Establishing and Developing the Weak-Axis Strength of Plates Subjected to Applied Loads

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ABSTRACT

When a plate is subjected to applied loads and significant out-of-plane deformation, the demand on the connection may exceed that derived from calculations due to the applied loads only. Where inelastic behavior is acceptable, the intent may be to ensure ductile behavior. For plates with fillet-welded edge connections, this can be accomplished by sizing the fillet welds to develop the strength of the plate.

One specific example of this arises when a brace in a special concentrically braced frame (SCBF) is subject to compression and buckles out-of-plane. Bending of the gusset plate may demand more of the gusset plate edge connection than the calculated forces that result on the gusset edge due to the brace force specified in Section F2.6c.2 of AISC 341-16 (AISC, 2016). If not accounted for in the weld size, the uncalculated weak-axis moment on the welds from out-of-plane bending of the gusset plate might cause rupture of the fillet welds to govern the behavior of the system.

The method provided in this paper is suitable to determine the minimum size of fillet welds necessary to prevent weld rupture as out-of-plane deformations occur. It can be used for fillet-welded gusset plate edges in SCBFs to satisfy the exception provided in Section F2.6c.4 of AISC 341-16.

Keywords: gusset plate, fillet welds, special concentrically braced frame, hinge, seismic, connection.

INTRODUCTION

When a plate is subjected to applied loads and significant out-of-plane deformation, the demand on the connection may exceed that derived from calculations due to the applied loads only. Where inelastic behavior is acceptable, the intent may be to ensure ductile behavior. For plates with fillet-welded edge connections, this can be accomplished by sizing the fillet welds to develop the strength of the plate.

One specific example of this arises when a brace in a special concentrically braced frame (SCBF) is subject to compression and buckles out-of-plane. Resulting weak-axis bending of the gusset plate may demand more of the gusset plate edge connection than the calculated forces that result on the gusset edge due to the brace force specified in Section F2.6c.2 of AISC 341-16 (AISC, 2016). If not accounted for in the weld size, the uncalculated weak-axis moment on the welds from out-of-plane bending of the gusset plate might cause rupture of the fillet welds to govern the behavior of the system.

Section F2.6c.4 of AISC 341-16—a new provision at the time of writing of this paper—provides a fairly simple approach that can be used to determine an appropriate fillet weld size to preclude this concern. It allows the required shear strength for the welds to be taken equal to $0.6R_yF_yt_p/\alpha_s$ (i.e., the expected shear strength of the plate), where these variables are defined in AISC 341-16. A user note is provided to further simplify this for common steel grades and double-sided fillet welds: A weld size of $0.62t_p$ is sufficient for 36-ksi gusset plates and 70-ksi weld metal, and a weld size of $0.74t_p$ is sufficient for 50-ksi gusset plates and 70-ksi weld metal.

An exception also is provided to recognize that a weak-axis flexural hinge in the gusset plate edge can be used to protect the fillet welds. It recognizes that the forces present from the brace consume a portion of the strength of the gusset plate edge. This paper explains how to calculate the portion consumed, the remainder of the gusset plate strength that must be developed, and the corresponding fillet weld size that will do so. It also shows that this approach will produce a smaller required fillet weld size than the other option provided in Section F2.6c.4 of AISC 341-16.

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Paper No. 2015-31

INTERACTION ON THE GUSSET PLATE EDGE

A yield mechanism can be used in the gusset plate to determine the maximum weak-axis bending moment that can exist in the presence of the gusset plate edge forces that result from the force in the brace. Using the generalized interaction equation recommended by Dowswell (2015), the total utilization of the gusset plate in shear, compression, and strong- and weak-axis bending can be expressed as:

$$\left(\frac{P_u}{\phi R_y P_y}\right)^2 + \left(\frac{V_u}{\phi R_y V_p}\right)^4 + \left[\left(\frac{M_{ux}}{\phi R_y M_{px}}\right)^{1.7} + \left(\frac{M_{uy}}{\phi R_y M_{py}}\right)^{1.7}\right]^{0.59} \leq 1 \tag{1}$$

where

P_u = gusset edge compression force due to brace compression force specified in section F2.6c.2, kips

V_u = gusset edge shear force due to brace compression force specified in Section F2.6c.2, kips

M_{ux} = gusset edge strong-axis moment due to brace compression force specified in Section F2.6c.2, kip-in.

M_{uy} = gusset edge weak-axis moment due to deformations from brace buckling, kip-in.

$\phi R_y P_y$ = $0.9 R_y F_y L t_p$, the expected compression strength of the gusset place edge, kips

$\phi R_y V_p$ = $1.0(0.6 R_y F_y L t_p)$, the expected shear strength of the gusset plate edge, kips

$\phi R_y M_{px} = \dfrac{0.9 R_y F_y L t_p^2}{4}$, the expected strong-axis flexural strength of the gusset plate edge, kip-in.

