# Stability Design of Cross-Bracing Systems for Frames

ERIC M. LUI and XIAORAN ZHANG

## ABSTRACT

In this study, the inelastic load-carrying capacity of the compression diagonal of a typical cross-bracing system used in concentrically braced frames under gravity and wind loads is investigated, taking into consideration its interaction effect with the tension diagonal. Depending on the lateral stiffness of the tension diagonal, the compression diagonal can be fully or partially braced by the tension diagonal at their intersection point. An expression for the transition lateral stiffness, $k_{st}$, that demarcates the fully and partially braced conditions is derived. When the compression diagonal is fully braced, its maximum load-carrying capacity is a function of its member slenderness, $L/r$, only. However, when the compression diagonal is partially braced, its load capacity is dependent upon both its member slenderness, $L/r$, and the lateral stiffness, $k_s$, of the tension diagonal. Once the equations for the maximum load-carrying capacity of the compression diagonal are established, design guidelines are proposed and design examples are given to demonstrate how the proposed guidelines can be used for the design of cross-bracing systems in steel frames. The consideration of the lateral bracing effect will result in a more economical and logical design for such bracing systems.

**Keywords:** cross-bracing systems, steel frames, inelastic analysis, stability design.

## INTRODUCTION

Cross braces are used in concentrically braced steel frames to provide resistance against excessive sway caused by horizontal loads. They are also used in industrial buildings to resist crane surge and in roof trusses to account for load reversal under wind uplift (Kitipornchai and Finch, 1986).

Under a lateral load, one member of this cross-bracing system is often under tension, while the other is subjected to compression. However, in conventional design of cross-bracing systems for wind load, a common yet conservative assumption is that only the tension diagonal resists the applied lateral load. The contribution of the compression diagonal to resist frame sway is neglected (El-Tayem and Goel, 1986). Although this assumption can simplify the design, the result is an overdesign of the bracing system. A somewhat less conservative approach is to design the compression diagonal as a column supported at midspan by the tension diagonal. Timoshenko and Gere (1961) derived the relationship for the elastic buckling load of a column braced at mid-point. However, the nonlinear relationship is rather complicated and is therefore difficult to apply in a design situation.

To achieve a more concise and practical method for design, Picard and Beaulieu (1987, 1988) carried out a series of analytical and experimental studies to establish the relationship between the ultimate strength of the bracing system and the internal forces in both diagonals. They recommended the use of an effective length factor $K$ of 0.5 applied to the full length of the member in the design of the compression diagonal. Nevertheless, their research is limited to the condition in which no out-of-plane translational movement is experienced by either diagonal at their intersection point.

Stoman (1989) provided a set of effective length spectra for cross bracing within the elastic range. However, Stoman's study did not provide any formula to quantify the lateral support to the compression diagonal by the tension diagonal. Moon et al. (2008) proposed values for the effective length factor $K$ for use in an elastic design of the compression diagonal and checked the validity of the proposed $K$ factors with the AISC equation for the inelastic case. However, no direct mathematical relationship relating the inelastic ultimate strength of the compression diagonal with other system parameters is given. The objectives of this article are, therefore, to investigate the interaction effect between the compression and tension diagonals of a typical cross-bracing system under combined gravity and wind loads and to develop equations suitable for use in the design of such system.

If the compression and tension diagonals are connected at their intersection point and if all the connections are properly designed for strength and ductility (Sabelli and Hohbach, 1999), the ultimate strength of the cross bracing is mostly controlled by the out-of-plane buckling capacity of the compression diagonal. This out-of-plane buckling capacity can be more accurately determined if the tension diagonal is

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taken into consideration in design. Even though the tension diagonal may not always have the necessary lateral stiffness to enable the compression diagonal to buckle in its second mode, it often provides sufficient out-of-plane restraint to the compression diagonal at the intersection point so that the ultimate strength of the compression diagonal will be higher than that predicted based on its first buckling mode (i.e., by ignoring the bracing effect provided by the tension diagonal altogether).

