Space Frame Buckling

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THE LARGE dome structure is being used more and more because of its structural efficiency, its low cost, and its beauty. The Houston Astrodome (see Fig. 1) is an illustration of future trends in large covered areas. Domes have been built using structural shapes, tubing, shells with stiffeners and composite materials. Buckling analysis is one of the most important considerations facing the designer of such a structure. If buckling failure occurs, it is often sudden and often results in complete collapse of the structure. The failure of the 300-ft diameter dome in Bucharest, Rumania[1], is but one example of such a collapse.

The purpose of this report is to show that the designer can calculate the general buckling load in the nonlinear stress-strain range and can take into account the important factors in buckling such as yield strength, edge conditions, construction and fabrication tolerances, and residual stresses in structural members.

The Hencky deformation theory of plasticity has been used for over twenty years in the design of column, plate, and flange structures.[2] The theory, which has been verified experimentally, gives a plasticity reduction factor that takes into account the nonlinear effects. The buckling load is obtained by multiplying the elastic buckling stress by the plasticity reduction factor. The plasticity reduction factor is E_T/E for columns and E_s/E for flanges,[2, 3] where E = elastic modulus, E_T = tangent modulus, E_s = secant modulus.

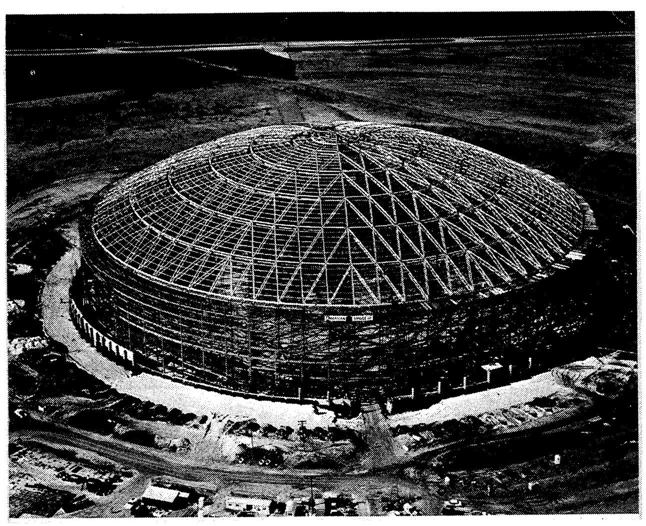

Fig. 1. Houston Astrodome

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The plasticity reduction factor for a spherical unstiffened shell derived by Bijlaard[4] using a differential equation approach is

$$\eta = \frac{(E_s E_T)^{1/2}}{E} \tag{1}$$

The author has derived the reduction factor for an unstiffened shell by using a von Karman type energy approach coupled with the Hencky deformation theory. The result is

$$\eta = \frac{3}{4}\left(E_T + \frac{E_s}{3}\right)\frac{1}{E} \tag{2}$$

Equations (1) and (2) give approximately the same results, as illustrated in Fig. 2.

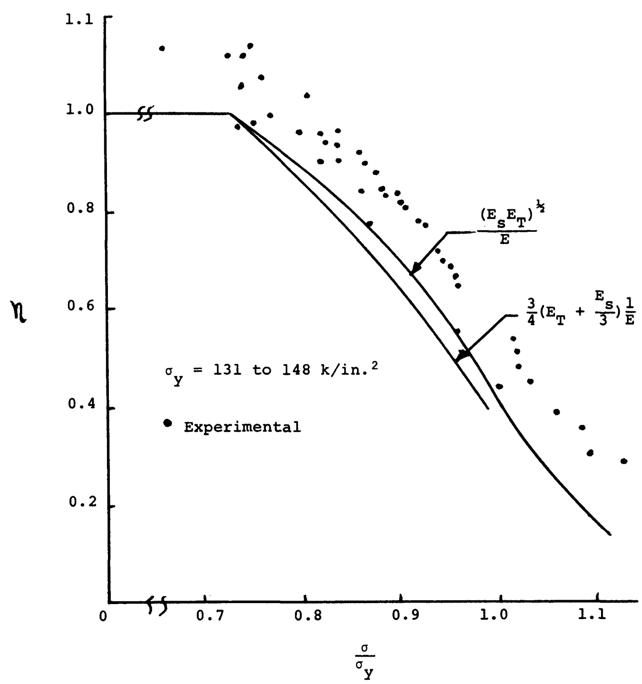

Fig. 2. Plasticity reduction factors for machined models

Edge conditions and deviations from a near perfect shape often reduce the buckling load considerably.[6, 7] The author has theoretically derived the general critical buckling stress of a dome by combining the von Karman energy approach and Hencky deformation theory with the previously published stability theory for orthotropic or space framed domes.[8] The result is

$$\sigma_{cr} = \frac{4}{3}\,\eta E \frac{t_m}{R}\left\{\left[0.210\left(\frac{\Delta}{t_m}\right)^2 + 0.0715\left(\frac{t_B}{t_m}\right)^3\right]^{1/2} - 0.459\,\frac{\Delta}{t_m}\right\} \quad (3)$$

where

η = Plasticity reduction factor given by Eq. (2).

