

Design Curves for Plastic Design of Uniformly Loaded Steel Beams

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CONVENTIONAL methods of plastic design are relatively complex when applied to uniformly loaded beams. A curve, developed in this paper, provides a simple, quick, direct method for the design of beams subject to such uniform loads. The beams may have either constant or variable cross sections. A second curve facilitates determination of cover plate lengths.

THE DESIGN CURVE

Figure 1 shows a free body diagram for one span of a continuous structure subject to a uniform load. The three plastic hinge moments required to produce a mechanism are called M_{p1} (left support), M_{p2} (interior of span), and M_{p3} (right support). To further simplify the problem, these plastic moments are expressed as a parameter times the simple beam bending moment, $wL^2/8$. Thus,

$$M_{p1} = C_L wL^2/8 \quad (1)$$

$$M_{p2} = C_M wL^2/8 \quad (2)$$

$$M_{p3} = C_R wL^2/8 \quad (3)$$

The subscripts L , M , and R on the parameters C represent respectively: left, mid-span, and right.

The design curve is based on these definitions of the plastic hinge moments and two fundamentals: (1) The shear must be zero if M_{p2} is to be greater than any other bending moment in the neighborhood, and (2) M_{p2} is the plastic bending moment, expressed as $C_M wL^2/8$.

At a distance x from the left support, the shear is

$$V_x = C_L wL/8 - C_R wL/8 + wL/2 - wx \quad (4)$$

At the point where V_x equals zero, Eq. (4) reduces to an equation for distance x , so that

$$x = (C_L - C_R + 4)L/8 \quad (5)$$

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The bending moment at distance x from the left support is

$$M_x = -C_L wL^2/8 + C_L wLx/8 - C_R wLx/8 + wLx/2 - wx^2/2 \quad (6)$$

At the point where the shear V_x is zero, x is defined by Eq. (5) and M_x by $C_M wL^2/8$. Thus, Eq. (6) can be written

$$C_M wL^2/8 = -C_L wL^2/8 + C_L wLx/8 - C_R wLx/8 + wLx/2 - wx^2/2$$

or

$$x^2/2 - Lx(C_L - C_R + 4)/8 + (C_M + C_L)L^2/8 = 0 \quad (7)$$

Using Eq. (5), Eq. (7) reduces to

$$x^2 = (C_M + C_L)L^2/4 \quad (8)$$

so that

$$x = L\sqrt{C_M + C_L}/2 \quad (9)$$

From Eq. (5), it is possible to relate C_R to C_L and x . From Eq. (9), x is related to C_M and C_L . The curve of Fig. 2 showing the relationship between C_L , C_M , C_R , and x was constructed in the following manner: A value was assumed for x , for instance $0.4L$. Values for C_M ranging from 0.1 upward by tenths were then utilized to find the respective values of C_L . From C_L and x the value

