Design of Horizontal Life Lines in Personal Fall Arrest Systems

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ABSTRACT

Personal fall arrest systems have become common in construction, maintenance, and many other activities including recreation. Many use a horizontal lifeline (HLL), often a steel cable. Their design is governed by Occupational Safety and Health Administration (OSHA) regulations that require "supervision by a qualified person" and a factor of safety of at least two. In contrast to a vertical lifeline that can be analyzed as a linear spring, a HLL has non-linear elastic behavior. The analysis is complicated by the fact that both the maximum arresting force and the geometric shape of the HLL at the stopping point are initially unknown. Some suggestions for estimating the arresting forces are known to be grossly in error for the general case. Greater confidence in a design is realized when arresting forces are found by rational analysis. This paper gives a summary of regulations, a reiterative method of analysis, a discussion of the limit states, and some appropriate modifications in the case of unacceptable behavior. The effects of assumptions used in the analysis are discussed in the conclusion.

Keywords: horizontal lifeline, personal fall arrest systems, Occupational Safety and Health Administration, harness and lanyard, steel cable, galvanized aircraft cable, life safety, fall protection.

INTRODUCTION

Personal fall arrest systems (PFASs) are a topic of great importance in the construction industry as well as recreational facilities such as high ropes challenge courses. The Occupational Safety and Health Administration (OSHA) governs the design of such systems used by employees, and few engineers or lawyers would concede that any person in the same environment should have less protection simply because they are not employees. Many component parts of PFASs are readily available in high-quality, ready-made products, partly because the loads they are expected to sustain and the way they are used have little variation from site to site, and their designs can be tested before manufacture. Such is not the case for a horizontal lifeline (HLL). Kits for constructing HLLs within certain limitations are commercially available but are not for universal use. Some rely on proprietary components such as "impact attenuators" that make their analysis difficult. HLLs should be validated for use in the specific case before they are placed in service. The engineer responsible for the specification of HLLs may want to check the design of a proposed HLL or complete a design from scratch.

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One publication by the Association for Challenge Course Technology (ACCT, 2012) referring to a PFAS using an HLL with a relatively short span (as opposed to a zipline) supported by guyed poles, states that "a single person may generate a vertical load up to 500 lbf (2.2 kN) under normal operating conditions" and goes on to analyze an HLL with 5% sag under the assumed arresting load. This HLL is similar to the one analyzed in Example 2 herein, where the calculated arresting force of 1.57 kip does not agree with the arresting force recommended in the ACCT standard.

In a field test performed on August 26, 2014 (Jacobs, private correspondence), that used a plastic dummy filled with 300 lb of steel ball bearings and water arrested by an HLL of newly installed, non-prestretched ⅜-in. galvanized aircraft cable (GAC) with a specified initial sag of 6% spanning 25.7 ft between guyed poles, the arresting forces recorded by a load cell for three drops were 763, 862 and 807 lb. The field test does not agree with the arresting force recommended in the ACCT standard.

Designers looking for guidance may find statements such as, "If the lifeline is tight, it won't sag much when a fall occurs, but the impact force on the lifeline will be high." This appeared in a text used for training (OSHAcademy, n.d.) until a recent modification. Possibly, this refers to tension in the HLL, but, even as a rule of thumb, it is untrue and possibly dangerous if interpreted as the arresting force on the falling person.

Perceiving a need for clarity, a rational method for design of HLLs adaptable to many configurations is presented herein with examples.

DEFINITIONS AND DESIGN CRITERIA

Not every provision of OSHA 1926 (n.d. a), which governs many details in a PFAS, will be included in this paper. The focus will be on compliance with OSHA 1926.502(d)(8): "Horizontal lifelines shall be designed, installed, and used, under the supervision of a qualified person, as part of a complete personal fall arrest system, which maintains a safety factor of at least two." Referring to OSHA 1926.32(m), "Qualified" means one who, "by possession of a recognized degree, certificate, or professional standing, or who by extensive knowledge, training, and experience, has successfully demonstrated his ability to solve or resolve problems relating to the subject matter, the work, or the project." The design of HLLs is within the discipline of structural engineering in that "safe design and construction require that loads and stresses must be computed and the size and strength of parts determined by mathematical calculations based upon scientific principles and engineering data" (State of Illinois, 1989).

To describe a PFAS, these three definitions appear in the OSHA (n.d. b) glossary:

> Personal fall arrest system: A system used to stop an employee in a fall from a working level. It consists of an anchorage, connectors, a body harness, and may include a lanyard, deceleration device, lifeline, or suitable combinations of these.
>
> Lanyard: A flexible line of rope, wire rope, or strap which generally has a connector at each end for connecting the body belt or body harness to a deceleration device, lifeline, or anchorage.
>
> Lifeline: A component consisting of a flexible line connected vertically to an anchorage at one end (vertical lifeline), or connected horizontally to anchorages at both ends (horizontal lifeline), and which serves as a means for connecting other components of a personal fall arrest system to the anchorage.

