A Graphical Design Aid for Selecting Standard W-Shape Steel Beam-Columns with Minimum Weight

MOHAMMAD ALI SA'ADAT and MOHAMMAD REZA BANAN

ABSTRACT

This paper presents an alternative technique for selecting the lightest W-section for beam-column members. Following the AISC procedure, the number of required steps to find a feasible section highly depends on the initial trial section. A new graphical technique based on the AISC interaction formula for designing steel beam-columns is proposed. By employing the newly developed diagrams, the new approach quickly leads to the lightest feasible section for a member subjected to combined biaxial bending and axial force. The number of computational steps in the proposed method is slightly reduced from the current AISC procedure. Some advantages of the new proposed method are converging to the solution in fewer steps, imposing no limit on L_b, considering the effect of C_b, considering biaxial bending by simply using a modification factor and covering cases where L_b is not equal to KL.

Keywords: beam-column, design aid, graphical solutions.

INTRODUCTION

Prior to the third edition of the AISC *Load and Resistance Factor Manual of Steel Construction* (2001), the procedure for design of steel beam-columns was based on computing an approximate equivalent axial load then selecting a trial shape from the column load tables and finally checking the compliance of the section with those requirements specified by the 1999 AISC LRFD *Specification*.

Aminmansour (2000) rewrote the 1999 AISC LRFD interaction equations for beam-columns and developed design-aid tables in terms of three parameters—*b, m* and *n*—which, after adoption by the third edition AISC LRFD *Manual* (2001) in Part 6, became the parameters p, b_x and b_y, respectively. Hereinafter, we will refer to this method as either the AISC method or the current method. In the same year, Keil (2000) presented graphical design aids for $C_b = 1$. The curves were developed for a standard case where $KL = L_b = 0$. For other cases, he provided some linear transformation factors such that the same graphs developed for the standard case could be used. Hosur and Augustine (2007) improved Keil's technique to consider the effect of C_b (with a value between 1.0 and 2.3).

Although all these works are referred to as graphical design aids, they all require using some tables during the course of design. In each of these methods, the graphical design aid is used in the last step in place of using interaction equations.

The graphical design aids proposed by Keil (2000) and Hosur and Augustine (2007) could not handle cases with biaxial bending.

Mohammad Ali Sa'adat, Senior Student, Department of Civil and Environmental Engineering, School of Engineering, Shiraz University, Shiraz, Iran. Email: malisaadat92@gmail.com

Mohammad Reza Banan, Associate Professor, Department of Civil and Environmental Engineering, School of Engineering, Shiraz University, Shiraz, Iran (corresponding). Email: banan@shirazu.ac.ir

PROPOSED METHOD COMPARED WITH AISC METHOD

The proposed method as an alternative to the current method enjoys some advantages in terms of the following points of view.

- *Visualization:* Graphical design aids provide a visual representation of the design procedure, to some extent.
- *Least weight:* Following the current method, the selected shape may not be the lightest one, which must be verified by trying other shapes (at the expense of additional computation). It is important to remember, however, that the least weight solution may not always be the least expensive solution.
- *Initial depth:* The current method is more like a checking guide because the designer must know the initial depth of the W-shape before starting the design process.
- *Initial trial section independence:* Tables presented in Part 6 of the 14th edition of the AISC *Manual* (2011) are sorted by depth, and shapes with the same depth are sorted by their corresponding weights in descending

order. As a general rule, it is understood that between two shapes with the same weight, the deeper one has higher flexural strength. Thus, a good practice is picking a depth for the section—say, W12—and then finding a shape that satisfies the interaction equation. And as a final step, search among deeper and lighter shapes to find the lightest one if desired.

There are some cases, however, that violate this simple rule. For instance, a W8×31 has higher flexural strength than a W16×31 for $L_b > 13.5$ ft and higher axial strength for all KL (considering $C_b = 1$ and $F_y = 50$ ksi). Examples like this could be a source of confusion in selecting the best section. This case will be discussed later in Example 1.