The amount of increase in the buckling capacity is a function of the slenderness ratios, $L/r$, boundary conditions and effective stiffness (stiffness accounting for the axial force effect) of both the compression and tension diagonals. In addition, geometrical imperfections and inelastic behavior of the cross-braced members will also have an effect on system stability. The objective of this study is to investigate the effects these factors have on the ultimate strength of a typical cross-bracing system. This study will focus on cross-bracing systems primarily used for concentrically braced frames, that is, a symmetrical system in which the intersection point occurs at the braces' half-lengths and that the connection provides full continuity (e.g., the use of welded or fully bolted connections) for both braces as shown in Figure 1. In the current analysis, it is assumed that system behavior is controlled by out-of-plane buckling of the braces as depicted in Figure 2. As will be discussed in more detail in a later section, the symmetric out-of-plane buckling mode corresponds to the partially braced condition and the anti-symmetric out-of-plane buckling mode corresponds to the fully braced condition. Unless both diagonals are subjected to the same compressive force, one will provide lateral support to the other. Normally, the brace that is providing the lateral support is in tension, but even when both braces are in compression, the brace that is subjected to a lower compressive force can still brace the one with a higher compressive force, although the amount of lateral support that can be relied upon in this scenario will undoubtedly be lower.

## CROSS-BRACING SYSTEM MODEL

In reference to a diagonal cross-bracing system shown in Figure 3, if the beam-column joints to which the ends of the cross-bracing system are connected are braced against out-of-plane deflections, the support conditions of the cross-bracing system can conservatively be idealized as pinned. If we denote $P$ as the compressive force acting on the compression diagonal with length $L$ and flexural rigidity $EI$, and $\overline{P}$ as the tensile force acting on the tension diagonal with length $\overline{L}$

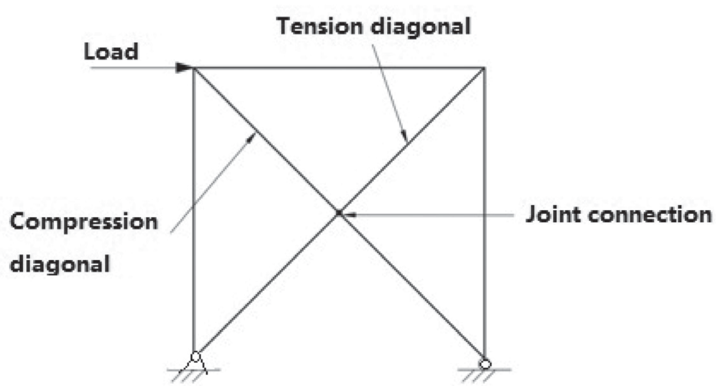


*Fig. 1. Steel frame with a cross-bracing system.*

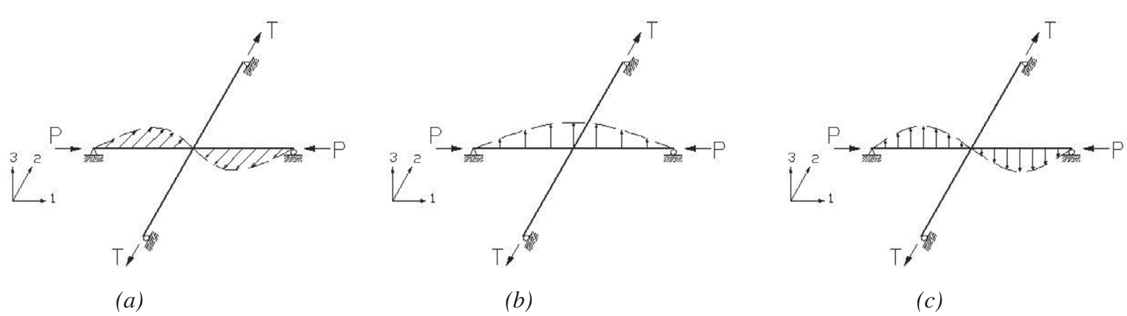


*Fig. 2. Buckling modes of a cross-bracing system: (a) in-plane; (b) symmetric out-of-plane; (c) anti-symmetric out-of-plane.*

