TECHNICAL NOTE

Two-Way Bending of Base Plates under Uniaxial Moment Loading—Alternative Approach

EDWARD R. HANINGER and BRUCE M. TONG

ABSTRACT

This Technical Note presents an alternate model for two-way bending design of column base plates under uniaxial moment loading and is based on the design premises of AISC Design Guide 1, *Base Plate and Anchor Rod Design* (Fisher and Kloiber, 2006). Two-way bending, in this paper, refers to bending of a column base plate in the direction perpendicular to the primary direction of bending and is also called side bending. When two-way bending governs, which is commonly the case, this procedure results in more efficient base plates that more closely reflect available strengths. A sample calculation is also provided.

Keywords: base plate, steel design, column base, two-way bending, side bending.

INTRODUCTION

Two-way bending will commonly govern the required thickness of column base plates subjected to compressive loads, with or without an applied moment. For the purposes of this paper, the term *two-way bending* refers to bending of a column base plate perpendicular to the primary direction of bending. Two-way bending is particularly important for wide base plates and narrow column flanges. The focus of this paper is the situation where the applied moment dominates the plate stresses. A diagram of a column base plate subject to a uniaxial moment about the column's strong axis is shown in Figure 1.

AISC Design Guide 1, *Base Plate and Anchor Rod Design* (Fisher and Kloiber, 2006), provides a design procedure for determining the required thickness of a base plate. In this guide, hereafter referred to as Design Guide 1, a design procedure is provided for the m and n cantilever lengths, as shown in Figure 1. The cantilever length for two-way bending, n, is as defined in the *Manual of Steel Construction*, 14th edition (AISC, 2011) and Design Guide 1. As noted in Figure 1, the cantilever length, n, used to determine the two-way bending force is based on 0.8 times the width of a wide flange column. The 0.8 factor would also apply to pipe columns. A factor of 0.95 would be used for rectangular column sections. Wide flanges are commonly used for columns, so this shape will be used to demonstrate the procedure in this paper.

The primary method of analysis used in Design Guide 1 is to assume a rectangular stress block for compression on the concrete as shown in Figure 2. The rectangular compression block is an effective design method and is also used in this paper.

THE PROBLEM

Design Guide 1 presents a design procedure that considers bending caused by compression at the bearing interface and bending caused by tension on the anchor rods on the tension side. However, Design Guide 1 uses a simple and conservative means of designing the base plate for the common bending case when the cantilever lengths perpendicular to the primary load direction are greater than the cantilever lengths in the direction of load ($n > m$). This failure mechanism is called two-way bending in this paper.

Design Guide 1 provides direction for two-way bending only in notes at the ends of Sections 3.3.2 and 3.4.2, where it is recommended that n be substituted for m in Equations 3.3.14a-1, 3.3.14b-1, 3.3.15a-1 and 3.3.15b-1 when n is larger than m. This procedure functionally sets the effective bending width, b_{eff}, of the plate equal to the compression length Y ($b_{eff} = Y$). This is reasonable when the base plate is in full compression. However, for cases where the bearing length is small, such as in Figure 2, this can lead to overly conservative results because resistance to bending would utilize a larger effective plate width. Moreover, for the case $Y < n$, the note in these Design Guide 1 sections could lead to inaccurate design results. The objective of this paper is to present an improved procedure for design of the n cantilever length.

Edward Haninger, P.E., S.E., Director I, Design Engineering, Fluor Corporation, Irvine, CA (corresponding). E-mail: ed.haninger@fluor.com

Bruce Tong, P.E., Principal Engineer, Fluor Corporation, Irvine, CA. E-mail: bruce.tong@fluor.com

Table 1. Roark Values for K_m as a Function of c/a at $z = 0$	
c/a	K_m
1.0	0.509
0.75	0.428
0.5	0.370
0.25	0.332

This procedure results in an effective plate width, which is then used to determine section properties for the strength of the plate in flexure.