Furthermore, in Part 1 of the 14th edition AISC *Manual* (2011), dimensions and properties, after categorizing shapes by their depth, the first subcategory is based on flange width and then by weight in a descending order. Because, for the same depth, the strength is more dependent on flange width rather than weight (e.g., a W40×372 has higher flexural strength than a W40×392 for $L_b > 10.5$ ft in spite of its lighter weight because of wider flanges), there is the possibility of not selecting the lightest section.

- *Speed:* Some reasons that make the current method relatively lengthy, which increases the possibility of errors, are as follows.
  - There are two interaction equations H1-1a and H1-1b in the 2010 AISC *Specification*, and one has to find out which one to use. Sometimes this step could be forgotten. So, if equation H1-1b must be used, then the user may not follow the appropriate required modifications.
  - Finding the lightest shape involves trying sections of different depths (and not necessarily deeper ones).
  - When $C_b \neq 1$, one has to check extra tables or diagrams to find or compute plastic moment capacity and consequently to recalculate coefficient b_x.

In the proposed new method, there is no need to decide which interaction formula is appropriate. The curves are ready to use. When the plotted curve is solid, the W-shape assigned to that curve is the lightest possible shape for the given combination of loads applied to the member. When the curve is represented by a dashed line, a lighter W-shape exists. There are no additional tables or diagrams. In most cases, the calculations can be done easily without using a calculator.

DEVELOPMENT OF THE PROPOSED DESIGN AIDS

The criterion for checking the adequacy of a member as a beam-column is its compliance with AISC *Specification* equations H1-1a or H1-1b. These interaction equations are based on the required and provided axial compressive and flexural strengths of a member.

If $\dfrac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{\phi_c P_c} + \frac{8}{9}\left(\frac{M_{rx}}{\phi_b M_{cx}} + \frac{M_{ry}}{\phi_b M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1a)}$$

If $\dfrac{P_r}{P_c} \leq 0.2$

$$\frac{P_r}{2\phi_c P_c} + \left(\frac{M_{rx}}{\phi_b M_{cx}} + \frac{M_{ry}}{\phi_b M_{cy}}\right) \leq 1.0 \qquad \text{(H1-1b)}$$

Required strengths P, M_{rx} and M_{ry} are determined from analysis. So, the provided strengths will govern the selection process of an appropriate member. A closer look at the AISC equations reveals that for a beam-column subjected to uniaxial bending about major axis ($M_y = 0$), with uniform moment factor ($C_b = 1$) and having unbraced length L_b (for the flexural design strength) the same as effective length KL (for the axial compressive design strength), these two equations can be rewritten as follows. Note that M_{cx} is the flexural capacity about the major axis for $C_b = 1$.

If $\dfrac{P_r}{P_c} \geq 0.2$

$$P_r = \phi_c P_c \left[1 - \frac{8}{9}\left(\frac{M_{rx}}{\phi_b M_{cx}}\right)\right] \qquad (1)$$

If $\dfrac{P_r}{P_c} \leq 0.2$

$$P_r = 2\phi_c P_c \left(1 - \frac{M_{rx}}{\phi_b M_{cx}}\right) \qquad (2)$$

These interaction equations can be plotted with two straight lines. Figure 1 shows the interaction curve for a section—in this case, a W44×230. Above and below each curve, additional parameters are included. All curves are developed for $F_y = 50$ ksi.

Remember that P_c is calculated based on buckling about the minor axis. If buckling occurs about the major axis, the effective length, $(KL)_x$, has to be modified as follows:

$$(KL)' = \frac{(KL)_x}{\left(r_x/r_y\right)} \tag{3}$$

PROCEDURE FOR USING PROPOSED DESIGN AID

In this section, we explain a unified procedure suitable for designing beam-column members (subjected to either compression or tension) by using the proposed design aid.

Beam-Column (Compression and Flexure)

While designing a beam-column, six different cases could happen, discussed in detail as follows.

Case 1. $M_y = 0$, $C_b = 1$ and $KL = L_b$

In this case, we have a beam column with uniaxial bending about the major axis. The only step that leads to the most economical section for this member is to locate a point with known P_r and M_{rx} on the graph corresponding to the given L_b. All sections above that point will be adequate for the given loading.