These are a few pertinent OSHA (n.d. a) requirements regarding free fall, arresting distance and force:

> 1926.502(d)(12) Self-retracting lifelines and lanyards which automatically limit free fall distance to 2 feet (0.61 m) or less shall be capable of sustaining a minimum tensile load of 3,000 pounds (13.3 kN) applied to the device with the lifeline or lanyard in the fully extended position.
>
> 1926.502(d)(16) Personal fall arrest systems, when stopping a fall, shall:
>
> > 1926.502(d)(16)(ii) limit maximum arresting force on an employee to 1,800 pounds (8 kN) [about 6 g's] … ;
> >
> > 1926.502(d)(16)(iii) be rigged such that an employee can neither free fall more than 6 feet (1.8 m), nor contact any lower level;
> >
> > 1926.502(d)(16)(iv) bring an employee to a complete stop and limit maximum deceleration distance an employee travels to 3.5 feet (1.07 m); and …
> >
> > 1926.502(d)(16)(v) have sufficient strength to withstand twice the potential impact energy of an employee free falling a distance of 6 feet (1.8 m), or the free fall distance permitted by the system, whichever is less.
>
> Note: If the personal fall arrest system meets the criteria and protocols contained in Appendix C to subpart M, and if the system is being used by an employee having a combined person and tool weight of less than 310 pounds (140 kg), the system will be considered to be in compliance with the provisions of paragraph (d)(16) of this section. If the system is used by an employee having a combined tool and body weight of 310 pounds (140 kg) or more, then the employer must appropriately modify the criteria and protocols of the Appendix to provide proper protection for such heavier weights, or the system will not be deemed to be in compliance with the requirements of paragraph (d)(16) of this section.

DESIGN OVERVIEW

Consider a person fitted with a harness connected by a lanyard running along a horizontal lifeline anchored at both ends to supporting structures that may be rigid or elastic, thus protected from falling to a surface below his position at work. There is usually some slack in both the lanyard and the HLL, allowing some distance of vertical free fall from initial position until the slack is taken up. From this point, the arresting (or decelerating) force increases through the arresting distance from zero to a maximum value at the stopping point. The shape of a HLL under its own weight is a catenary, changing to a shallow V when the slack is taken up, and a deepening V as the fall is arrested. The HLL is a nonlinear spring. After reaching the low point, there will be some rebound and dissipation of energy, reducing the dynamic forces to zero. This subsequent behavior is not investigated because all the forces within the PFAS are greatest at the low point.

The design problem is stated thus: Given the weight of the falling person, the free-fall distance, and the spring constants of the supports, determine the proper material properties, span and initial sag of the HLL so that the arresting distance and maximum arresting force are acceptable and the limit states of the HLL are not exceeded. The method of solution given here will assume a value of tension in the HLL, determine the deflected shape that satisfies statics and compatibility, and then check that limit states (including restrictions on arresting force and stopping distance) are satisfied. All objects and parts of the system are assumed to be stationary at the onset of a fall and again at the instant of lowest position when the fall is arrested. At these two instants, the kinetic energy is zero. The change in potential energy will be compared with the change in strain energy within the PFAS. The principle of conservation of energy requires that these two must be equal. If not, the assumed tension in the HLL is adjusted until the correct solution is found. Assumptions, including those already stated, are as follows:

- The weight of the HLL is small compared with the falling person and may be treated as a concentrated load equal to half the cable weight coincident with the location of the falling object, or even neglected.
- Persons are represented by a rigid object having mass at a single point.
- Except for the falling object, inertia within the PFAS is ignored.
- Kinetic energy is zero at the onset of a fall and at the low point.
- No energy is dissipated before reaching the low point of a fall.
- Lanyards, ropes and cables, and horizontal supports of HLLs are linearly elastic.
- The tension in a HLL is the same throughout its length.
- Only one object falls at any given time.
- The spring constant of lanyards is large enough that strain energy within a lanyard can be neglected.
- The trajectory of a falling object and its lanyard are vertical and located at midspan of the HLL.
- There are no other objects supported by a HLL at the time of a fall.

Falls may occur anywhere in the span of an HLL. There are reports that one person falling may precipitate subsequent falls by one or more others. It is not unreasonable to suppose more conservatively that a lifeline may support some previously fallen person while another fall is arrested.

The designer can specify the cable used for the HLL, but there may be a strong preference for some product available from existing stock. The supports may be at predetermined points. Supports may be rigid, or there may be a horizontal spring constant associated with the supports, especially with support by poles or long HLLs passing through intermediate vertical supports. The body harness and lanyard may be the product preferred by the user. The designer should consider the worst case.