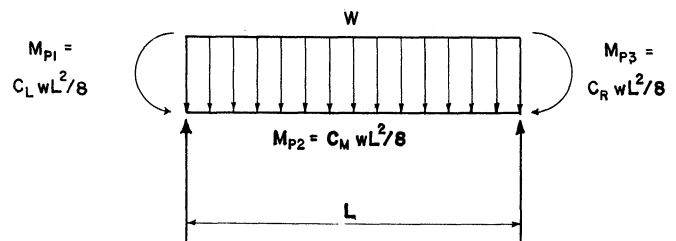


Fig. 1. "Free body" of single span uniformly loaded

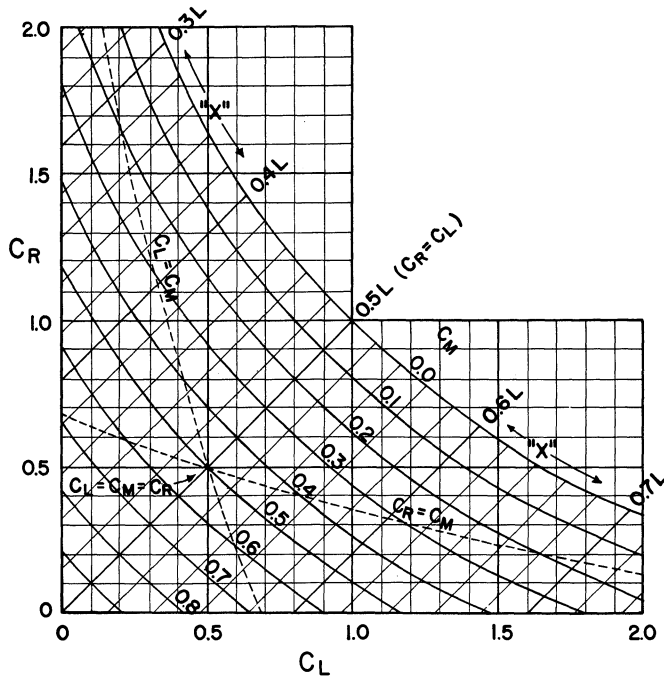


Fig. 2. Plastic design curve for uniform load

of C_R was then found. Once these points were plotted, the value of x was changed and the procedure repeated.

On the curve, the x values are shown in terms of tenths of the span length L , and are found along lines making 45° angles with axes. The C_M values, varying from 0.1 to 0.9, are found along the curved lines. C_R and C_L are plotted, respectively, on the vertical and horizontal axes.

To facilitate use of the chart, a dashed line has been drawn where C_L is equal to C_M , representing the condition when M_{p1} is equal to M_{p2} , that is, for instance, where a cover plate extends from the support to a position beyond the maximum positive moment. A similar dashed line is drawn for the case when C_R is equal to C_M . C_L is, of course, equal to C_R when x is equal to $0.5L$. These three lines (two dashed and $x = 0.5L$) intersect at the point $C_R = 0.5$, $C_M = 0.5$, $C_L = 0.5$, and $x = 0.5L$, representing the case of a beam with a constant cross section.

EXAMPLE 1: CONSTANT SECTION

A fixed end beam is laterally stabilized throughout and loaded by a 2 kip/ft working load as shown in Fig. 3. This is a very simple case when EI is constant and the statical method immediately signifies that $M_p = (wL^2/8)/2 = 3.4(40)^2/16 = 340$ kip-ft and $Z_R = 113.3$ in.³, requiring a 21W55.

A chart solution of the problem, although trivial, is based on the fact that $C_L = C_M = C_R$, which establishes each of them as 0.5 and the location of the intermediate plastic hinge as $x = 0.5L$.

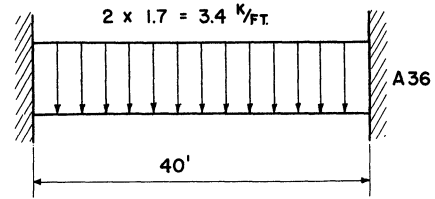


Fig. 3. Fixed end beam—uniform load

EXAMPLE 2: MID-SPAN COVER PLATE

Using the load and span of Fig. 3, a design using a mid-span cover plate can also be performed readily. Assume an 18W45 (A36) base section having a plastic section modulus of 89.6 in.³ and an allowable bending moment of 268.8 kip-ft. For this beam, $wL^2/8$ is 680 kip-ft; therefore, $C_L = C_R = 268.8/680 = 0.396$. Using Fig. 2, C_L and C_R indicate that C_M is about 0.60 and x is $0.5L$. Therefore, the plastic bending moment at $L/2$ is $0.60(680) = 408$ kip-ft, requiring a section modulus of 136.0 in.³. Thus, cover plates must be used which increase beam capacity by $136.0 - 89.6 = 46.4$ in.³.