Case 2. $M_y \neq 0$, $C_b = 1$ and $KL = L_b$

In this case with biaxial bending, we can rewrite Equations 1 and 2 as follows.

$$\frac{M'}{\phi_b M_{cx}} = \frac{M_{rx}}{\phi_b M_{cx}} + \frac{M_{ry}}{\phi_b M_{cy}} = \frac{M_{rx}M_{cy} + M_{ry}M_{cx}}{\phi_b M_{cx} M_{cy}} \tag{4}$$

where M' is an equivalent bending moment, defined such that the designer could use the same diagrams as before.

Solving Equation 4 yields:

$$M' = M_{rx} + r_b M_{ry} \tag{5}$$

where

$$r_b = M_{cx}/M_{cy} \tag{6}$$

In this equation, the new variable r_b is the ratio of flexural capacity about the major axis $C_b = 1$ to flexural capacity about the weak axis. The recommended procedure for use of this method is to split the diagram into equal parts. In every step, add M_{ry} to the modified (or original) M_{rx} to compute new M' (Figure 2). For the nth step, we have

$$M' = M_{rx} + (n+1)M_{ry} \tag{7}$$

Those sections in the range of $(M_{rx} + M_{ry}, M_{rx} + (n+1)M_{ry})$ that satisfy $1 \leq r_b \leq n+1$ could be the answer. Finally, the equivalent bending moment, M', should be computed using Equation 5. Compliance with AISC interaction equations can be checked by using the developed diagrams.

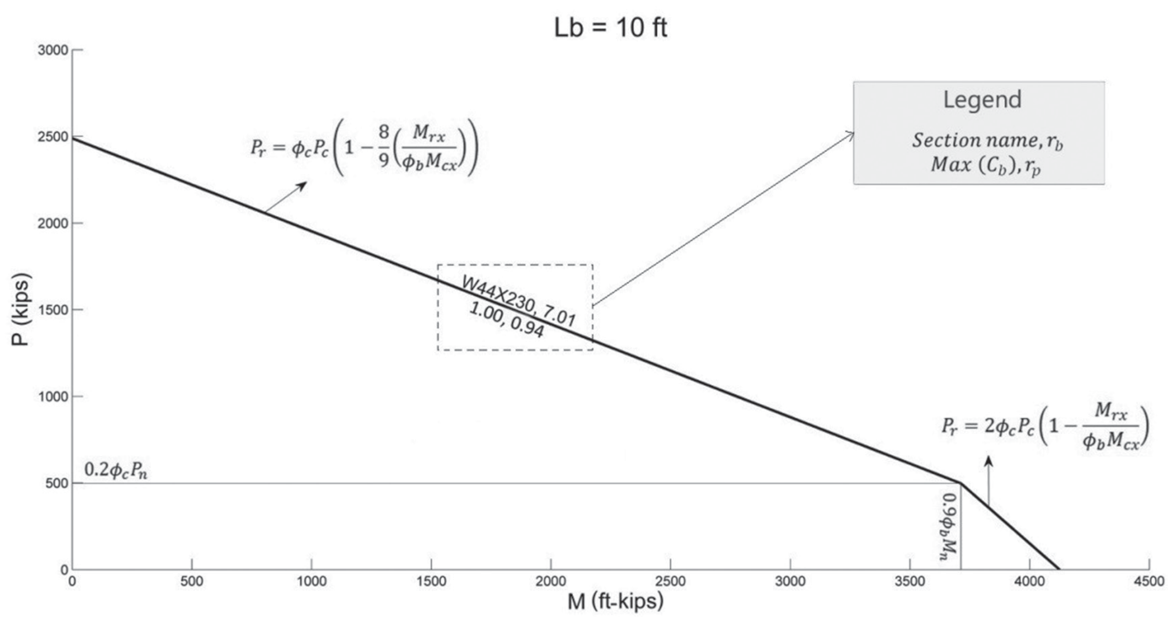

Fig. 1. Interaction curve for W44×230 *(L_b = KL = 10 ft and F_y = 50 ksi).*

Case 3. $M_y = 0$**,** $C_b \neq 1$ and $KL = L_b$

In the case where C_b assumes values other than 1.0, we just need to calculate the transformed bending moment, M' (Equation 8), and then adjust the diagrams with given P and calculated M. Because the bending moment capacity of a member cannot exceed the plastic moment capacity when a shape is compact and when either yielding controls or compression flange yielding governs the strength for a shape with noncompact web, the maximum value that C_b can take is specified for each section on the diagrams.