A designer may have more discretion in specifying the sag of the HLL. Too little sag results in high cable tension, even for small loads. Too much may result in excessive arresting force. From experience, 10 to 12% of span is needed for the initial V-shaped sag at midspan. This is converted to unloaded catenary sag for installation and perhaps for a specification for installation tension in the unloaded HLL, measurable by a mechanical tensionometer. Installations with less initial sag, 3% or less, are possible, depending on loads and load combinations, configuration and use of special equipment. A breakaway retainer may be used to tighten the HLL until a fall occurs. Whatever the initial sag, its effect should be determined by analysis.

Galvanized aircraft cable (rarely used in aircraft) may be chosen for HLLs because of its flexibility and other desirable properties. GAC 7 × 19 meeting ASTM A1023 (ASTM, 2009) and federal specification RR-W-410E (GSA, 2007) is available in diameters from 3⁄32 to 3⁄8 in. The 7 × 19 construction means there are 7 strands of 19 wires each in the cable. The wires should be individually galvanized before they are assembled in the cable. Swaged connections can develop 100% of the tensile breaking strength; other mechanical connections can develop not more than 80%.

The properties of rope and cable used in HLLs are not the same as the properties of solid bars of the same material. Because of the spaces between the fibers, the effective cross-section area may be something like 60% of the gross area. The nominal area is the area of a circumscribed circle. The nominal area is greater than the metallic area. Because the arrangement of fibers or wires tightens under tension, the effective modulus of elasticity of a steel cable may be as low as 18,000 to 20,000 ksi. In addition to elastic behavior, there is some amount of inelastic elongation of newly made cable due to compaction when first loaded, perhaps 1%. If prestretched cable is not specified, the initial sag can be made somewhat less to compensate. For the analysis presented herein, the most important properties are the metallic area and the effective modulus of elasticity, or their product (called AE in this paper), and the tensile breaking strength. The value of AE may be obtained from the manufacturer, or by testing, preferably before analysis. The ASTM International standards A931, "Standard Test Method for Tension Testing of Wire Ropes and Strand" (ASTM, 2013); A603, "Standard Specification for Zinc-Coated Steel Structural Wire Rope" (ASTM, 2014a); and A586, "Standard Specification for Zinc-Coated Parallel and Helical Steel Wire Structural Strand" (ASTM, 2014b), are pertinent. The required value (not the minimum value) of AE should be specified by the designer, and certification of conformance should be required. Compliance ultimately depends on testing. Methods for estimating AE are available from suppliers but not recommended. If estimated values of AE are used, expedited redesign might be necessary later when confronted by the properties of the material actually available, which may be inconvenient at that time.

While every configuration will have a solution to the energy equation, the limit states may not be satisfied in some cases. If acceptable limit states cannot be achieved by changing the initial sag or using a different cable, the introduction of devices acting as springs (hereinafter called springs) and dampers in HLLs and lanyards may give better results. Springs in lanyards and dampers are discussed but not analyzed in this paper.

PROCEDURE FOR ANALYSIS

Determine the configuration of the HLL and gather the information needed to begin the analysis. The equations given with the following definitions are for the arresting force of an object falling at midspan of an HLL. Referring to Figures 1, 2 and 3:

1. The following are known when the analysis begins:
   - AE = HLL cable property
     = metallic area times the effective modulus of elasticity, kips
   - Do = free-fall distance, same as slack in PFAS lanyard, ft
   - Ho = initial span of HLL, ft
   - So = initial V-shaped sag at rest, ft
   - Tn = cable breaking strength, kips
   - Wo = weight of a falling person with equipment, kips
2. Find the initial length of the unloaded cable, Lo, and the cable stiffness, Ke.

$$Lo = \sqrt{4(So)^2 + (Ho)^2} \tag{1}$$

$$Ke = \frac{1}{\left(\dfrac{Lo}{AE} + \dfrac{1}{Ks}\right)}, \text{ if spring is included} \tag{2}$$

$$Ke = \frac{AE}{Lo}, \text{ if spring is not included} \tag{3}$$

where
Ks = stiffness of a spring included in a HLL, kip/ft

3. Select a trial value of T, the cable tension when the fall is arrested. This may be a first guess or a better estimate based on previous iterations of the solution.
4. Find the elongation, e, and cable length, L, under load.

$$e = \frac{LoT}{AE} \tag{4}$$

$$L = e + Lo = Lo\left(1 + \frac{T}{AE}\right) \tag{5}$$

5. Find the horizontal reactions, F.

$$F = T\frac{Ho}{L} \tag{6}$$

[Use Equation 6 for Figures 1 and 2. Use Equation 17 for Figure 3.]