$\phi R_y M_{py} = \dfrac{0.9 R_y F_y L t_p^2}{4}$, the expected weak-axis flexural strength of the gusset plate edge, kip-in.

R_y = ratio of the expected yield stress to the specified minimum yield stress, F_y

F_y = specified minimum yield stress, ksi

L = length of fillet welds on gusset plate edge, in.

t_p = gusset plate thickness, in.

Because all variables in Equation 1 except M_{uy} are known for a given gusset plate, it is convenient to rewrite it at the point of equivalency as follows:

$$M_{uy\ max} = \frac{0.9 R_y F_y L t_p^2}{4}\left[\left(1 - P'^2 - V'^4\right)^{1.7} - M_x'^{1.7}\right]^{0.59} \tag{2}$$

where

$$P' = \frac{P_u}{0.9 R_y F_y L t_p} \tag{3}$$

$$V' = \frac{V_u}{0.6 R_y F_y L t_p} \tag{4}$$

$$M'_x = \frac{4 M_{ux}}{0.9 R_y F_y L^2 t_p} \tag{5}$$

Equation 2 provides the maximum weak-axis moment, $M_{uy\,max}$, that can be delivered to the welds by the gusset plate in the presence of P_u, V_u, and M_{ux}.

In the preceding formulations, the expected yield strength, $R_y F_y$ is used rather than the minimum specified yield strength, F_y. To not use R_y would reduce the denominator in these terms—and thereby also reduce the calculated value of the remaining weak-axis flexural demand, $M_{uy\,max}$, that must be developed. $R_y F_y$ also is used in the denominator of the weak-axis flexural strength ratio as provided in Section A3.2 in AISC 341-16 because all of these ratios are determined for the same element in the interaction equation.

DESIGN REQUIREMENTS FOR GUSSET PLATE EDGE FILLET WELDS

The four required strengths at the gusset plate edge (P_u, V_u, M_{ux} and $M_{uy\,max}$) can be used to design fillet welds that will fully develop the edge of the gusset plate and preclude weld rupture. All four effects produce shear on the effective throat of the fillet weld. The effects of P_u, M_{ux} and $M_{uy\,max}$ all are oriented transverse to the fillet weld, while the effect of V_u is oriented parallel to the weld axis. Accordingly, the maximum weld force per unit length due to the combination of all four effects can be expressed as follows:

$$f_u = \sqrt{f_{uv}^2 + \left(f_{up} + f_{um_x} + f_{um_y}\right)^2} \tag{6}$$

where

$$f_{uv} = \frac{V_u}{2L}, \text{ kips/in.} \tag{7}$$

$$f_{up} = \frac{P_u}{2L}, \text{ kips/in.} \tag{8}$$

$$f_{um_x} = \frac{2 M_{ux}}{L^2}, \text{ kips/in.} \tag{9}$$

$$f_{um_y} = \frac{M_{uy\ max}}{\left(t_p + 0.5w\right) L}, \text{ kips/in.} \tag{10}$$

w = weld size, in.

The corresponding weld design strength per inch is:

$$\phi r_n = 1.392D\left(1.0 + 0.5\sin^{1.5}\theta\right) \tag{11}$$

where

$$\theta = \tan^{-1}\left(\frac{f_{up} + f_{um_x} + f_{um_y}}{f_{uv}}\right) \tag{12}$$

Here, D is the number of sixteenths in the weld size, and the basic weld strength of $1.392D$ is determined as explained in Part 8 of the AISC *Steel Construction Manual* (AISC, 2011). Because the quantity in Equation 11 must equal or exceed the quantity in Equation 6, the minimum weld size can be determined as:

$$D_{min} = \frac{f_u}{1.392\left(1.0 + 0.5\sin^{1.5}\theta\right)} \tag{13}$$

This weld size is sufficient to develop the gusset plate; therefore, it is unnecessary to also apply the "Richard" 1.25 weld ductility factor (AISC, 2011), which is used to address hot spots in cases where the welds do not develop the gusset plate.

Equation 10 is based on a moment arm equal to the distance between centroids of the effective throats of the fillet welds taken at 45 degrees in the welds ($t_p + 0.5w$); see Figure 1. Although some references illustrate the use of the weld area centroids—a moment arm of $t_p + 0.67w$ in this case—the calculations in this paper are made relative to the centroids of the effective throats, and the moment arm of $t_p + 0.5w$ is used for consistency.