t_m = Effective membrane or stretching thickness of the orthotropic dome.[9] For the equilateral reticulated dome, $t_m = 2A/\sqrt{3}l$, and for the framed dome, $t_m = A/d$, where A is the area of the member, l is the length of the member, and d is the spacing between members.

t_B = Effective bending thickness of the dome. For a reticulated dome, $t_B = [9\sqrt{3}\ (I/l)]^{1/3}$, and for a framed dome, $t_B = (12I/d)^{1/3}$, where I is the moment of inertia of the member.

Δ = Deflection just prior to buckling which may be due to edge conditions, fabrication and erection tolerances, etc. Reference 6 gives one method of calculating Δ.

Several test programs have generally confirmed Eq. (3) for the effects of deflections in the elastic case.[6, 7] Figure 3 shows the reasonably good agreement for the unstiffened shell case, based upon Navy tests. Reference 6 shows good agreement for the orthotropic shell-like structural case.

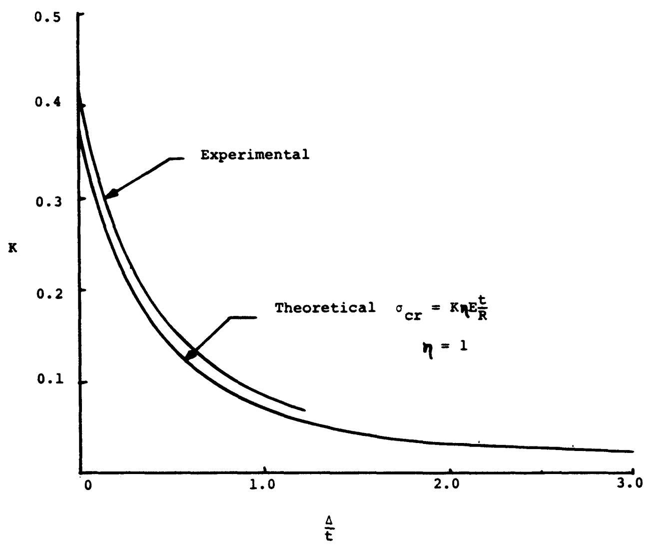

Fig. 3. Effect of deflections on dome buckling

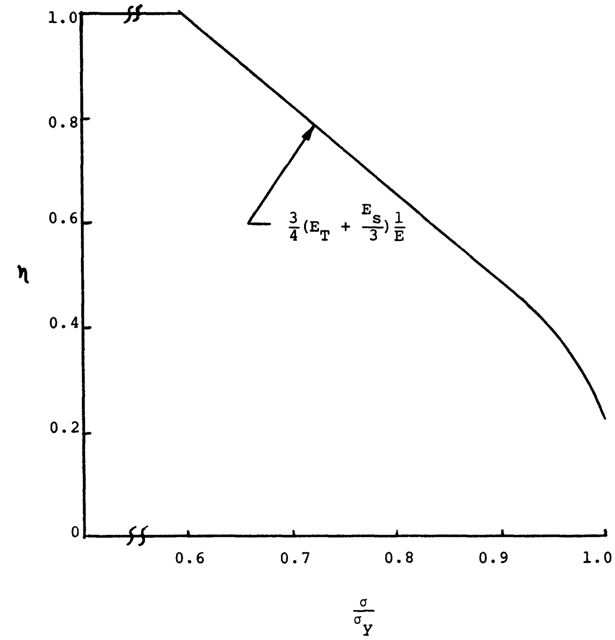

Fig. 4. Plasticity reduction factors for structural steel members

Designers have long recognized the importance of residual stresses on column buckling. Equations such as those presented in the AISC *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings* account for residual stresses and the effects of yield strength on these stresses. Yield strengths and residual stresses also have a significant effect on framed and stiffened dome buckling.

The plasticity reduction factors for framed domes based on stub column tests given in Ref. 10 are shown in Fig. 4. In these tests on wide flange structural shapes, the measured elastic limit was about 0.6 of the yield strengths. The plasticity reduction factor for a dome at a stress of 0.9 of the yield is about 0.5, which indicates a buckling stress one-half that calculated by elastic theory. Deflections and erection tolerances reduce this factor even more.

In summary, Eq. (3) (with a factor of safety) gives the designer a tool that can be used to estimate the effects of plasticity, deflections, construction and fabrication tolerances, and residual stresses on the buckling strengths of framed and stiffened domes.

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