6. Find the vertical reactions, R, and the arresting force, P.

$$P = 2R = 2\sqrt{T^2 - F^2} \tag{7}$$

7. Find the distance, H, between loaded supports, which may be elastic.

$$H = Ho - F\left(\frac{1}{K_1} + \frac{1}{K_2}\right) \tag{8}$$

where
K_1, K_2 = stiffness of horizontal supports, kip/ft

8. Find the sag, S.

$$S = \frac{\sqrt{L^2 - H^2}}{2} \tag{9}$$

9. Find the strain energy, U.

$$U = \left(\frac{1}{2}\right)\left[\frac{T^2}{Ke} + F^2\left(\frac{1}{K_1} + \frac{1}{K_2}\right)\right] \tag{10}$$

10. Find the change in potential energy, W.

$$W = Wo(Do + S - So) \tag{11}$$

11. Compare U and W. They will be equal when the solution is found.
    - If $U > W$, return to step 3 and decrease the trial tension T.
    - If $U < W$, return to step 3 and increase the trial tension T.
    - If $U = W$ (or close enough), the value of T is correct. Go on to check the limit states.

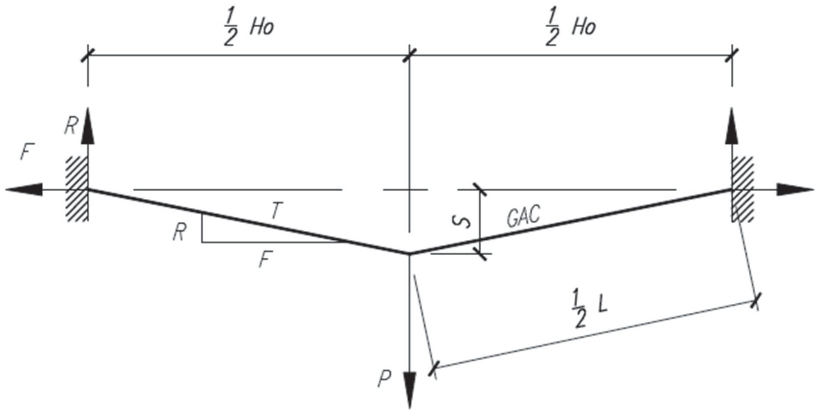

Fig. 1. HLL for Example 1.

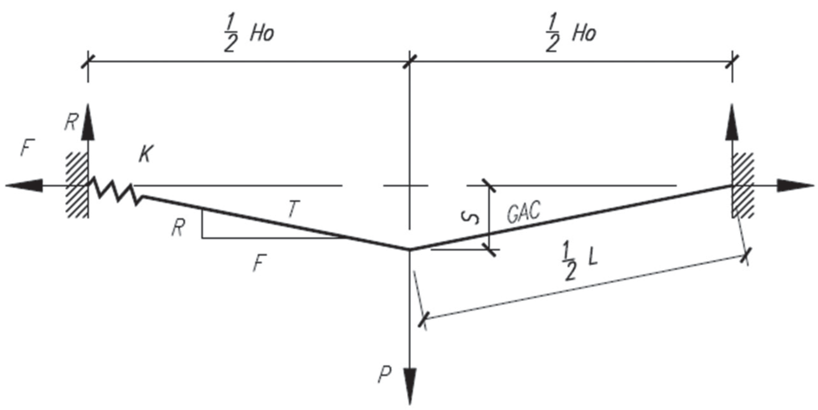

Fig. 2. HLL with spring.

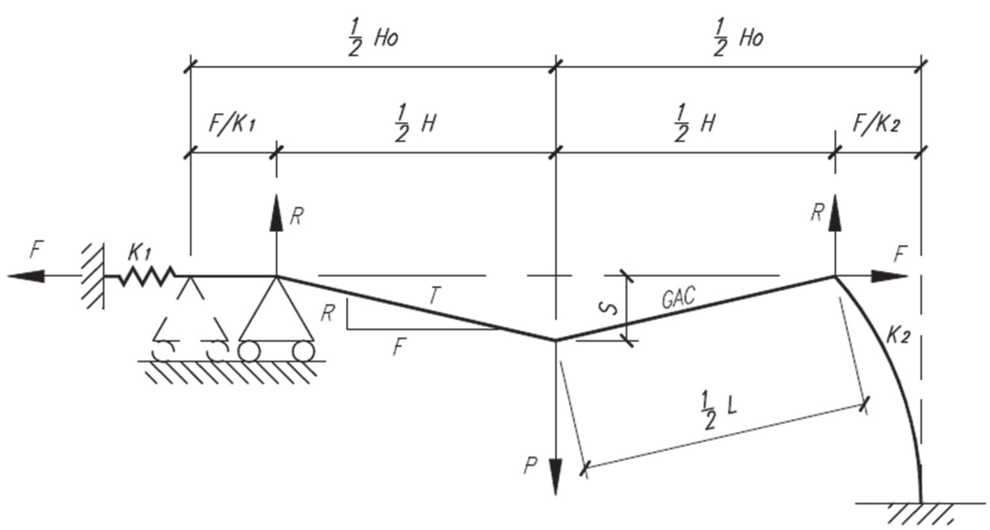

Fig. 3. HLL with elastic horizontal supports.