$$M' = \frac{M}{C_b} \tag{8}$$

Case 4. $M_y \neq 0$**,** $C_b \neq 1$ and $KL = L_b$

In this case, modification will be as given in Equation 9. The procedure to select the lightest cross-section will be the same as the procedure used in case 3.

$$M' = \frac{M_{rx}}{C_b} + r_b M_{ry} \tag{9}$$

Case 5. $M_x = 0$ and $M_y \neq 0$

This case is similar to case 2, except $M_{rx} = 0$. The same procedure as defined for case 2 must be followed.

Case 6. $KL \neq L_b$

For this case, we have

$$\frac{P_r'}{P_{c_{\text{at } L_b}}} = \frac{P_r}{P_{c_{\text{at } KL}}} \tag{10}$$

where P_r' is an equivalent axial load defined such that the same diagrams developed before can be used. Now, Equation 10 will be rearranged as follows:

$$\frac{P_r'}{P_r} = \frac{P_{c_{\text{at } L_b}}}{P_{c_{\text{at } KL}}} = \frac{\dfrac{P_{c_{\text{at } L_b}}}{P_{c_{\text{at } KL=0}}}}{\dfrac{P_{c_{\text{at } KL}}}{P_{c_{\text{at } KL=0}}}} \tag{11}$$

By introducing $r_{p \text{ at } \beta} = \dfrac{P_{c_{\text{at } \beta}}}{P_{c_{\text{at } KL=0}}}$, which is the ratio of axial compression capacity at $KL = \beta$ to axial compression capacity at $KL = 0$, the equivalent axial load, P_r', can be computed as given in Equation 12.

$$P_r' = \frac{r_{p \text{ at } L_b}}{r_{p \text{ at } KL}} P_r \tag{12}$$

To select the lightest section, we have to compute equivalent axial load and locate the corresponding point on the proper curve.

Beam-Column (Tension and Flexure)

If the beam-column is subjected to tension instead of compression, either of the two limit states of tension yielding and tension rupture might govern.

Case 1. Tension yielding governs the tensile capacity of the member: The tension yielding strength, $\phi F_y A_g$, is equal to compression strength for $KL = 0$; therefore, we should follow the same procedure as explained for the case with $KL \neq L_b$. Because $r_{p \text{ at } KL=0} = 1$, the axial load must be modified to P_r' as follows:

$$P_r' = \frac{r_{p \text{ at } L_b}}{r_{p \text{ at } KL=0}} = r_{p \text{ at } L_b} P_r \tag{13}$$

Case 2. Tension rupture governs the tensile capacity of the member. By assuming shear lag factor $U = 0.75$, we must simply modify the axial load and compute the equivalent axial force, P_r', as follows:

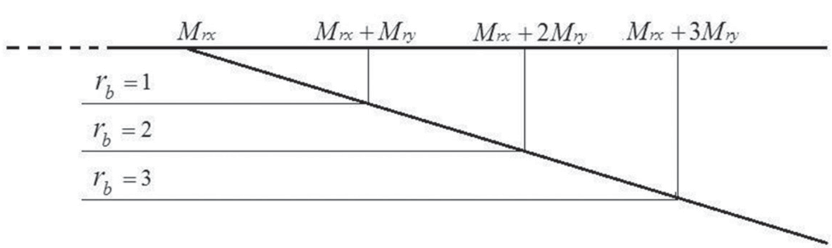

Fig. 2. Distribution of r_b *for different values of* M'.