EXAMPLE 3: THREE UNEQUAL PLASTIC BENDING MOMENTS

Figure 3 could represent a free-body diagram of one span of a continuous structure. In such a case, it could be appropriate to have a 16W50 through the left support and a coverplated 16W45 through the right support, with the maximum positive bending moment carried by the 16W45 without cover plate. For this case, with $wL^2/8 = 680$ kip-ft.:

$$16W45: Z_{p2} = 82.0; M_{p2} = 246.0;$$

$$C_M = 246.0/680 = 0.362$$

$$16W50: Z_{p1} = 92.7; M_{p1} = 278.1;$$

$$C_L = 278.1/680 = 0.410$$

Then, from Fig. 2, $x = 0.44L$ and $C_R = 0.89$, making $M_{p3} = 0.89(680) = 605$ kip-ft.

The final free-body diagram is shown in Fig. 4 and the results may be readily checked by either the statical or the mechanism method.

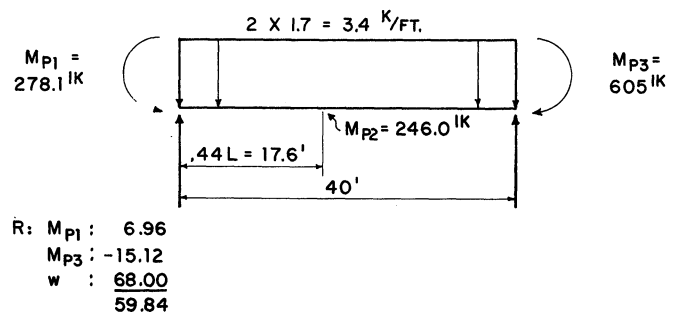


Fig. 4. Free body diagram—three unequal plastic bending moments

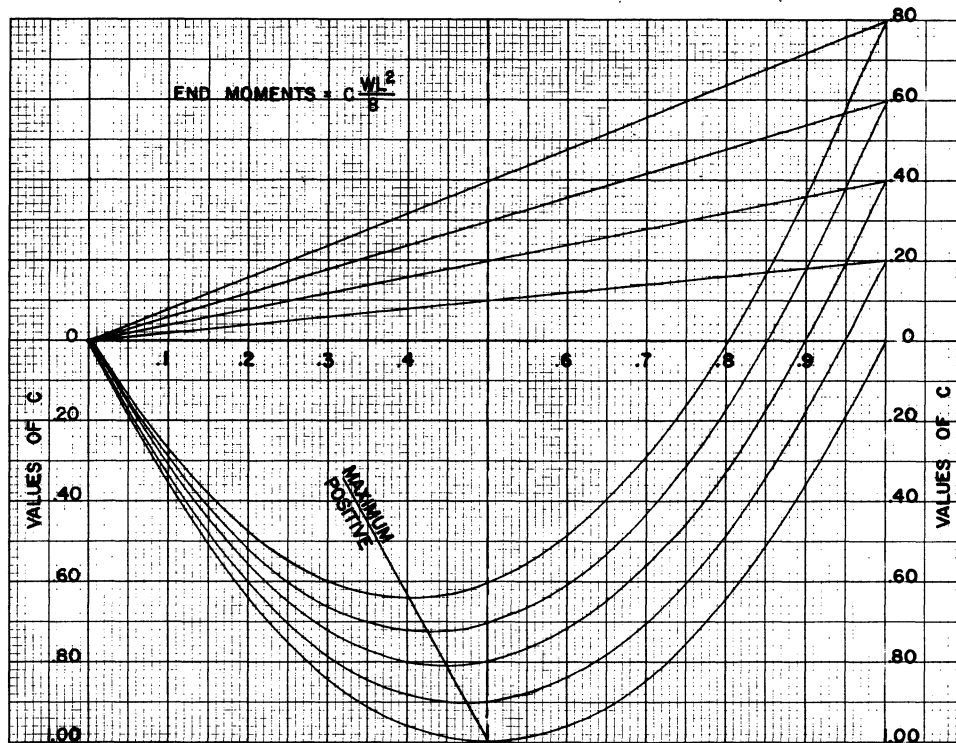


Fig. 5. Composite uniform bending moment curves with end moments

UNIFORM BENDING MOMENT WITH UNIFORM LOAD

Figure 5 is a set of curves which are very useful in determining the theoretical point of cutoff for cover plates. To use these curves:

- (1) Using only C_L and C_R , plot the lowest C value downward on the left axis and project a horizontal line. This horizontal line represents zero bending moment.
- (2) Plot the difference between the highest and lowest of these C values upward on the right axis.
- (3) Sketch in the bending moment diagram using the relative position of the point on the right axis as a guide.
- (4) The bending moment diagram is now complete to scale and can be used readily by converting all bending moments to equivalent C values by dividing by $wL^2/8$.