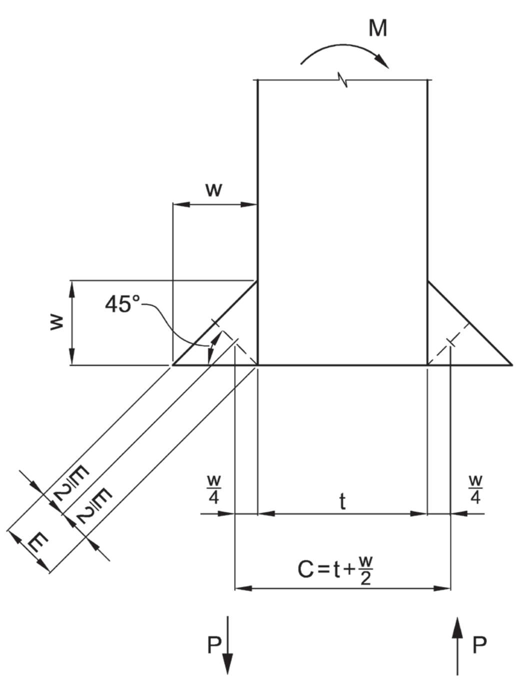

Fig. 1. Moment arm for weak-axis bending.

This equation is directly useful when checking a design, as shown in Examples 1 and 2 at the end of this paper, but requires iteration when designing a connection. Iteration can be minimized by assuming a weld size; a reasonable starting assumption is $w = t_p/2$, in which case, $t_p + 0.5w = 1.25t_p$. Alternatively, the weld size can be ignored in design and the moment arm taken as t_p if the resulting penalty is not objectionable.

COMPARISON TO TEST DATA

Table 1 shows the available testing (Johnson, 2005; Roeder, 2015) by which the suitability of the foregoing method can be judged. Predicted and actual test results are shown as GY and BR for gusset yielding and brace rupture and WR for weld rupture. The former is the desired behavior; the latter is undesirable.

Note that weld tearing as testing progresses is not preventable and should not be confused with weld rupture. The geometric deformations of the specimens during testing will cause tearing starting at the ends of the welds. As long as the specimen remains viable and the brace continues to function through the testing until the brace fractures, it is an acceptable result. The key concept here is that the weld tearing cannot be unstable and result in complete weld rupture. Rather, weld tearing must be stable so that gusset yielding can occur, and ultimately, brace rupture will limit the test.

Tests are available on both sides of the prediction point of the method, and as can be seen in Table 1, the prediction of the method provided in this paper is correct for all tests shown. The testing by Roeder (2015) and the two edge connections in test HSS 01 (Johnson, 2005) are the most relevant because they bound the prediction and all have a weld size within 1⁄16 in.—the smallest increment of weld size—of the prediction of the method.

Table 1 also shows a comparison to the use of the $0.6R_yF_yt_p/\alpha_s$ provision from Section F2.6c.4 of AISC 341-16, which is easier to use but requires a larger weld size than the method provided in this paper. Adjusting from weld size to weld volume to better reflect the impact on the cost of welding, the difference is from 125% to 300% in the cases shown in Table 1.

ASD DERIVATION

All of the foregoing information was presented using LRFD equations. Following are the similar equations for an ASD solution:

Table 1. Test Data and Comparison of Method Predition to Actual Results

				Johnson HSS 01		Johnson HSS 02		Johnson HSS 03		Johnson HSS 04		Johnson HSS 05	
	Roeder SCBF 1	Roeder SCBF 2	Roeder SCBF 3	(beam edge)	(column edge)	(beam edge)	(column edge)	(beam edge)	(column edge)	(beam edge)	(column edge)	(beam edge)	(column edge)
General Parameters													
L, in.	20	18	17.375	33	29	24	20	24	20	24.5	20.375	24	20
t_p, in.	0.5	0.625	0.625	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.375	0.375
w, in.	5⁄16	¼	¼	3⁄16	3⁄16	½	½	7⁄16	7⁄16	7⁄16	7⁄16	5⁄16	5⁄16
F_{exx}, ksi	70	70	70	70	70	70	70	70	70	70	70	70	70
F_y, ksi	50	50	50	50	50	50	50	50	50	50	50	50	50
R_y	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
Gusset Edge Forces													
P_u, kips	84.8	83.6	83.5	64.3	49.1	71.5	54.6	71.5	54.6	71.3	54.5	68.2	52.1
V_u, kips	116	106	92.8	139	124	115	97.7	115	97.7	116	100	109	93.2
M_{ux}, kip-in.	7	59.1	0	6.43	0	7.15	0	7.15	0	10.7	6	6.82	0
Calculated Parameters													
P'	0.171	0.150	0.155	0.0787	0.0684	0.120	0.110	0.120	0.110	0.118	0.108	0.153	0.140
V'	0.352	0.286	0.259	0.255	0.259	0.290	0.296	0.290	0.296	0.287	0.297	0.367	0.377
Mx'	0.00283	0.0236	0	0.000954	0	0.00201	0	0.00201	0	0.00288	0.00234	0.00255	0
$M_{uy\,max}$, kip-in.	59.1	84.4	81.6	101	88.9	72.6	60.6	72.6	60.6	74.2	61.8	40.0	33.4
f_{uv}, kips/in.	2.90	2.94	2.67	2.11	2.14	2.40	2.44	2.40	2.44	2.37	2.45	2.27	2.33
f_{up}, kips/in.	2.12	2.32	2.40	0.974	0.847	1.49	1.37	1.49	1.37	1.46	1.34	1.42	1.30
f_{umx}, kips/in.	0.0350	0.365	0	0.0118	0	0.0248	0	0.0248	0	0.0357	0.0289	0.0237	0
f_{umy}, kips/in.	4.50	6.25	6.26	5.16	5.16	4.04	4.04	4.21	4.22	4.22	4.22	3.14	3.14
f_u, kips/in.	7.26	9.41	9.07	6.49	6.38	6.05	5.93	6.21	6.09	6.18	6.10	5.12	5.02
θ, radians	1.16	1.25	1.27	1.24	1.23	1.16	1.15	1.17	1.16	1.18	1.16	1.11	1.09
D_{min}, 16ths	3.63	4.62	4.44	3.19	3.14	3.02	2.97	3.09	3.04	3.07	3.05	2.58	2.55
w_{min}, in.	¼	5⁄16	5⁄16	¼	¼	¼	3⁄16	¼	¼	¼	¼	3⁄16	3⁄16
Results													
Is $w \geq w_{min}$?	Yes	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
By how much?	1⁄16" >	1⁄16" <	1⁄16" <	1⁄16" <	1⁄16" <	¼" >	5⁄16" >	3⁄16" >	3⁄16" >	3⁄16" >	3⁄16" >	⅛" >	⅛" >
Prediction	GY & BR	WR	WR	WR	WR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR
Actual Behavior	GY & BR	WR	WR	WR	WR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR	GY & BR
Method works?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Alternative $0.6R_yF.0_yt_p/\alpha$ Comparison													
D_{min}, 16ths	5.93	7.41	7.41	5.93	5.93	5.93	5.93	5.93	5.93	5.93	5.93	4.45	4.45
w_{min}, in.	⅜	½	½	⅜	⅜	⅜	⅜	⅜	⅜	⅜	⅜	5⁄16	5⁄16
Size increase	⅛	3⁄16	3⁄16	⅛	⅛	⅛	3⁄16	⅛	⅛	⅛	⅛	⅛	⅛
% vol. increase	125%	156%	156%	125%	125%	125%	300%	125%	125%	125%	125%	178%	178%

$$\left(\frac{P_a\Omega}{R_yP_y}\right)^2+\left(\frac{V_a\Omega}{R_yV_p}\right)^4+\left[\left(\frac{M_{ax}\Omega}{R_yM_{px}}\right)^{1.7}+\left(\frac{M_{ay}\Omega}{R_yM_{py}}\right)^{1.7}\right]^{0.59}\le 1 \tag{1a}$$

where

P_a = gusset edge compression force due to brace compression force specified in Section F2.6c.2, kips

V_a = gusset edge shear force due to brace compression force specified in Section F2.6c.2, kips

M_{ax} = gusset edge strong-axis moment due to brace compression force specified in Section F2.6c.2, kip-in.

M_{ay} = gusset edge weak-axis moment due to deformations from brace buckling, kin-in.

$\dfrac{R_yP_y}{\Omega}=\dfrac{R_yF_yLt_p}{1.67}$, the expected compression strength of the gusset plate edge, kips

$\dfrac{R_yV_p}{\Omega}=\dfrac{0.6R_yF_yLt_p}{1.5}$, the expected shear strength of the gusset plate edge, kips

$\dfrac{R_yM_{px}}{\Omega}=\dfrac{R_yF_yL^2t_p}{4(1.67)}$, the expected strong-axis flexural strength of the gusset plate edge, kip-in.

$\dfrac{R_yM_{py}}{\Omega}=\dfrac{R_yF_yLt_p^2}{4(1.67)}$, the expected weak-axis flexural strength of the gusset plate edge, kip-in.

$$M_{ay\ max}=\frac{R_yF_yLt_p^2}{4(1.67)}\left[\left(1-P'^2-V'^4\right)^{1.7}-M_x'^{1.7}\right]^{0.59} \tag{2a}$$

where

$$P'=\frac{1.67P_a}{R_yF_yLt_p} \tag{3a}$$

$$V'=\frac{1.5V_a}{0.6R_yF_yLt_p} \tag{4a}$$

$$M'_x=\frac{4(1.67)M_{ax}}{R_yF_yL^2t_p} \tag{5a}$$

$$f_a=\sqrt{f_{av}^2+\left(f_{ap}+f_{am_x}+f_{am_y}\right)^2} \tag{6a}$$

where

$$f_{av}=\frac{V_a}{2L},\ \text{kips/in.} \tag{7a}$$

$$f_{ap}=\frac{P_a}{2L},\ \text{kips/in.} \tag{8a}$$

$$f_{am_x}=\frac{2M_{ax}}{L^2},\ \text{kips/in.} \tag{9a}$$

$$f_{am_y}=\frac{M_{ay\ max}}{\left(t_p+0.5w\right)L},\ \text{kips/in.} \tag{10a}$$