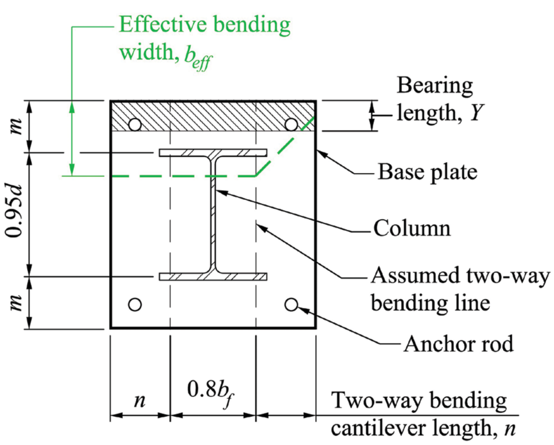

Fig. 1. Base plate two-way bending terms.

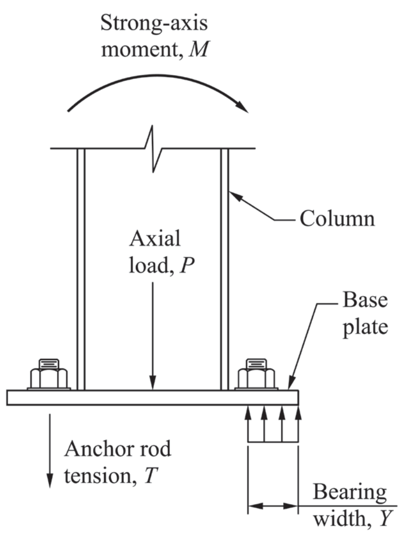

Fig. 2. Rectangular stress block forces.

RESEARCH AND DEVELOPMENT

Roark

In *Roark's Formulas for Stress and Strain* (Young and Budynas, 2001), Section 8.11, "Beams of Relatively Great Width," a case is presented of a very wide cantilever plate under a concentrated load. The bending stress σ at any point is expressed by the equation

$$\sigma = K_m \left(\frac{6P}{t^2} \right) \tag{1}$$

where P is the concentrated load, t is the plate thickness and K_m is a dimensionless coefficient that depends on the location of the point in question. Values for K_m are presented in a table in which the rows indicate the ratio c/a and the columns indicate the ratio z/a and where c and a are as shown in Figure 3. The dimension a used in Roark's tables corresponds to n in base plate two-way bending design.

Roark presents the bending factor K_m in the context of a single concentrated load. A review of the values of K_m as a function of c for $z = 0$ indicates that the effective factor for a line load along the X-axis for all values of c would average approximately 0.4. A table of Roark's values for K_m for varying values of c/a at $z = 0$ is shown in Table 1, and a diagram showing an equivalent line load is shown in Figure 4.

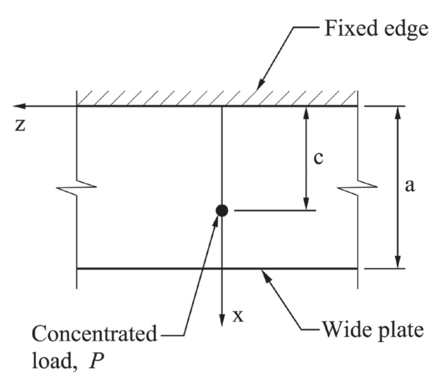

Fig. 3. Roark case of a concentrated load on a cantilever.

An average value of $K_m = 0.4$ implies that the effective width would have been about $2.5n$, or $1.25n$ on each side. To account for the unbalanced or twisting forces caused by the asymmetrical load, this coefficient was then reduced to 1.0, leaving a reduced value of $1.0n$ for the additional width effective for bending. This value was then added to the z-direction distance from the centroid of the loaded area to the edge of the loaded area to arrive at the final equation for b_{eff}:

$$b_{eff} = \frac{Y}{2} + n \tag{2}$$

For the purposes of establishing an upper limit to this theory, we will mirror the b_{eff} calculation to the opposite side of the X-axis, giving us a total width of $2.0n$. Therefore, the equation for b_{eff} will be limited to the condition $Y \le 2n$, beyond which we will revert to Design Guide 1's original $b_{eff} = Y$.