12. Use T to check the limit states.

   a. If $Tn/2 > T$ the cable strength is o.k. Hardware connecting the cable to its supports would also be checked.

   b. Stopping distance $Ds = S - So$ is o.k. if less than 3.5 ft.

   c. Free fall is the slack in the lanyard. Free fall is o.k. if slack Do is less than 6 ft.

   d. The total fall $(Do + Ds)$ is o.k. if it is less than the distance to obstacles.

   e. Arresting force P is o.k. if it is less than 1,800 lb.

Because there may be many iterations of the solution before converging to the correct value of T and because a number of configurations may need to be analyzed before arriving at an acceptable design, a spreadsheet is suggested. A spreadsheet should allow changing the initial trial value of T and the increment in T, testing perhaps 16 values of T at a time.

Example Problem 1

Given:

A person is connected by a body harness and lanyard to an HLL of ⅜-in.-diameter 7 × 19 GAC; breaking strength = 14.4 kips, effective rigidity AE = 1,004 kips, span = 30 ft and weight = 310 lb, including equipment (consistent with OSHA). The HLL is connected to rigid horizontal and vertical supports as shown in Figure 1. Slack in the lanyard and HLL will be taken up after a free fall of 2 ft. The initial V-shaped sag is 3.0 ft when the slack is taken up. Analyze the system for a fall at midspan and determine the catenary sag and tension for installation. Ignore the weight of the cable.

1. The following are known when the analysis begins:

   AE = 1,004 kips
   Do = 2.00 ft free fall
   Ho = 30.0 ft
   So = 3.00 ft = 10% H_o
   Tn = 14.4 kips for ⅜-in.-diameter 7 × 19 GAC, ASTM A1023
   Wo = 0.310 kips, consistent with OSHA for a person with equipment

Solution

2. Find the length of the unloaded cable and find the cable stiffness using Equations 1 and 3:

$$\begin{aligned} Lo &= \sqrt{4(So)^2 + (Ho)^2} \\ &= \sqrt{4(3.00\text{ ft})^2 + (30.0\text{ ft})^2} \\ &= 30.5941\text{ ft} \end{aligned}$$

$$\begin{aligned} Ke &= \frac{AE}{Lo} \\ &= \frac{1{,}004\text{ kips}}{30.5941\text{ ft}} \\ &= 32.8168\text{ kip/ft} \end{aligned}$$

3. Try a cable tension of T = 7.20 kips when the fall is arrested. This may be a first guess or a better estimate based on previous iterations of the solution.

4. Find the elongation and cable length under load using Equations 4 and 5:

$$e = \frac{LoT}{AE}$$
$$= \frac{(30.5941 \text{ ft})(7.20 \text{ kips})}{1{,}004 \text{ kips}}$$
$$= 0.2194 \text{ ft}$$

$$L = e + Lo$$
$$= 0.2194 \text{ ft} + 30.5941 \text{ ft}$$
$$= 30.8135 \text{ ft}$$

5. Find the horizontal reactions using Equation 6:

$$F = T\frac{Ho}{L}$$
$$= (7.20 \text{ kips})\left(\frac{30.0 \text{ ft}}{30.8135 \text{ ft}}\right)$$
$$= 7.0099 \text{ kips}$$

6. Find the arresting force using Equation 7:

$$P = 2\sqrt{T^2 - F^2}$$
$$= 2\sqrt{(7.20 \text{ kips})^2 - (7.0099 \text{ kips})^2}$$
$$= 3.2870 \text{ kips}$$

7. Supports are immoveable:

$$H = Ho$$
$$= 30.0 \text{ ft}$$

8. Find the sag using Equation 9:

$$S = \frac{\sqrt{L^2 - H^2}}{2}$$
$$= \frac{\sqrt{(30.8135 \text{ ft})^2 - (30.0 \text{ ft})^2}}{2}$$
$$= 3.5168 \text{ ft}$$

9. Find the strain energy using Equation 10:

$$U = \frac{1}{2}\left(\frac{T^2 Lo}{AE}\right)$$
$$= \frac{1}{2}\left(\frac{(7.20 \text{ kips})^2 (30.5941 \text{ ft})}{1{,}004 \text{ kips}}\right)$$
$$= 0.7898 \text{ kip-ft}$$

Table 1. Example 1 *T* Trials							
T **(kips)**	***F*** **(kips)**	***P*** **(kips)**	***L*** **(ft)**	***H*** **(ft)**	***S*** **(ft)**	***U*** **(kip-ft)**	***W*** **(kip-ft)**
7.100	6.913	3.236	30.810	30.00	3.510	0.768	0.778
7.150	6.962	3.261	30.812	30.00	3.514	0.779	0.779
7.200	7.010	3.287	30.814	30.00	3.517	0.790	0.780

