayed-tube.

At this point, a designer may question whether higher column strength could have been attained had a cable prestrain other than $\varepsilon_{s_{spec}}$ = 0.002 been chosen. It behooves the designer to do so as our previous work clearly indicates (see Figure 5, plot 1b). Thus, had the optimum prestrain at $\varepsilon_{s_{spec}}$ = 0.000888 been found first and selected for use, this new design would have provided N_o = 71.4 kips at P = 0, P_{cr} = 446 kips and N_{cr} = 449 kips from SPM. Now using the AISC *Specification* obtain F_e = 24.9 ksi, F_{cr} = 20.7 ksi, N_n = 374 kips, and finally, P_n = 357 kips. The latter value exceeds that of the previous design, P_n = 253 kips, by a substantial margin. This is due to the much smaller cable residual tension 39.2 kips at optimum prestrain $\varepsilon_{s_{spec}}$ = 0.000888 versus 139 kips at $\varepsilon_{s_{spec}}$ = 0.002. Even when a designer would likely not know the optimum cable prestrain at the outset, evaluating a few values of cable prestrain may help enhance P_n as desired.

SUMMARY AND CONCLUSIONS

1. Adding cross-arms and cables to a slender steel tube transforms it into a cable-stayed column with a substantially larger axial compressive strength. A simple approximate formula is given that evaluates the minimum tube slenderness for which this transformation may be justified. Because the formula is based only on strength considerations, and does not account for additional costs involved in the transformation, the result should be considered a low estimate.

2. A cable-stayed column with two cross-arms at the mid-third points of its height may resist a larger applied compression load for the same size cables and prestraining than an identical column with only one set of cross-arms at mid-height.

3. The amount of cable prestraining is important. Too little prestrain must be avoided to prevent total cable slackening and premature column failure under applied load. Too much prestrain may cause overstressing of the steel tube and a subsequent reduction in column strength. An optimum value for cable prestraining that provides the highest column strength for a given set of cables can be calculated.
4. Once the stay cables are pretensioned, they impose an initial compression force on the tube. The applied load increases the tube compression force but reduces the cable initial tension. The axial compression force in the tube is always larger than the applied load by the amount of residual tension in the cables. It is the axial force in the tube, and not the applied load causing column buckling, that must be used to verify the nominal strength of the cable-stayed column.
5. Premature failure caused by total cable slackening due to low prestraining can and must be avoided. Failure of cable-stayed tubed caused by buckling under increasing axial load may occur in both the symmetric (one-lobe) and anti-symmetric (two-lobe) modes. The critical load for both modes must be calculated to ascertain which one governs. Although counterintuitive to conventional column design, it is a fact that the anti-symmetric mode may govern design.
6. Buckling calculations for cable-stayed columns are easily accomplished using the stiffness probe method (SPM), which is presented here for the first time. SPM is conceptually based on the fact that the local structural stiffness at the point of application of a perturbation force (or moment) in the presence of an artifice spring (translational for a perturbation force and rotational for a perturbation moment) degrades from a maximum for the unloaded column to zero at the buckling load. This measurement of the stiffness under controlled deformations as the structure approaches instability allows for a very accurate calculation of the buckling load. SPM has been successfully used in various structures to predict elastic instability generated not just by compression loads as shown herein, but also by masses forced to vibrate at their natural frequencies.

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SYMBOLS

A	Cross-sectional area of core (tube), in.^{2}
A_{ca}	Cross-sectional area of cross-arm, in.^{2}
A_g	Gross cross-sectional area of core (tube), in.^{2}
A_s	Cross-sectional area of each stay cable, in.^{2}
E	Modulus of elasticity of core (tube), ksi
E_{ca}	Modulus of elasticity of cross-arm, ksi
E_s	Modulus of elasticity of cable, ksi
F_{cr}	Critical compressive stress used to calculate nominal strength, ksi
F_e	Critical elastic buckling stress of core (tube), ksi
F_u	Specified minimum tensile strength of the type of steel used, ksi
F_y	Specified minimum yield stress of the type of steel used, ksi
I	Moment of inertia of core (tube), in.^{4}
K_{aug}	Stiffness of augmented column, kip/in.
K_{col}	Column stiffness, kip/in.
K_{rot}	Rotational spring stiffness, kip-in./rad.
K_{spr}	Translational spring stiffness, kip/in.
L	Length of core (tube), in.
N	Compression force in tube due to applied load, kips
N_{cr}	Compression force in tube at buckling, kips
N_E	Euler buckling force for core (tube) alone, kips
N_n	Nominal design force for core (tube), kips
N_o	Axial force in core (tube) at $P = 0$, kips
N_y	Yield force for core (tube) cross-section, kips
P	Applied axial load at column ends, kips
PF	Perturbation force, kips
PM	Perturbation moment, kip-in.
P_{cr1}	Governing critical buckling load for cable-stayed column, kips
P_{cr2}, P_{cr3}	Buckling loads for unattainable modes 2 and 3, respectively, kips
P_n	Nominal compressive strength, kips
$P_{n,\,tube}$	Nominal compressive strength of unstayed-tube, kips
P_{sl}	Axial load causing cables to fully slacken ($T = 0$), kips

P_u Ultimate strength of cable-stayed column, kips

SER Strength enhancement ratio

$T(P)$ Tension force in stay cable at a given load P, kips

T_o Initial tension in stay cables, kips

T_{res} Residual tension in stay cables, kips

b_{ca} Horizontal length of cross-arm between cable attachments, in.

k Effective length factor for compression members

kL/r Equivalent slenderness ratio of cable-stayed core (tube)

r Radius of gyration of the tube, in.

Ω Safety factor

α Angle between the core (tube) and stay cables, deg. or rad.

δ Transverse displacement of the artifice translational spring, in.

ε_s Strain in cable

$\varepsilon_{S_{ini}}$ Initial strain in cable

$\varepsilon_{S_{spec}}$ Specified strain in cable

λ Control quantity used for steel column design

θ Nodal rotation of the artifice rotational spring, rad.

ϕ Strength reduction factor

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