Finite Element Analyses

Finite element analyses were performed to provide confirmation of the proposed equation. The analyses included both (linear) elastic and (nonlinear) plastic methods. The base plate was modeled as a simple cantilever plate, fixed at the assumed bending line described earlier, with a constant uniform load. In doing so, two simplifying assumptions are made, which are consistent with current design methods:

1. The effective fixed edge is a straight line at $0.8b_f$ This simplifies the effect of the flange tips and the off-center loaded area.
2. The loading is uniform over the bearing area. This neglects the interaction effect between the plate and foundation surface.

The base plate used for both the elastic and plastic analysis is a 1-in.-thick base plate under partial compression loading. The plate's plan dimensions and the assumed cantilever length beyond the bending line are shown in Figure 5.

Elastic Analysis

A simple analysis, based on Design Guide 1's assumptions, was performed using elastic plate elements in RISA-3D (2012). Uniform loads were applied over various load widths to represent varying lengths of the compression block, Y. In each analysis, the effective width, b_{eff}, was determined by comparing the results of similar loading on two different RISA-3D models: (1) a cantilever plate of width 22 in. loaded with a uniform load of width Y along one edge and (2) a cantilever plate of width Y loaded with the same uniform load of width Y. The two models were created using shell elements 0.5 in. × 0.5 in. × 1.0 in. thick. RISA uses the MITC4 plate element described in K.J. Bathe's self-published book, *Finite Element Procedures*, which includes bending and shear effects. The effective width was determined by comparing the maximum moments found in the two runs, which is equivalent to comparing the results at the point of initial yield. The ratio of the maximum plate moments in the two runs was then used to determine b_{eff} by:

$$b_{eff} = \frac{\text{Maximum moment, 22-in.-wide model}}{\text{Maximum moment, } Y\text{-in.-wide model}} \times Y$$

The results of the elastic analysis are shown in Figure 6: Four data points correspond to the four pairs of analysis runs, and a curve was fitted to the results. The curve connecting these points also passes through the known theoretical point at the upper right corner, where the effective width is equal to the plate width for full compression on the plate.

In addition to the elastic analysis results, Figure 6 also plots the proposed equation for b_{eff}, the value of b_{eff} used in Design Guide 1 and the plastic analysis results discussed in the next section.

Figure 6 shows a wider divergence between the elastic and $b_{eff} = Y$ curves at smaller bearing widths. This indicates that two-way action is most significant in this lower range. The values for the elastic curve are lower than for the proposed formula for b_{eff} due to the limitation of elastic analysis

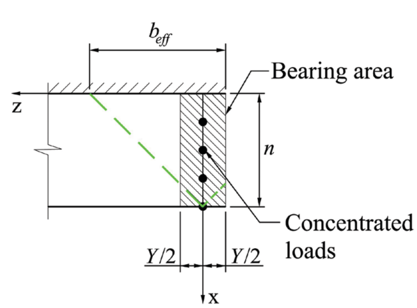

Fig. 4. Roark case adapted for line load.

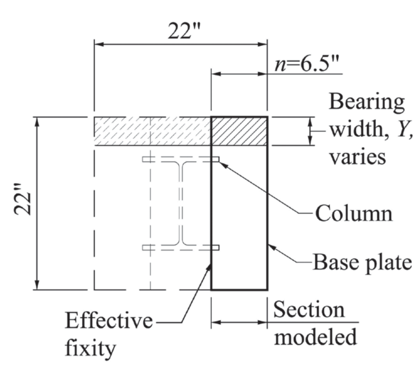

Fig. 5. Base plate used in analyses.

and the effects of twisting. As we shall see in the following plastic analysis, the elastic analysis results are based on first yield and are not representative of the true strength of the plate.