10. Find the change in potential energy using Equation 11:

$$
\begin{aligned}
W &= Wo(Do + S - So) \\
&= (0.310 \text{ kips})(2.00 \text{ ft} + 3.5168 \text{ ft} - 3.00 \text{ ft}) \\
&= 0.7802 \text{ kip-ft}
\end{aligned}
$$

11. Because $U > W$, the trial value of T is too great. Other values of T may be tested by returning to step 3. The energy equation balances within 2%; try other values of T using an increment less than $2\%\ T/2$, say, 0.05 kip. The results of trials of nearby values of T made with a spreadsheet are summarized in Table 1. From these results, we see that for values of $T < 7.15$ kip, the calculated strain energy U is less than the change in potential energy W. For values of $T > 7.15$ kip, the calculated strain energy U is greater than the change in potential energy W. The correct value of T is 7.15 kip, resulting in $U = W = 0.779$ kip-ft.

12. The following limit states will be checked using $T = 7.15$ kip.

a. The cable strength is checked:

$Tn/2 = 7.20 \text{ kip} > T = 7.15 \text{ kips}$ **o.k.** (but very close)

b. Stopping distance is checked:

$$
\begin{aligned}
Ds &= S - So \\
&= 3.514 \text{ ft} - 3.00 \text{ ft} \\
&= 0.514 \text{ ft} < 3.50 \text{ ft} \quad \textbf{o.k.}
\end{aligned}
$$

c. Free fall is limited by lanyard slack adjustment:

$Do = 2.00 \text{ ft} < 6.00 \text{ ft}$ **o.k.**

d. The total fall ($Do + Ds$) must be checked against distance to obstacles:

$Do + Ds = 2.00 \text{ ft} + 0.514 \text{ ft} = 2.514 \text{ ft}$

e. The arresting force is checked:

$P = 3.261 \text{ kip} = 3{,}261 \text{ lb} > 1{,}800 \text{ lb.}$ **N.G.**

The arresting force is too high. Reducing the initial sag will reduce the arresting force but increase the cable tension, which is already very close to the limit. The arresting force can also be reduced by introducing a spring in the lanyard or the HLL, in the horizontal supports of the HLL or by damping (e.g., using a manufactured load limiting energy dissipating device). Manufactured damping devices often operate on the principle that deforming or ripping material dissipates energy, and the length of the device is extended in the process. The energy dissipated by a damping device would be included in U, and the elongation associated with a damping device would be considered in calculating W. The force required to activate the device, the energy dissipated, and the maximum extension would be specified by the manufacturer.

Springs in the Horizontal Lifeline

If a spring is introduced in the HLL attached to rigid supports as in Figure 2, the arresting force will be softened. The effective HLL stiffness, *Ke*, of this combination of spring stiffness, *Ks,* and cable stiffness, *AE/Lo,* is given by:

$$Ke = \frac{1}{\left(\frac{Lo}{AE} + \frac{1}{Ks}\right)} \tag{12}$$

The analysis can proceed as in Example 1 using this modified value of *Ke*.

To realize a spring in the HLL, the HLL may take the form of a loop running through sheaves (pulleys) at the two supports. This configuration would nearly double the length of cable and reduce *Ke* by nearly one-half. Other configurations to increase the length of cable are also possible.

Elasticity in the Horizontal Supports

Elasticity in one or both horizontal supports where the horizontal reaction causes the vertical support to move also softens the arresting force. This is the case of an HLL supported by vertical elastic poles. A vertical elastic pole may be a column fixed at the base and free at the top, or it may be guyed at the top. The right support in Figure 3 is a vertical elastic pole, where deflection at the top is proportional to force. The horizontal span of the cable changes under load, and the slope of the cable and the horizontal reaction cannot be determined as in Example 1. It is useful to develop an equation to quickly evaluate the horizontal reaction.

Suppose the horizontal supports have spring constants K_1 and K_2. Recall that *L* is the length of the elongated cable having tension *T*, and let *H* be the cable span after movement of the supports.

The horizontal reaction is determined:

$$F = T\left(\frac{H}{L}\right) \tag{13}$$

$$FL = TH \tag{14}$$

The cable span after movement of support due to *F* is:

$$H = Ho - F\left(\frac{1}{K_1} + \frac{1}{K_2}\right) \tag{15}$$

Substituting *H* from Equation 15 into Equation 14:

$$FL = T\left[Ho - F\left(\frac{1}{K_1} + \frac{1}{K_2}\right)\right] \tag{16}$$

Solving for *F*:

$$F = \frac{Ho}{\left(\frac{L}{T} + \frac{1}{K_1} + \frac{1}{K_2}\right)} \tag{17}$$

Example Problem 2

Given:

Repeat Example 1, but one support has a horizontal spring constant of $K_2 = 2.00$ kip/ft and the initial sag is 5%.