Table 1. Average Relative Errors and Standard Deviation for Interpolation between Diagrams		
	Average Relative Error	**Standard Deviation**
M_{cx}	0.000957	0.00245
P_c	0.002669	0.006418
M for constant value of P*	0.002316	0.003973

* Error for this case is dependent on value of P_r, so for calculating error, the value of P_r has been changed from $0.1P_c$ at $KL = 0$ to $0.9P_c$ at $KL = 0$ for each section, and error has been calculated for different KL. The overall average relative error and standard deviation are reported.

$$\frac{\phi_t F_y A_g}{\phi_t F_u A_e} = \frac{(0.9)(50)A_g}{(0.75)(65)(0.75)A_g} = 1.23 \tag{14}$$

which leads to

$$P_r' = (1.23)\left(r_{p \text{ at } L_b}\right)P_r \tag{15}$$

Beams

To design a beam subjected to uniaxial or biaxial bending, we can follow the same procedure as that given for a beam-column with $P_r = 0$.

Columns

To design a column, the procedure is the same as the one given for a beam-column, except M_{rx} and M_{ry} will be zero.

Interpolation

If no diagram for a specific L_b is provided, we can conservatively design the member for a larger L_b. But if more accuracy is desired, we can compute M for the given L_b and P by using linear interpolation between two values for M corresponding to a smaller and a larger value for L_b. Then the section must be checked for the given loads.

The error and corresponding standard deviation for M_{cx} and P_c from different cases are computed and summarized in Table 1. Because these values are small, we can conclude that interpolation between diagrams with different L_b is permitted.

Because C_b, r_b and r_p are explicitly related to M_{cx} and P_c, interpolation is permitted for these parameters as well.

EXAMPLES

To show the validity of the new developed method, some problems will be simultaneously solved by using the AISC and the proposed methods. For each case, it is assumed that all given loads are determined through a second-order analysis. Therefore, no modification on either P or M is required.

Example 1. [Variation of Aminmansour (2000) Example 1]

Given: $P_u = 179$ kips, $M_{ntx} = 47.6$ ft-kips, $L_b = (KL)_y = 10$ ft. Select the lightest W-shape of ASTM A992 steel.

To follow the proposed method, locate the corresponding point on the appropriate diagram (Figure 4) and move to the right to find the answer.

First section is W80×31, and because the curve is solid, the designer will be assured that it is the most economical section.

Example 2. [Aminmansour (2000) Example 2]

Given: $P_u = 400$ kips, $M_{ntx} = 250$ ft-kips, $M_{nty} = 80$ ft-kips, $L_b = (KL)_y = 14$ ft. Select the lightest W14 shape of ASTM A992 steel.

In Figure 5, look for W14s.

Step 1. $330 \le M' \le 410$ and $r_b \le 2$ $\therefore$ No answer

Step 2. $410 \le M' \le 490$ and $r_b \le 3$

W14×99 with $r_b = 2.06 \rightarrow M' = 250 + (2.06)(80) = 415$ kip-ft $\rightarrow$ Good