Plastic Analysis

In order to get a better estimate of the true strength of a base plate in two-way bending, a basic plastic analysis was performed with a nonlinear program. A series of runs was made on two plates in SAP2000 (2011): one 2 in. wide, and the other 22 in. wide, with uniform loading of width 2 in. (Y) on each. The two models were created using shell-type layered shell elements 0.5 in. × 0.5 in. × 1.0 in. thick. Layered shell elements in SAP2000 use the thick-plate (Mindlin/Reissner) formulation for bending behavior, which includes the effects of transverse shear deformation. The material stress-strain properties used a common 50-ksi material with an initial elastic portion, then a plateau above yield with strain hardening starting after 1.5% strain. In each run, the uniform load was increased and the resulting deflection at the plate corner was recorded. The progression of plate yield is shown in Figure 7. The force-deflection plot of these runs is shown in Figure 8.

The plastic limit of the 22-in.-wide plate was found to occur at a load of 5.0 ksi, and the plastic limit of the 2-in.-wide plate was found to occur at 0.8 ksi, indicating a b_{eff} of $5.0/0.8 = 6.25$ at $Y = 2$ in. Figure 7 shows that first yielding occurs at the top, loaded edge of the plate. This first yield point is also indicated in Figure 8 and occurs relatively early due to twisting of the plate under load. The twist is caused by the centroid of the load being eccentric from the centroid of the plate resistance.

The curve for two-way bending starts out much steeper and thus stiffer than the one-way bending curve. This leads to the higher overall strength demonstrated with two-way action.

The analysis was then repeated for other values of Y. As the width of the loaded area became wider and wider, the yield line migrated lower and lower on the plate until, eventually, the entire length of the base plate was involved in resisting two-way bending moment—in this case, after $Y = 12$ in. The results of the plastic analysis series are also shown on Figure 6. The results of the plastic analysis demonstrate a significantly higher strength than predicted by elastic methods. The plastic curve is significantly above the proposed method at all points and is considered sufficiently conservative for now.

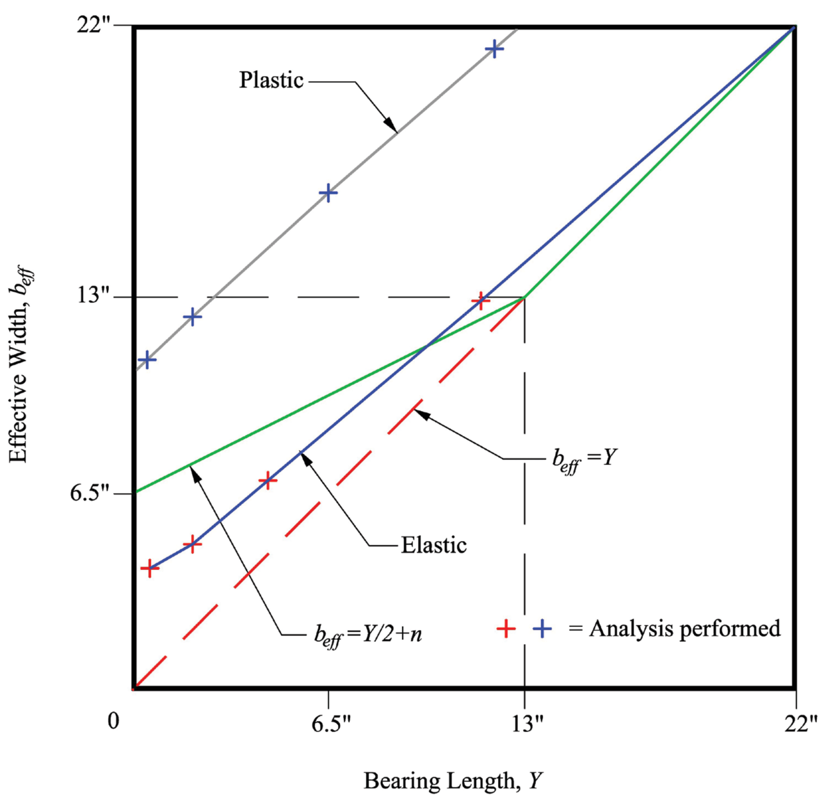

Fig. 6. Analysis results, effective width vs. bearing length.