$$\frac{r_n}{\Omega}=0.928D\left(1.0+0.5\sin^{1.5}\theta\right) \tag{11a}$$

where

$$\theta=\tan^{-1}\left(\frac{f_{ap}+f_{am_x}+f_{am_y}}{f_{av}}\right) \tag{12a}$$

$$D_{min}=\frac{f_a}{0.928\left(1.0+0.5\sin^{1.5}\theta\right)} \tag{13a}$$

CONCLUSIONS

The method provided in this paper is suitable to determine the minimum size of fillet welds necessary to prevent weld rupture as out-of-plane deformations occur. It can be used for fillet-welded gusset plate edges in special concentrically braced frames (SCBFs) to satisfy the exception provided in Section F2.6c.4 of AISC 341-16.

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Example 1. ASTM A36 Gusset Plate Using Theoretical Derivation

Consider the gusset plate-to-beam flange fillet welds for the upper gusset plate illustrated in Figure 2 [Figure 5-33 of the AISC *Seismic Design Manual* (AISC, 2012)] and the loads for Case 2 (a 444-kip brace force; see page 5-230). Determine if the ⅜-in. fillet welds used are acceptable for the new criterion. (Note that this example in the 2nd Edition AISC *Seismic Design Manual* illustrates the requirements in the 2010 AISC *Seismic Provisions*, which did not contain a requirement like that in Section F2.6c.4 of AISC 341-16 and illustrated in this paper.)

From the example:

$L = 25.75$ in. $t_p = 0.75$ in. $F_y = 36$ ksi $R_y = 1.3$ $F_{EXX} = 70$ ksi

$V_u = 216$ kips $P_u = 193$ kips $M_{ux} = 0$ kip-in.

The maximum weak-axis flexural moment that can exist in the presence of V_u, P_u, and M_{ux} can be calculated as follows:

$$P' = \frac{P_u}{0.9R_yF_yLt_p} = \frac{193\text{ kips}}{0.9(1.3)(36\text{ ksi})(25.75\text{ in.})(0.75\text{ in.})} = 0.237\text{ kip} \tag{3}$$

$$V' = \frac{V_u}{0.6R_yF_yLt_p} = \frac{216\text{ kips}}{0.6(1.3)(36\text{ ksi})(25.75\text{ in.})(0.75\text{ in.})} = 0.398\text{ kip} \tag{4}$$

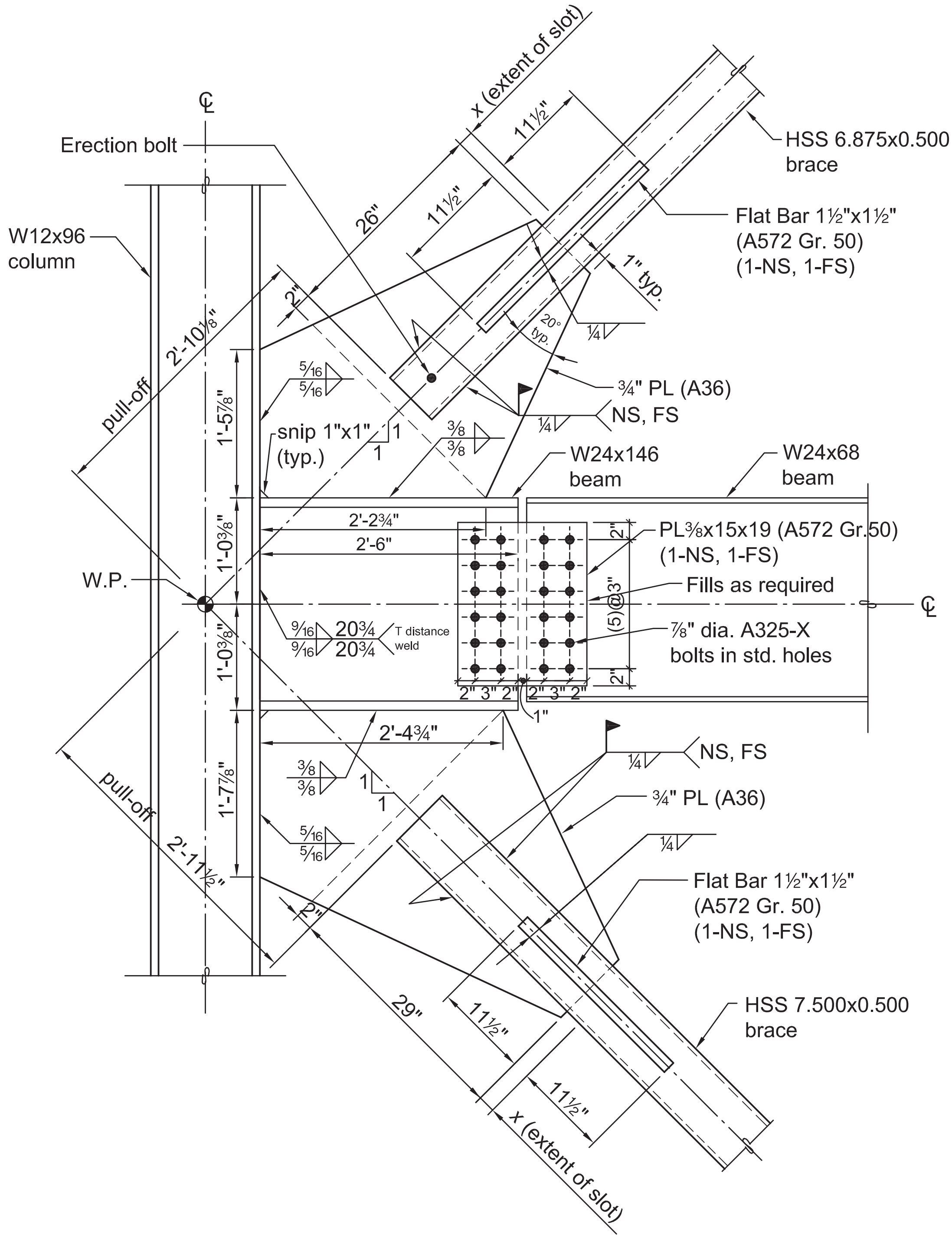
$$M'_x = \frac{4M_{ux}}{0.9R_yF_yL^2t_p} = 0\text{ kip-in.} \tag{5}$$

