Admissible Loads of Gable Frames

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THE MECHANISM method and the statical method have been presented in standard texts[1, 2, 3, 4] for plastic analysis and design of gable frames. AISC[5] devised charts for both calculating plastic moments and locating plastic hinges of various gable frames subjected to uniform forces with or without wind load. It is understandable that those charts cannot be applied to frames subjected to concentrated loads, simply because the collapse load computed under the concentrated forces is a lower bound on the collapse load of the same structure with equivalent distributed force.[6] Pincus[7] discussed a generalized superposition method in plastic analysis of gable frames for which tedious procedures similar to that of the mechanism method are used to search for cancellation of plastic hinges.

In this report, two domains of admissible loads of general gable frames are presented, in which the concentrated forces are independent and the structural dimensions are arbitrary. For given numerical values of structural dimensions and forces of a particular problem, the plastic moment and collapse mode can be simply determined from the appropriate domain; or for the given numerical values of structural dimensions and the magnitude of plastic moment, the possible collapse load of the frame may also be found. It is interesting to note that the condition of a structural collapse may be studied from the relationship of magnitude as well as direction of the independent forces in the domain.

EQUATIONS FOR DOMAINS OF COLLAPSE LOADS

Let the hinge-supported gable frame shown in Fig. 1 be subjected to a set of independent forces, wind load F_1 and gravity load F_2, in which the structural dimensions are arbitrary as L, CL, and QCL. The sign convention consists of placing a dashed line as positive moment on the tension side of all members in the structure sketched. Using dimensionless factors $f_1 = F_1L/M_p$; $f_2 = F_2L/M_p$; $m_i = M_i/M_p$, where i = number of moments sketched, the equations of equilibrium associated with the elementary mechanisms shown in Figs. 2a to 2d are:

$$-m_1 + 2m_2 - m_3 = 0.5f_2 \tag{1}$$

$$-m_3 + 2m_4 - m_5 = 0.5f_2 \tag{2}$$

$$m_1 - m_5 = Cf_1 \tag{3}$$

$$-m_1 + 2m_3 - (1 + 2Q)m_5 = f_2 \tag{4}$$

The elementary mechanisms of the fix-supported gable frame shown in Fig. 3 are similar to those of Figs. 2a to 2d, of which the equations of equilibrium can be expressed as:

$$-m_2 + 2m_3 - m_4 = 0.5f_2 \tag{5}$$

$$-m_4 + 2m_5 - m_6 = 0.5f_2 \tag{6}$$

$$-m_1 + m_2 - m_6 + m_7 = Cf_1 \tag{7}$$

$$-m_2 + 2m_4 - (1 + 4Q)m_6 + 2Qm_7 = f_2 \tag{8}$$

By using the method of inequalities,[1] the equations which completely characterize the restrictions on the collapse load of Fig. 1 may be expressed as:

$$f_2 = 4 + Q \tag{9}$$

$$f_2 = -4 - Q \tag{10}$$

$$f_1 = 2/C \tag{11}$$

$$f_1 = -2/C \tag{12}$$

$$3Cf_1 + 2f_2 = 8 + 2Q \tag{13}$$

$$-3Cf_1 - 2f_2 = 8 + 2Q \tag{14}$$

$$3Cf_1 + 2QCf_2 - 2f_2 = 8 + 2Q \tag{15}$$

$$-3Cf_1 - 2QCf_2 + 2f_2 = 8 + 2Q \tag{16}$$

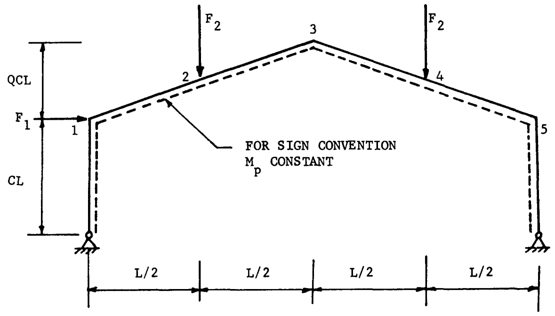

Fig. 1. Hinge-supported gable frame

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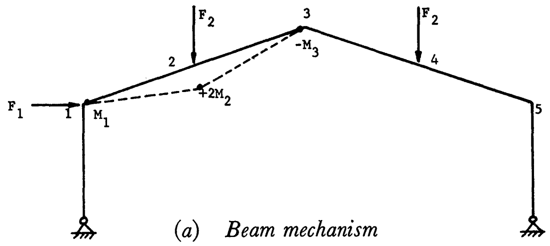
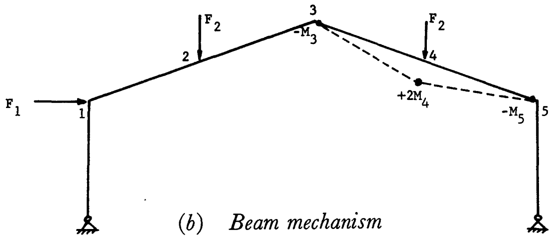
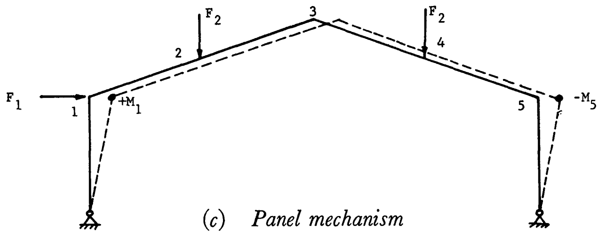


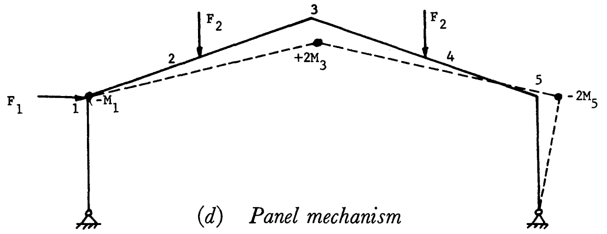
(a) *Beam mechanism*



(b) *Beam mechanism*



(c) *Panel mechanism*



(d) *Panel mechanism*

Fig. 2. Elementary mechanisms

Similarly, the equation which describes the restrictions on the collapse load of Fig. 3 can be formulated in Equations (17) to (26).