Comparison to Tests

Recent full scale tests were reviewed to shed additional light on the above analysis. Gomez, Deierlein and Kanvinde (2010) provided a detailed report on the testing with a summary by Kanvinde and Deierelein (2011). From a review of the test results, two comments can be made. First, tests demonstrated that base plates can continue to provide resistance well beyond the yield point. This indicates that the post-yield behavior seen in the preceding nonlinear analysis can be counted on to take the design loading. The second point concerns the observed plate bending behavior. The theory indicated by Design Guide 1 is that the effective bending width is equal to the rectangular stress block width ($b_{eff} = Y$). Using this theory and the dimensions of the test specimens, calculations indicate that two-way bending should have governed. However, the only compression-related bending reported was bending in the primary direction across the entire width of the plate parallel to the column flange. It should be noted that during actual tests, the bearing pressure is expected to decrease toward the end of the cantilever due to flexibility of the plate. This will tend to make the actual plate moment less than that resulting from the assumed uniform load.

Although two-way bending behavior was not one of the goals of the tests, the observed end bending failure mode lends credence to the proposed model. This indicates that the following recommended design procedure for two-way bending should be considered in design of base plates.

RECOMMENDED DESIGN PROCEDURE

Based on the preceding analysis and discussion, the authors of this paper recommend the use of an effective width, b_{eff}, rather than Y when determining the base plate thickness

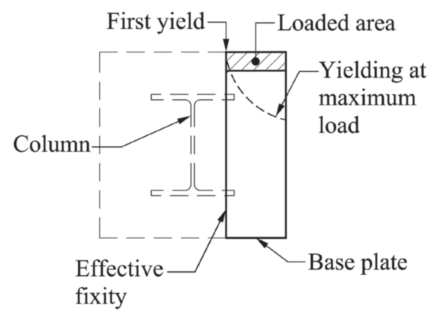

Fig. 7. Plastic analysis–yield progression for Y = 2 in.

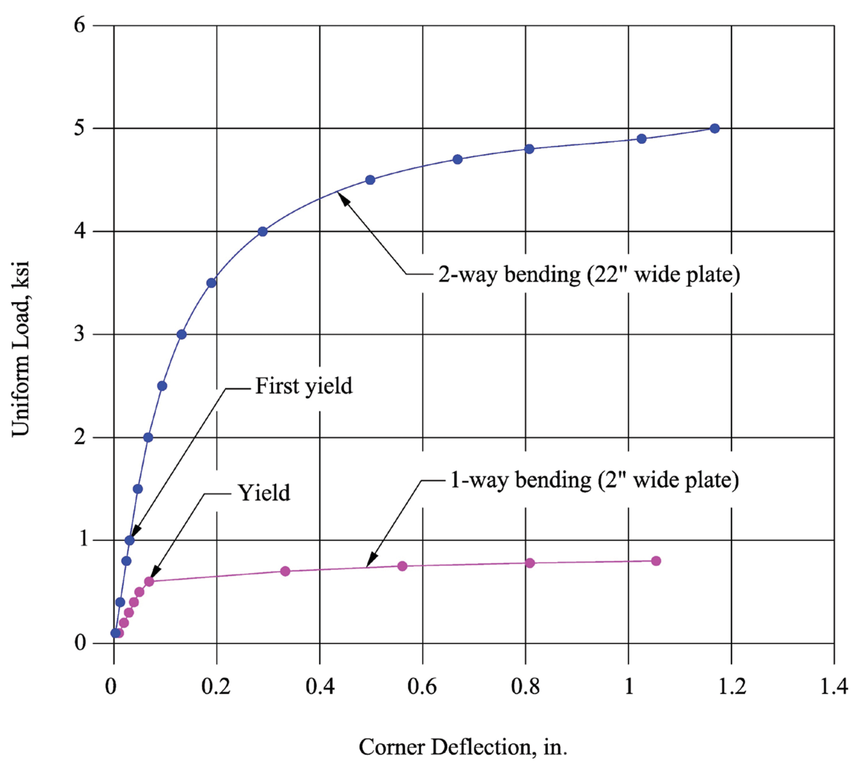

Fig. 8. Plastic load-deflection diagram of plate at Y = 2 in.

required for two-way bending. The effective width b_{eff} recommended is:

$$b_{eff} = \frac{Y}{2} + n \qquad \text{for } Y < 2n \tag{3}$$

and

$$b_{eff} = Y \qquad \text{for } Y \geq 2n \tag{4}$$

This is shown graphically in Figure 9 in terms of plate variables. The effective width for two-way bending, b_{eff}; the rectangular stress block width, Y; the lateral cantilever, n; and the length of base plate, N, are defined in Figure 10.