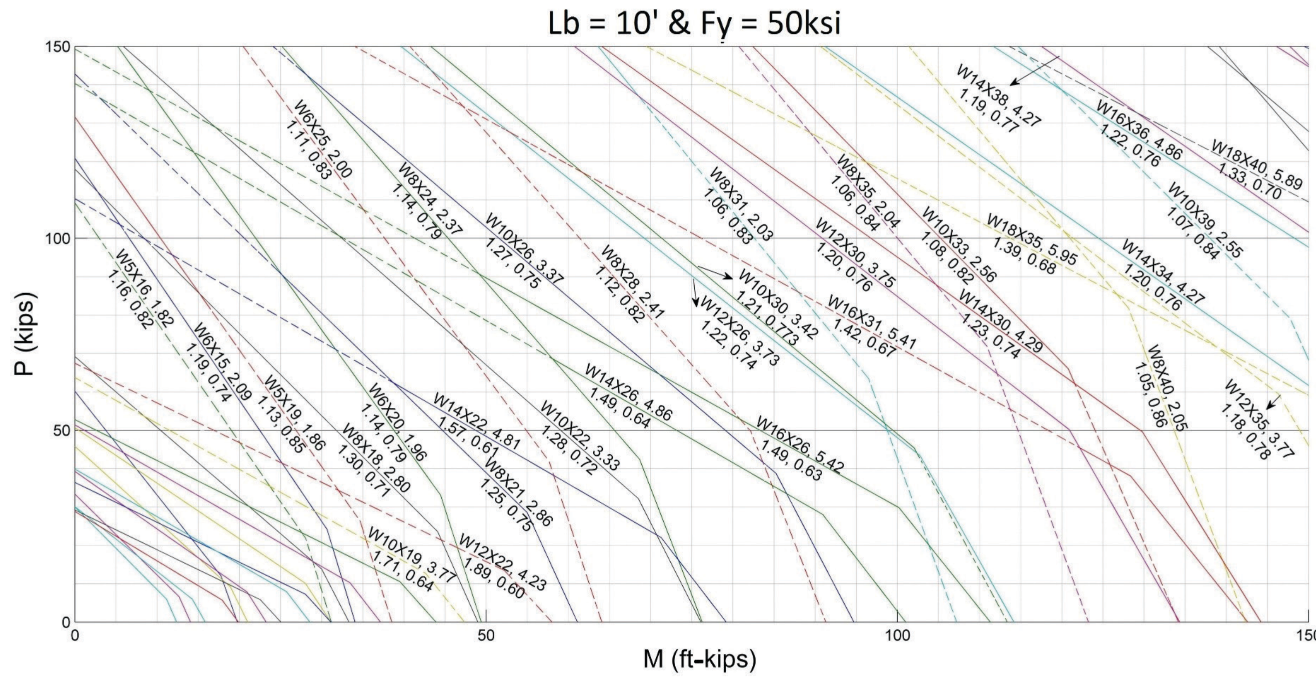

Fig. 3. Proposed design aid (part 1).