$$f_2 = 4 + 2Q \tag{17}$$

$$f_2 = -4 - 2Q \tag{18}$$

$$3Cf_1 + 2f_2 = 14 + 4Q \tag{19}$$

$$-3Cf_1 - 2f_2 = 14 + 4Q \tag{20}$$

$$f_1 = 4/C \tag{21}$$

$$f_1 = -4/C \tag{22}$$

$$(1 + 2Q)Cf_1 - 2f_2 = 6 + 4Q \tag{23}$$

$$-(1 + 2Q)Cf_1 + 2f_2 = 6 + 4Q \tag{24}$$

$$2QCf_1 - 2f_2 = 8 + 4Q \tag{25}$$

$$-2QCf_1 + 2f_2 = 8 + 4Q \tag{26}$$

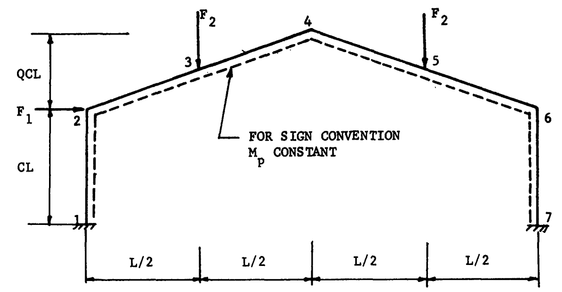


Fig. 3. Fix-supported gable frame

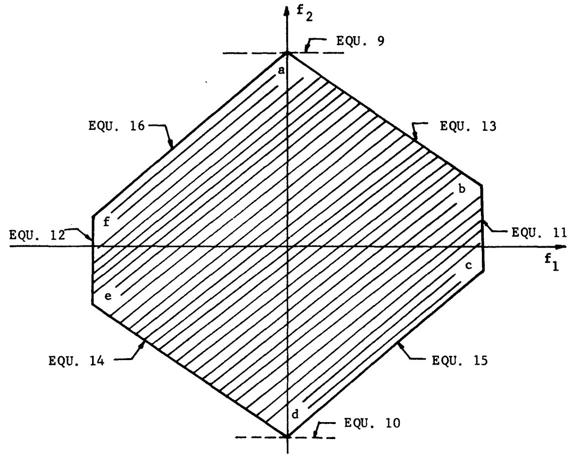


Fig. 4. Domain of admissible loads of Fig. 1

The restrictions of Equations (9) to (16) are now represented by a convenient geometrical description as a domain of admissible loads shown in Fig. 4; Equations (17) to (26) are also geometrically described by means of the domain sketched in Fig. 5.

EXAMPLES

The use of domains as aids to plastic design and analysis is illustrated in numerical examples with the following assumed information:

$L = 30$ ft; $C = 0.5$; $Q = 0.5$;
$F_1 = 20$ kips; $F_2 = 40$ kips; Steel = A36

Example 1—Find the plastic moment, M_p, of a hinge-supported gable frame.

Solution: Since F_1 and F_2 are both positive, the admissible load must be in the first quarter of the domain shown in Fig. 4. It is found that f_1 and f_2 at the discontinuous point, b, are 4 and 1.5, respectively. Therefore Equation (13) should be used, from which $f_1 = 9/5.5$, or $M_p = 20 \times 30 \times 5.5 \times 1.4/9 = 513$ kip-ft. It may be noted that this result agrees with Reference 7.

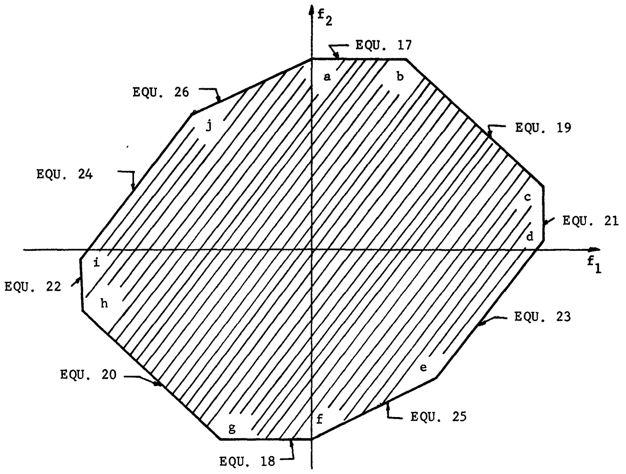


Fig. 5. Domain of admissible loads of Fig. 3

Example 2—Find the plastic moment, M_p, of the frame in Example 1 by using AISC charts[5] with the equivalent uniform load of 40/30 = 1.33 kips-ft.

Solution: Since the ratio of the moment due to horizontal load and that due to vertical load is

$$20(15)/1.33(60)(30) = 0.125$$

the plastic moment obtained from the chart is

$$M_p = 0.05(1.33)(60)(60)(1.85) = 443 \text{ kip-ft}$$

Therefore the required plastic modulus for the equivalence uniform load is only 86.3 percent of that required for concentrated load.

Example 3—The plastic moment of the fix-supported gable frame with the same assumed information as Example 2 may be obtained from the domain shown in Fig. 5. The coordinates f_1 and f_2 at point **c** have been calculated as 8 and 2. The collapse load is bounded by Equation (19), from which one may obtain $f_1 = 16/5.5$ or $M_p = 20(30)\ (5.5)(1.4)/16 = 289$ kip-ft.

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