Using the earlier b_{eff}, the plate thickness required to resist two-way bending is determined from Equation 5:

$$t_{p(req)} = n\sqrt{\frac{2f_{p(max)}Y}{\phi F_y b_{eff}}} \tag{5}$$

where

$f_{p(max)}$ = maximum design bearing stress in concrete using LRFD (strength design)

ϕ = resistance factor in bending = 0.9

F_y = specified minimum yield stress of the base plate

Equation 5 combines both loading and plate section properties and can be derived from Equation 3.3.14a-1 in Design Guide 1 by substituting n for m, $f_{p(max)}$ for f_p and ϕ for 0.9. Then, because the quantity under the radical is related to the applied moment divided by the plate section properties, the quantity under the radical is multiplied by the ratio Y/b_{eff}.

CONCLUSION AND RECOMMENDATIONS

The following points can be drawn from the preceding discussion:

1. Two-way bending of base plates should be considered. Two-way bending will commonly govern the plate thickness required, particularly for narrow columns and/or wide base plates.
2. Two-way action should be considered in determining the plate resistance.
3. Twisting of a base plate under two-way action can become significant; however, first yielding of the plate is not representative of true strength.
4. A recommended design method incorporating two-way bending is given by Equations 3, 4 and 5.

This design method is simple enough to use in everyday design, yet effective. Although only limited analysis was performed, indications are that the method is conservative and is considered appropriate for design purposes at this time.

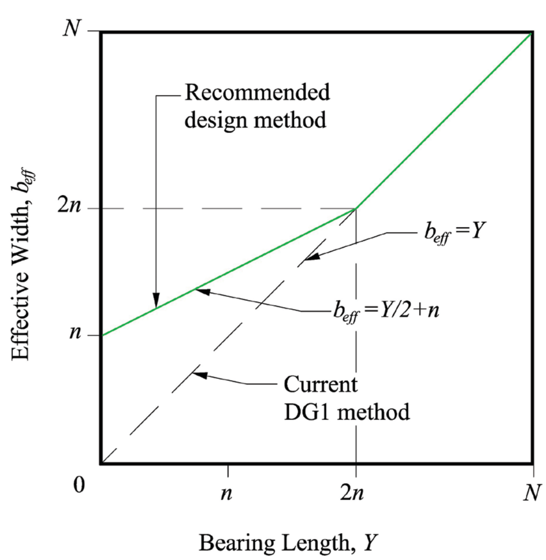

Fig. 9. Recommended design method for two-way bending.

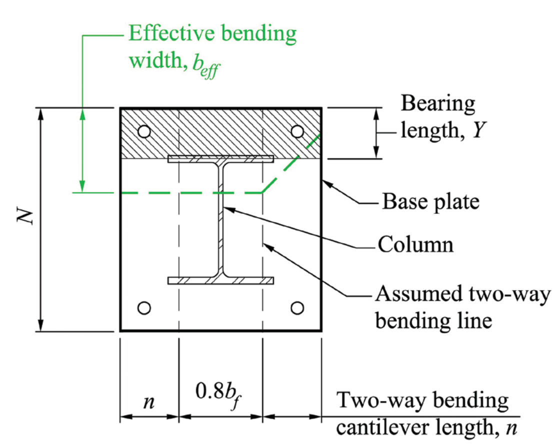

Fig. 10. Base plate terms.