1. The following are known when the analysis begins:
   - AE = 1,004 kips (must be obtained from the manufacturer or determined by testing)
   - Do = 2.00 ft (free-fall)
   - Ho = 30.0 ft
   - K_1 = ∞
   - K_2 = 2.00 kip/ft
   - So = 1.50 ft (= 5% *Ho*)
   - Tn = 14.4 kips for ⅜-in.-diameter 7 × 19 GAC, ASTM A1023
   - Wo = 0.310 kips

Solution:

2. Find initial length of cable using Equation 1:

$$Lo = \sqrt{4(So)^2 + (Ho)^2}$$
$$= \sqrt{4(1.50\text{ ft})^2 + (30.0\text{ ft})^2}$$
$$= 30.1496\text{ ft}$$

3. Try a cable tension of $T = 2.60$ kips when the fall is arrested.
4. Find the elongation and cable length under load using Equations 4 and 5:

$$e = \frac{LoT}{AE}$$
$$= \frac{(30.1496\text{ ft})(2.60\text{ kips})}{1{,}004\text{ kips}}$$
$$= 0.0781\text{ ft}$$

$$L = e + Lo$$
$$= 0.0781\text{ ft} + 30.1496\text{ ft}$$
$$= 30.2277\text{ ft}$$

5. Find the horizontal reactions using Equation 17:

$$F = \frac{Ho}{\left(\frac{L}{T} + \frac{1}{K_1} + \frac{1}{K_2}\right)}$$
$$= \frac{30.0\text{ ft}}{\left[\left(\frac{30.2277\text{ ft}}{2.60\text{ kips}}\right) + \left(\frac{1}{\infty}\right) + \left(\frac{1}{2.00\text{ kip/ft}}\right)\right]}$$
$$= 2.4741\text{ kips}$$

6. Find the arresting force using Equation 7:

$$P = 2\sqrt{T^2 - F^2}$$
$$= 2\sqrt{(2.60\text{ kips})^2 - (2.4741\text{ kips})^2}$$
$$= 1.5985\text{ kips}$$

7. Find the reduced span using Equation 15:

$$H = Ho - F\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$
$$= 30.0\text{ ft} - 2.4741\text{ ft}\left(\frac{1}{\infty} + \frac{1}{2.00\text{ kip/ft}}\right)$$
$$= 28.7629\text{ ft}$$

Table 2. Example 2 Results							
T **(kips)**	***F*** **(kips)**	***P*** **(kips)**	***L*** **(ft)**	***H*** **(ft)**	***S*** **(ft)**	***U*** **(kip-ft)**	***W*** **(kip-ft)**
2.55	2.428	1.556	30.226	28.786	4.610	1.572	1.584
2.60	2.474	1.599	30.228	28.763	4.648	1.632	1.596
2.65	2.519	1.643	30.229	28.740	4.685	1.692	1.607

8. Find the sag using Equation 9:

$$
\begin{aligned}
S &= \frac{\sqrt{L^2 - H^2}}{2} \\
&= \frac{\sqrt{(30.2277\text{ ft})^2 + (28.7629\text{ ft})^2}}{2} \\
&= 4.6478\text{ ft}
\end{aligned}
$$

9. Find the strain energy in the cable and horizontal supports using Equation 10:

$$
\begin{aligned}
U &= \left(\frac{1}{2}\right)\left[\frac{T^2 Lo}{AE} + F^2\left(\frac{1}{K_1} + \frac{1}{K_2}\right)\right] \\
&= \left(\frac{1}{2}\right)\left[\frac{(2.60\text{ kips})^2(30.1496\text{ ft})}{1{,}004\text{ kips}} + (2.4741\text{ kips})^2\left(\frac{1}{\infty} + \frac{1}{2.00\text{ kip/ft}}\right)\right] \\
&= 1.6318\text{ kip-ft}
\end{aligned}
$$

10. Find the change in potential energy using Equation 11:

$$
\begin{aligned}
W &= Wo(Do + S - So) \\
&= (0.310\text{ kips})(2.00\text{ ft} + 4.6478\text{ ft} - 1.50\text{ ft}) \\
&= 1.5958\text{ kip-ft}
\end{aligned}
$$

11. The energy equation balances within 3%. Other values of T may be tried by returning to step 3. The result of trials of nearby values of T made with a spreadsheet are summarized in Table 2. The energy equation will balance when T is between 2.55 and 2.60 kips. Either of these may be used to check the limit states, or T can be computed as precisely as desired by returning to step 3 for further reiteration. Were this to be done, the more precise value of $T = 2.57$ kip would be confirmed.