$$M_{uy\ max} = \frac{0.9R_yF_yLt_p^2}{4}\left[\left(1-P'^2-V'^4\right)^{1.7}-M_x'^{1.7}\right]^{0.59} = \frac{0.9(1.3)(36\text{ ksi})(25.75\text{ in.})(0.75\text{ in.})^2}{4}\left\{\left[1-\left(0.237\text{ kip}\right)^2-\left(0.398\text{ kip}\right)^4\right]^{1.7}-0^{1.7}\right\}^{0.59} = 140\text{ kip-in.} \tag{2}$$

The resultant weld force per inch can be calculated as follows:

$$f_{uv} = \frac{V_u}{2L} = \frac{216\text{ kips}}{2(25.75\text{ in.})} = 4.19\text{ kips/in.} \tag{7}$$

$$f_{up} = \frac{P_u}{2L} = \frac{193\text{ kips}}{2(25.75\text{ in.})} = 3.75\text{ kips/in.} \tag{8}$$



Note: A325-X bolts are to be pretensioned with slip-critical faying surfaces.

Fig. 2. Replication of Figure 5-33 of the AISC Seismic Design Manual, 2nd Ed.

$$f_{um_x} = \frac{2M_{ux}}{L^2} \tag{9}$$
$$= 0 \text{ kips/in.}$$

$$f_{um_y} = \frac{M_{uy\ max}}{\left(t_p + 0.5w\right)L} \tag{10}$$
$$= \frac{140 \text{ kip-in.}}{\left[0.75 \text{ in.} + 0.5\left(0.375 \text{ in.}\right)\right]\left(25.75 \text{ in.}\right)}$$
$$= 5.80 \text{ kips/in.}$$

$$f_u = \sqrt{f_{uv}^2 + \left(f_{up} + f_{um_x} + f_{um_y}\right)^2} \tag{6}$$
$$= \sqrt{\left(4.19 \text{ kips/in.}\right)^2 + \left(3.75 \text{ kips/in.} + 0 + 5.80 \text{ kips/in.}\right)^2}$$
$$= 10.4 \text{ kips/in.}$$
$$\theta = \tan^{-1}\left(\frac{f_{up} + f_{um_x} + f_{um_y}}{f_{uv}}\right)$$
$$= \tan^{-1}\left(\frac{3.75 \text{ kips/in.} + 0 + 5.80 \text{ kips/in.}}{4.19 \text{ kips/in.}}\right)$$
$$= 1.16 \text{ radians}$$

The required fillet weld size can be calculated as follows:

$$D_{min} = \frac{f_u}{1.392\left(1.0 + 0.5 \sin^{1.5}\theta\right)} \tag{13}$$
$$= \frac{\left(10.4 \text{ kips/in.}\right)}{1.392\left[1.0 + 0.5 \sin^{1.5}(1.16)\right]}$$
$$= 5.21 \text{ sixteenths}$$

The ⅜-in. fillet welds shown are adequate for the proposed criterion.