EXAMPLE PROBLEM

Determine the minimum base plate thickness required to resist two-way bending in a base plate with a factored axial load $P_u = 60$ kips and factored moment $M_u = 80$ kip-ft, using LRFD. Bending is about the strong axis of a W12×22 wide flange column with a flange width $b_f = 4.03$ in. Conservatively consider the ratio of the concrete to base plate area is unity, F_y of the base plate is 36 ksi and f'_c of concrete is 4 ksi. For the purposes of this example, assume the bearing value $f_{p(max)}$ and bearing width Y have been found to be 2.21 ksi and 1.91 in, respectively. The base plate is 20 in. wide (B) and 20 in. long. (See Figure 11.)

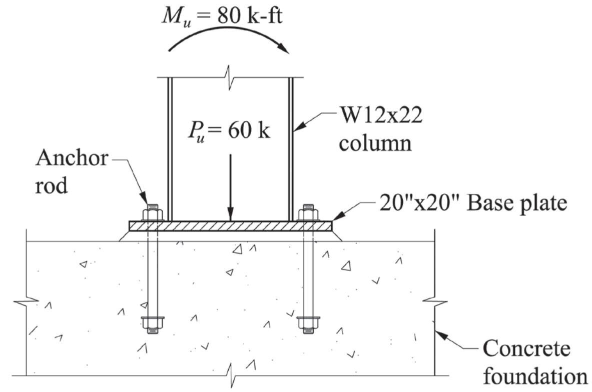

Fig. 11. Example.

Step 1. Determine the two-way bending cantilever:

$$\begin{aligned} n &= \frac{(B - 0.8b_f)}{2} \\ &= \frac{20\text{ in.} - 0.8(4.03\text{ in.})}{2} \\ &= 8.39\text{ in.} \end{aligned}$$

Step 2. For comparison, determine minimum base plate thickness for two-way bending per Design Guide 1. Use $b_{eff} = Y$:

$$\begin{aligned} t_{p(req)} &= n\sqrt{\frac{2f_{p(max)}Y}{\phi F_y b_{eff}}} \\ &= 8.39\text{ in.}\sqrt{\frac{2(2.21\text{ ksi})(1.91\text{ in.})}{(0.9)(36\text{ ksi})(1.91\text{ in.})}} \\ &= 3.10\text{ in.} \end{aligned}$$

Step 3. Per the new model:

Because $Y < 2n$, use

$$\begin{aligned} b_{eff} &= \frac{Y}{2} + n \\ &= \frac{1.91\text{ in.}}{2} + 8.39\text{ in.} \\ &= 9.35\text{ in.} \end{aligned}$$

$$\begin{aligned} t_{p(req)} &= n\sqrt{\frac{2f_{p(max)}Y}{\phi F_y b_{eff}}} \\ &= 8.39\text{ in.}\sqrt{\frac{2(2.21\text{ ksi})(1.91\text{ in.})}{(0.9)(36\text{ ksi})(9.35\text{ in.})}} \\ &= 1.40\text{ in.} \end{aligned}$$

It can be seen from this example that a significant reduction in plate thickness due to two-way bending can be used with the new model.

REFERENCES

AISC (2011), *Manual of Steel Construction,* 14th edition, American Institute of Steel Construction, Chicago, IL, pp. 14-4 to 14-9.

Fisher, J.M. and Kloiber, L.A. (2006), *Design Guide 1, Base Plate and Anchor Rod Design*, 2nd edition, 2nd printing, American Institute of Steel Construction, Chicago, IL.

Gomez, I.R., Deierlein, G.G. and Kanvinde, A.M. (2010), Exposed Column Base Connections Subjected to Axial Compression and Flexure, https://nees.org/resources/839.

Kanvinde, A.M. and Deierlein, G.G. (2011), "Recent Research on Column Base Plate Connections," *Modern Steel Construction*, Vol. 51, No. 4, pp. 42–44.

RISA-3D (2012), Software, Version 10.0.1, RISA Technologies, Foothill Ranch, CA.

SAP2000 (2011), Software, Version 15.1.0/1, Computers and Structures Inc., Walnut Creek, CA.

Young, W.C. and Budynas, R.G. (2001), *Roark's Formulas for Stress and Strain*, 7th edition, McGraw-Hill, New York, NY.