12. $T = 2.60$ kip will be used to check the limit states.

    a. The cable strength is checked:

    $Tn/2 = 7.20\text{ kips} > T = 2.60\text{ kips}$ **o.k.**

    b. The stopping distance is checked:

$$
\begin{aligned}
Ds &= S - So \\
&= 4.6478\text{ ft} - 1.50\text{ ft} \\
&= 3.1478\text{ ft} < 3.50\text{ ft}
\end{aligned}
$$

o.k.

Table 3. (*So*, *Sc*) Pairs	
Sc **(%)**	***So*** **(%)**
2.60	3.00
3.00	3.47
3.46	4.00
3.50	4.04
4.00	4.62
4.32	5.00
4.50	5.18
5.00	5.77
5.18	6.00
5.50	6.35
6.00	6.90
6.07	7.00
6.50	7.47
6.91	8.00
7.00	8.06
7.50	8.64
7.78	9.00
8.00	9.20
8.50	9.76
8.65	10.00
9.00	10.32
9.50	10.96
9.51	11.00
10.00	11.48
10.39	12.00

c. Free fall is limited by lanyard slack adjustment:

$Do = 2.00 \text{ ft} < 6.00 \text{ ft}$ **o.k.**

d. The total fall ($Do + Ds$) must be checked against distance to obstacles:

$Do + Ds = 2.00 \text{ ft} + 3.1478 \text{ ft} = 5.1478 \text{ ft}$

e. The arresting force is checked:

$P = 1.5990 \text{ kip} = 1{,}599 \text{ lb} < 1{,}800 \text{ lb}$ **o.k.**

When compared with Example 1, the arresting force has been reduced to an acceptable level by the elasticity of the horizontal support and reduction of initial sag. Tension in the HLL and the loads on its anchors are also significantly reduced.

Specifying Initial Sag in the HLL

As mentioned in the "Design Overview," the initial sag in the unloaded HLL will be a catenary. If the designer chooses to specify catenary sag, it only remains to relate the sag, *Sc,* of the unloaded catenary to the initial V-shaped sag, *So,* when the slack is first taken up during a fall. The length of the cable can be found from the span *Ho* and sag *So*, and then the catenary sag, *Sc,* can be calculated.

Results of calculating (*So, Sc*) pairs to two decimal places of precision are given in Table 3. From Table 1, the initial V-shaped sags *So* of 10% and 5% in the preceding examples correspond to catenary sags *Sc* of 8.65% and 4.32%, respectively, at time of installation. An approximate linearized expression may also be used.

$$Sc = 0.864So \tag{18}$$

For these same values of V-shaped sag *So*, Equation 18 gives catenary sag *Sc* of 8.64% and 4.32%, respectively.

CONCLUSION

A method of calculating the arresting force by rational analysis and comparing the state of the PFAS with the limit states of the system has been presented herein. Conservation of energy is tested for trial values of HLLs with cable tension *T.* Having identified the correct value of cable tension to any desired precision by reiterative analysis, the maximum arresting force and other items of interest can be compared with the limit states. When the results are unsatisfactory, changing the selection of cable, changing the initial sag, introducing a spring in the HLL or introducing elasticity in the horizontal supports are among the many available remedies. Examples and suggestions for analysis of modifications are given in this paper.

The energy absorbed within the body of a person has not been accounted for. The assumptions that no energy is dissipated by the PFAS, that lanyards and supports are very stiff, and that the falling object is a point mass all act to stiffen the system and increase the arresting force. Ignoring the inertia within the PFAS (e.g., inertia of an elastic support) acts to soften the system and decrease the arresting force. In the absence of damping, a PFAS would be oscillatory, but in every real PFAS, there will be many sources of damping whereby energy is absorbed and dissipated. Suggestions that a good estimate of the arresting force is two times the weight of a person are known from field tests to be inaccurate in the general case.

SYMBOLS

AE HLL cable property = Metallic area times the effective modulus of elasticity, kips

Do Free-fall distance, same as slack in PFAS lanyard, ft

F Horizontal reaction at support, kips

H Horizontal span after displacement of supports under load, ft

Ho Initial span of HLL, ft

K_1, K_2 Stiffness of horizontal supports, kip/ft

Ke Effective stiffness of the HLL, kip/ft

Ks Stiffness of a spring included in a HLL, kip/ft

L Total length of cable under load, ft

Lo Initial length of unloaded cable, ft

P Arresting force applied to cable, kip

R Vertical reaction at support, kip

S Sag of cable in the V configuration under load, ft

Sc Initial sag at rest in a catenary configuration, ft

So Initial V-shaped sag at rest, ft

T Tension in the HLL cable, kip

Tn Cable breaking strength, kip

U Strain energy, ft-kip

W External work = Change in potential energy, ft-kip

Wo Weight of a falling person with equipment, kip

e Elongation of cable under load *T*, ft

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