The theoretical length of coverplate for the beam of Example 3 is determined graphically in Fig. 6. The bending moment diagram is constructed as indicated above:

- (1) Because C_L is less than C_R , $C_L = 0.41$ is plotted downward on the left axis.
- (2) A horizontal line (base axis) is drawn through the point representing the axis of zero bending moment.
- (3) $C_R - C_L = 0.89 - 0.41 = 0.48$ is then plotted upward on the right axis.

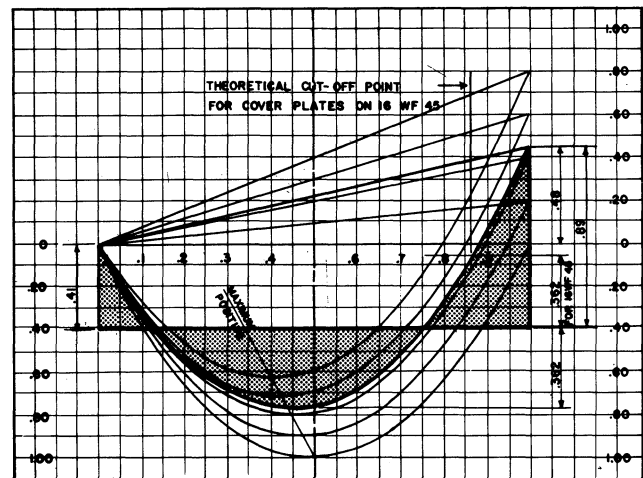


Fig. 6. Theoretical point of cut-off by bending moment curves

- (4) This point makes it possible to sketch the bending moment curve (heavy solid line) accurately "by eye," using the curves on the figure as guides.
- (5) Once the bending moment diagram is complete, the C value for the 16WF45 acting without cover plates, $C_M = 0.362$, is plotted upward from the base axis.
- (6) A horizontal line through this point intersects the bending moment diagram at $0.86L$, signifying the theoretical point of cut-off.

Table 1

1	2	3	4	5	6	7	8	9	
D_L (in.)	C_L	D_R (in.)	C_R	$C_{M(reqd.)}$	$x(L)$	$x(D_R - D_L)$	D_M	C_M	
12	0.253	24	0.634	0.57	0.455	5.45	17.45	0.42	N.G.
18	0.428	24	0.634	0.47	0.47	2.8	20.8	0.525	BIG
16	0.367	24	0.634	0.50	0.47	3.76	19.76	0.484	O.K.

EXAMPLE 4: TAPERED SECTION

In some cases, often for architectural reasons, a tapered section is preferred. For the beam of Fig. 3, it is decided that the limitations on depth will be between 12 and 24 in., with the web made from $\frac{3}{8}$ -in. thick plate and the flanges from $8 \times \frac{1}{2}$ -in. plate. The C values and plastic moments are plotted vs. depth in Fig. 7.

Although a more reasonable first trial can be assured by noting that a constant cross section must have a C of 0.5 and a corresponding depth of 20.2 in., the poorest first trial of $D_L = 12$ in. and $D_R = 24$ in. is used to illustrate convergence.