Now compare to the fillet weld size that would be required by the $0.6R_yF_yt_p/\alpha_s$ alternative provision given in Section F2.6c.4 of AISC 341-16:

$$D_{min} = \frac{0.6R_yF_yt_p / \alpha_s}{2 \times 1.392} = \frac{0.6(1.3)(36 \text{ ksi})(0.75 \text{ in.})/1.0}{2 \times 1.392} = 7.56 \text{ sixteenths}$$

These ½-in. fillet welds, though easier to determine, would require 78% more weld metal volume.

Example 2. ASTM A572 Grade 50 Gusset Plate Using Theoretical Derivation

Consider the gusset plate-to-beam flange fillet welds for the gusset plate illustrated in Figure 3 [Figure 6-1 of AISC Design Guide 29 (Muir and Thornton, 2014)] and the brace compression force of 783 kips (see page 296). Determine if the ½-in. fillet welds used are acceptable for the new criterion. (Note that this example in AISC Design Guide 29 illustrates the requirements in the 2010 AISC *Seismic Provisions*, which did not contain a requirement like that in Section F2.6c.4 of AISC 341-16 and illustrated in this paper.)

From the example:

$L = 34.25$ in. $t_p = 1$ in. $F_y = 50$ ksi $R_y = 1.1$ $F_{EXX} = 70$ ksi
$V_u = 493$ kips $P_u = 161$ kips $M_{ux} = 1{,}290$ kip-in.

The maximum weak-axis flexural moment that can exist in the presence of V_u, P_u, and M_{ux} can be calculated as follows:

$$P' = \frac{P_u}{0.9R_yF_yLt_p} \tag{3}$$

$$= \frac{161 \text{ kips}}{0.9(1.1)(50 \text{ ksi})(34.25 \text{ in.})(1 \text{ in.})}$$

$$= 0.0949 \text{ kip}$$

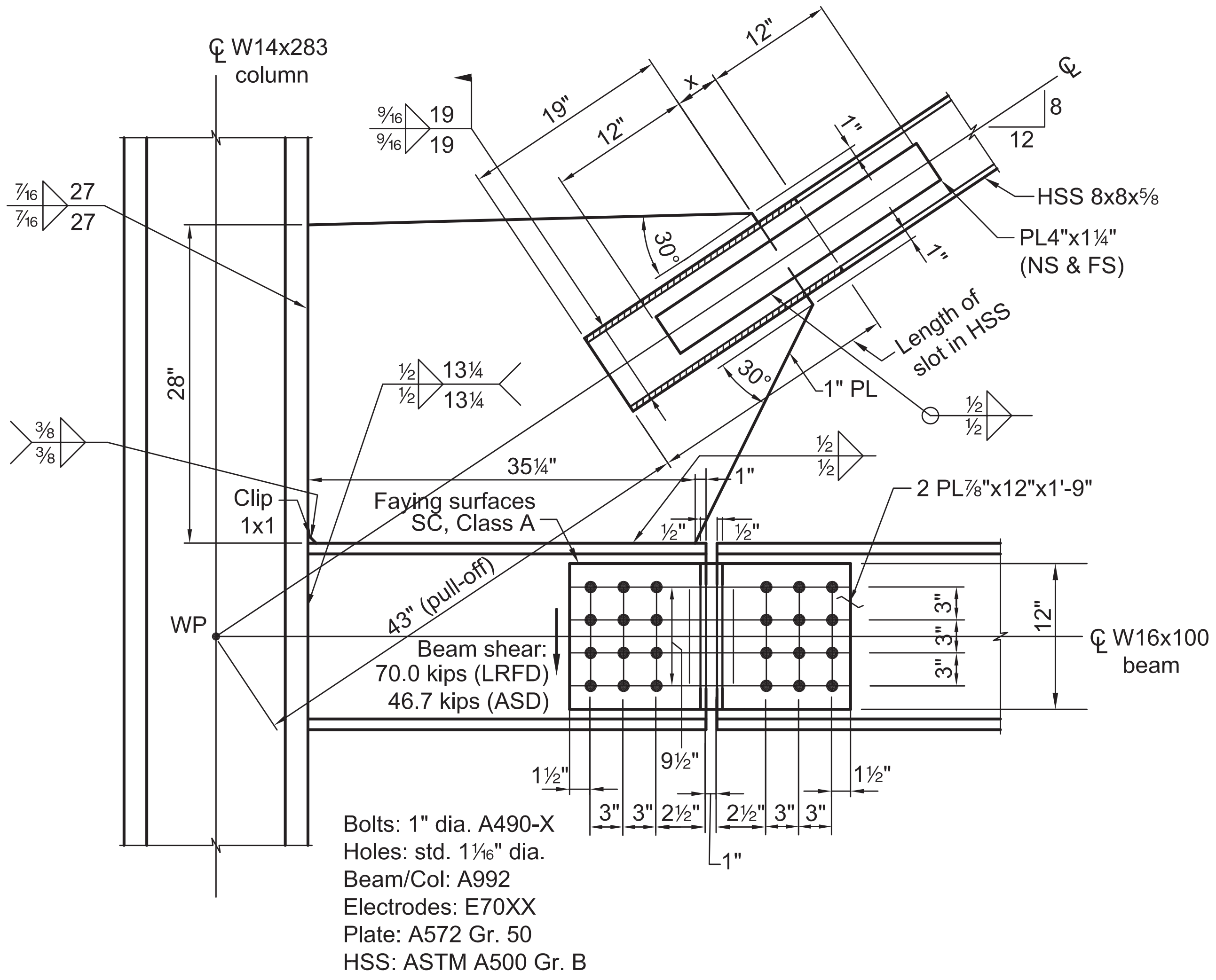


Fig. 3. Replication of Figure 6-1 of AISC Design Guide No. 29.