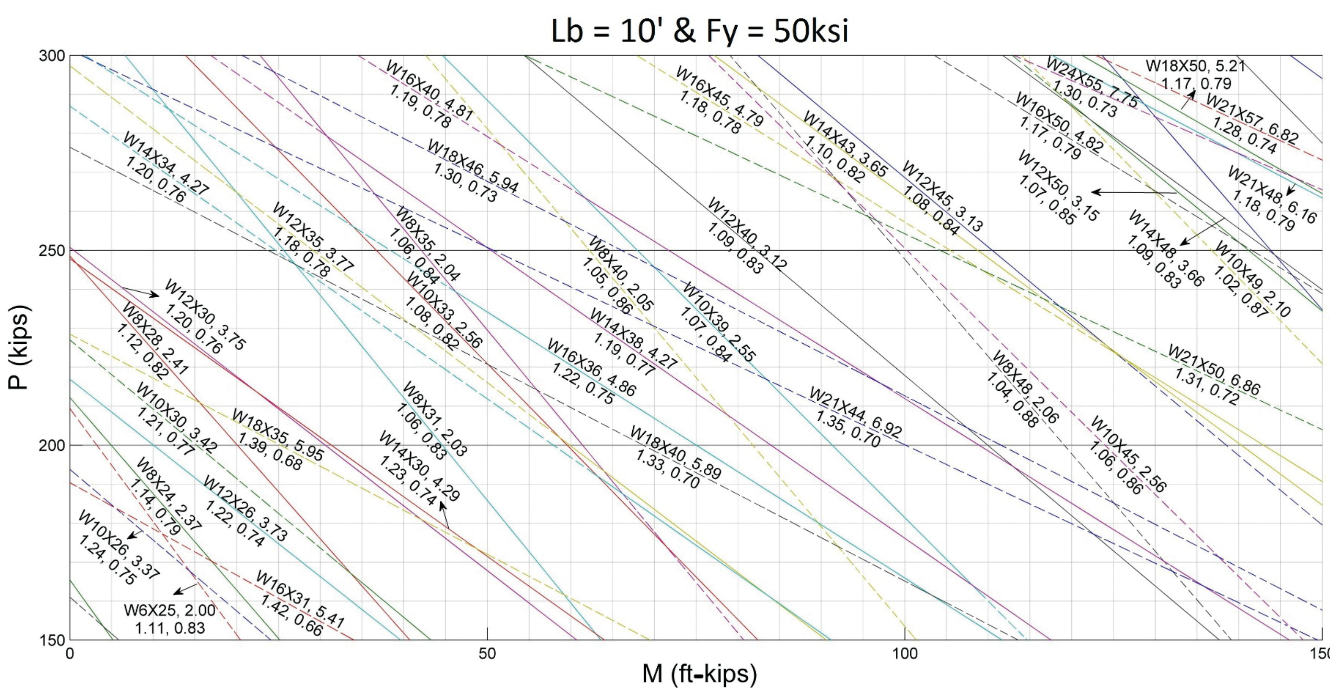

Fig. 4. Proposed design aid (part 2).

Example 3.

Given: P_u = 400 kips, M_{ntx} = 600 ft-kips, C_b = 1.5, $L_b = (KL)_y$ = 14 ft. Select the lightest W-shape of ASTM A992 steel.

Compute M' as follows:

$$M' = \frac{M}{C_b} = \frac{600}{1.5} = 400 \text{ ft-kips}$$

Then using Figure 5:

Step 1. Try W24×103 with C_b = 1.22

$$M' = \frac{600}{1.22} = 492 \text{ ft-kips} \rightarrow \text{N.G.}$$

Step 2. Try W30×99 with C_b = 1.23. This section will be acceptable with a C_b = 1.22. Because the curve is solid, it is the lightest shape.

Example 4.

Given: P_u = 300 kips, M_{ntx} = 375 ft-kips, M_{nty} = 50 ft-kips, C_b = 1.5, $L_b = (KL)_y$ = 14 ft. Select the lightest W-shape of ASTM A992 steel.

From Figure 5, we have:

Step 1. $\frac{375}{1.5} + 50 = 300 \leq M' \leq 350$ and $r_b \leq 2 \therefore$ No answer