The solution is best performed in tabular form (see Table 1). The first and third columns represent the trial depths. The second and fourth columns represent the C values read from Fig. 7 for these trial depths. Using the C_L and C_R values, Fig. 2 is used to determine $C_{M(reqd.)}$ shown in column 5 and x as a function of span length L shown in column 6. Column 7 represents the change in depth from left to right multiplied by x , signifying the increase in depth over the left-most section. Column 8 is the sum of columns 1 and 7 and indicates the depth

of section at xL . From Fig. 7, this depth yields the value of C_M in column 9, which is compared to the required value shown in column 5.

Although the C_M formulated is 0.484, which is less than the $C_{M(reqd.)}$ of 0.50, the solution is considered adequate within the accuracy of known data.

The validity of such a solution is indicated in Fig. 8 where the bending moment diagram has been plotted. The allowable bending moment is plotted as a straight line from C_L of 0.367 to C_R of 0.634, both positive downward. From this plot the value of M_p and its position is verified.

Actually, the C values of a tapered section do not vary linearly as is evidenced by Fig. 7. However, a moment's reflection suggests that the only nonlinear portion is due solely to the increased web area, normally rather insignificant. At any rate, an adequate and economical tapered section can be designed by using Fig. 2 in conjunction with a chart similar to Fig. 7. In any case of doubt, Fig. 5 can then be used to verify the section, plotting the actual allowable bending moments rather than a linear approximation if desired.

CONCLUSION

The design curves given in Figs. 2 and 5 provide a rapid and accurate means for plastic design of uniformly loaded beams. The beams may be tapered, coverplated, or both. The ends may be simply supported, continuous, or fixed. The accuracy of the results is readily verified by either the statical or the mechanism method.

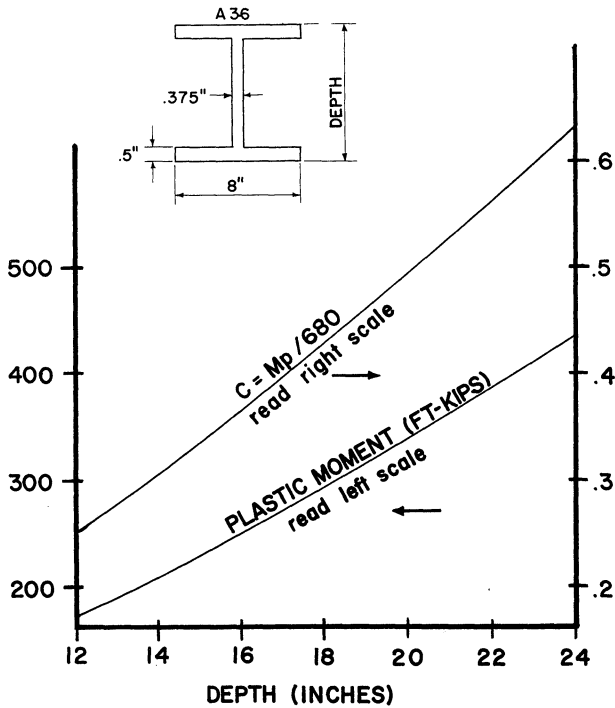


Fig. 7. C and M_p vs. beam depth

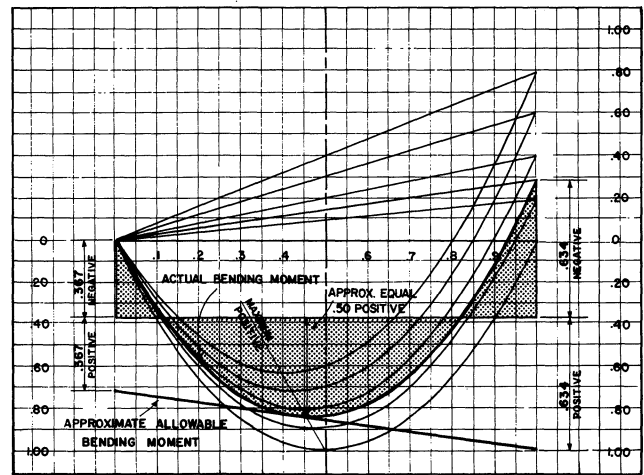


Fig. 8. Tapered section—actual vs. allowable bending moments