$$V' = \frac{V_u}{0.6R_yF_yLt_p} \tag{4}$$

$$= \frac{493 \text{ kips}}{0.6(1.1)(50 \text{ ksi})(34.25 \text{ in.})(1 \text{ in.})}$$

$$= 0.436 \text{ kip}$$

$$M'_x = \frac{4M_{ux}}{0.9R_yF_yL^2t_p} \tag{5}$$

$$= \frac{4(1{,}290 \text{ kip-in.})}{0.9(1.1)(50 \text{ ksi})(34.25 \text{ in.})^2(1 \text{ in.})}$$

$$= 0.0889 \text{ kip-in.}$$

$$M_{uy\ max} = \frac{0.9R_yF_yLt_p^2}{4}\left[\left(1-P'^2-V'^4\right)^{1.7}-M_x'^{1.7}\right]^{0.59} \tag{2}$$

$$= \frac{0.9(1.1)(50 \text{ ksi})(34.25 \text{ in.})(1 \text{ in.})^2}{4}\left\{\left[1-(0.0949 \text{ kip})^2-(0.436 \text{ kip})^4\right]^{1.7}-(0.0889 \text{ kip-in.})^{1.7}\right\}^{0.59}$$

$$= 400 \text{ kip-in.}$$

The resultant weld force per inch can be calculated as follows:

$$f_{uv} = \frac{V_u}{2L} \tag{7}$$

$$= \frac{493 \text{ kips}}{2(34.25 \text{ in.})}$$

$$= 7.20 \text{ kips/in.}$$

$$f_{up} = \frac{P_u}{2L} \tag{8}$$

$$= \frac{161 \text{ kips}}{2(34.25 \text{ in.})}$$

$$= 2.35 \text{ kips/in.}$$

$$f_{um_x} = \frac{2M_{ux}}{L^2} \tag{9}$$

$$= \frac{2(1{,}290 \text{ kip-in.})}{(34.25 \text{ in.})^2}$$

$$= 2.20 \text{ kips/in.}$$

$$f_{um_y} = \frac{M_{uy\ max}}{\left(t_p+0.5w\right)L} \tag{10}$$

$$= \frac{400 \text{ kip-in.}}{\left[1 \text{ in.}+0.5(0.5 \text{ in.})\right](34.25 \text{ in.})}$$

$$= 9.35 \text{ kips/in.}$$

$$f_u = \sqrt{f_{uv}^2+\left(f_{up}+f_{um_x}+f_{um_y}\right)^2} \tag{6}$$

$$= \sqrt{(7.20 \text{ kips/in.})^2+(2.35 \text{ kips/in.}+2.20 \text{ kips/in.}+9.35 \text{ kips/in.})^2}$$

$$= 15.7 \text{ kips/in.}$$

$$\theta = \tan^{-1}\left(\frac{f_{up} + f_{um_x} + f_{um_y}}{f_{uv}}\right)$$

$$= \tan^{-1}\left(\frac{2.35 \text{ kips/in.} + 2.20 \text{ kips/in.} + 9.35 \text{ kips/in.}}{7.20 \text{ kips/in.}}\right)$$

$$= 1.09 \text{ radians}$$

The required fillet weld size can be calculated as follows:

$$D_{min} = \frac{f_u}{1.392\left(1.0 + 0.5\sin^{1.5}\theta\right)} \tag{13}$$

$$= \frac{15.7 \text{ kips/in.}}{1.392\left[1.0 + 0.5\sin^{1.5}(1.09)\right]}$$

$$= 7.93 \text{ sixteenths}$$

The ½-in. fillet welds shown are adequate for the proposed criterion.

Now compare to the fillet weld size that would be required by the $0.6R_yF_yt_p/\alpha_s$ alternative provision given in Section F2.6c.4 of AISC 341-16:

$$D_{min} = \frac{0.6R_yF_yt_p/\alpha_s}{2 \times 1.392} = \frac{0.6(1.1)(50 \text{ ksi})(1 \text{ in.})/1.0}{2 \times 1.392} = 11.9 \text{ sixteenths}$$

These ¾-in. fillet welds, though easier to determine, would require 125% more weld metal volume.