Step 2. $350 \leq M' \leq 400$ and $r_b \leq 3 \therefore$ No answer

Step 3. $400 \leq M' \leq 450$ and $r_b \leq 4$

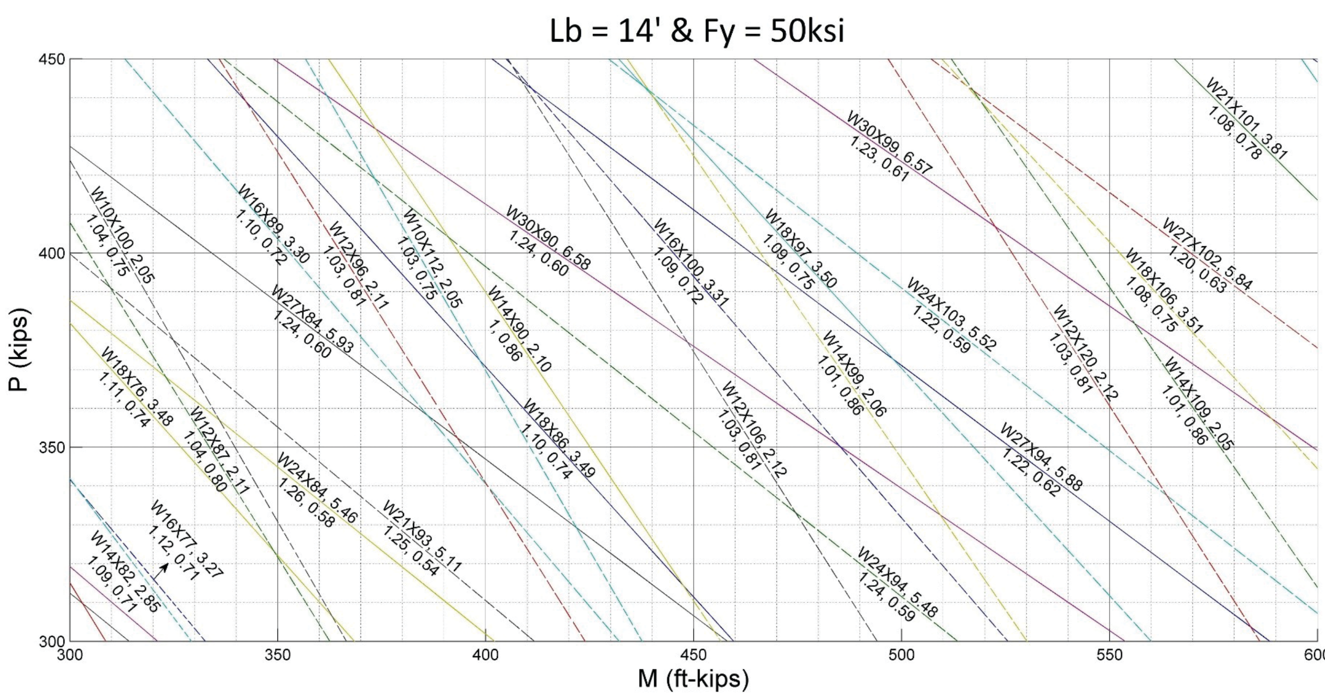

Fig. 5. Proposed design aid (part 3).

Try W12×96: $M' = \dfrac{375}{1.03} + (2.11)(50) = 470 \text{ ft-kips} \rightarrow \text{N.G.}$

Try W10×112: $M' = \dfrac{375}{1.03} + (2.05)(50) = 467 \text{ ft-kips} \rightarrow \text{N.G.}$

Step 4. $450 \le M' \le 500$ and $r_b \le 5$

W14×90 will not work because this section has the same r_b but a smaller C_b than the W12×96 has, so it has a lower capacity.

W18×86 will not work because of having a very large value for r_b.

Try W12×106: $r_b = 2.12$ and $C_b = 1.03$. Because it has the same values for C_b and r_b as a W12×96, no calculation is needed and it will work.

Step 5. $500 \le M' \le 550$ and $r_b \le 6$

Try W16×100: $M' = \dfrac{375}{1.09} + (3.31)(50) = 480 \text{ ft-kips} \rightarrow \text{Good}$

Try W14×99; $r_b = 2.06$ and $C_b = 1.01$

Step 6. $550 \le M' \le 600$ and $r_b \le 7$

Try W18×97: $M' = \dfrac{375}{1.09} + (3.5)(50) = 520 \rightarrow \text{Good}$

Because the next solid curve with the same weight lighter than W18×97 is W27×94—which is not adequate—the next lighter shape with a solid curve is W30×99. Thus, W18×97 is the lightest shape.

CONCLUSION

The AISC method is an accurate technique for determining the capacity of a section or to check the adequacy of a given section as a beam-column member. A design aid (Figures 3, 4 and 5) is proposed that leads to the minimum weight design for a beam-column with slightly less computational effort compared with the current method. It is important to remember that the least weight solution is not always the least cost solution; many other factors in the design and construction process must be considered as well. The proposed technique is simple, straightforward and covers the selection of beam-column members as well as the selection of individual beam or column members.

ACKNOWLEDGMENT

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REFERENCES

AISC (1999), *LRFD Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL.

AISC (2001), *Load and Resistance Factor Design Manual of Steel Construction*, 3rd ed., American Institute of Steel Construction, Chicago, IL.

AISC (2010), *Specification for Structural Steel Buildings*, ANSI/AISC 360-10, American Institute of Steel Construction, Chicago, IL.

AISC (2011), *Steel Construction Manual*, 14th ed., American Institute of Steel Construction, Chicago, IL.

Aminmansour, A. (2000), "A New Approach for Design of Steel Beam-Columns," *Engineering Journal*, AISC, Vol. 37, No. 2, pp. 41–72.

Hosur, V. and Augustine, B. (2007), "Graphical Design Aid for Beam-Columns (LRFD)," *Engineering Journal*, AISC, Vol. 44, No. 2., pp. 147–180.

Keil, W.J. (2000), "LRFD Beam-Column Graphical Design Aid," *Engineering Journal*, AISC, Vol. 37, No. 3, pp. 99–119.