and flexural rigidity $\overline{EI}$, the differential equations that govern the out-of-plane instability behavior of the first half (i.e., $0 \le x \le L/2$, $0 \le \bar{x} \le \bar{L}/2$) of the compression and tension diagonals (Figures 4a and 4b) can be written, respectively, as (Chen and Lui, 1987)

$$\frac{d^2 y}{dx^2} + k^2 y = \frac{Q}{2EI} x \tag{1}$$

$$\frac{d^2 \bar{y}}{d\bar{x}^2} - \bar{k}^2 \bar{y} = -\frac{\bar{Q}}{2\overline{EI}} \bar{x} \tag{2}$$

where $k^2 = \dfrac{P}{EI}$, $\bar{k}^2 = \dfrac{\bar{P}}{\overline{EI}}$, and $Q = \bar{Q}$ is the action/reaction internal force pairs acting at the midpoints of the two diagonals.

By using the pinned support condition and symmetry at midpoint, the solutions to the differential Equations 1 and 2 that satisfy the boundary conditions of $y_{x=0} = \bar{y}_{\bar{x}=0} = 0$, $\left.\dfrac{dy}{dx}\right|_{x=\frac{L}{2}} = \left.\dfrac{d\bar{y}}{d\bar{x}}\right|_{\bar{x}=\frac{\bar{L}}{2}} = 0$ are

$$y = -\frac{QL^3 \sin(kx)}{2(kL)^3 EI \cos\left(\frac{kL}{2}\right)} + \frac{QL^2 x}{2(kL)^2 EI} \qquad 0 \le x \le L/2 \tag{3}$$

$$\bar{y} = -\frac{\bar{Q}\bar{L}^3 \sinh(\bar{k}\bar{x})}{2(\bar{k}\bar{L})^3 \overline{EI} \cosh\left(\frac{\bar{k}\bar{L}}{2}\right)} + \frac{\bar{Q}\bar{L}^2 \bar{x}}{2(\bar{k}\bar{L})^2 \overline{EI}} \qquad 0 \le \bar{x} \le \bar{L}/2 \tag{4}$$

Because the two braces are connected at midpoint, compatibility requires that $y_{x=\frac{L}{2}} = \bar{y}_{\bar{x}=\frac{\bar{L}}{2}}$, and because $Q = \bar{Q}$, we have

$$\frac{Q}{y_{x=\frac{L}{2}}} = \frac{\bar{Q}}{\bar{y}_{\bar{x}=\frac{\bar{L}}{2}}} \tag{5}$$

and with respect to the compression diagonal, the above equation can be written as

$$\frac{2\,EIk^3}{\frac{kL}{2} - \tan\left(\frac{kL}{2}\right)} = k_s \tag{6}$$

where

$$\begin{aligned} k_s &= \frac{\bar{Q}}{\bar{y}_{\bar{x}=\frac{\bar{L}}{2}}} = \frac{2\,\overline{EI}\,\bar{k}^3}{\frac{\bar{k}\bar{L}}{2} - \tanh\left(\frac{\bar{k}\bar{L}}{2}\right)} \\ &= \left[\frac{\left(\bar{k}\bar{L}\right)^3}{24\left(\frac{\bar{k}\bar{L}}{2} - \tanh\left(\frac{\bar{k}\bar{L}}{2}\right)\right)}\right]\frac{48\overline{EI}}{\bar{L}^3} \\ &\approx \left[1 + \frac{\left(\bar{k}\bar{L}\right)^2}{10} - \frac{\left(\bar{k}\bar{L}\right)^4}{8400}\right]\frac{48\overline{EI}}{\bar{L}^3} \end{aligned} \tag{7}$$

is the lateral stiffness the tension diagonal is imparting to the compression diagonal when the system is experiencing out-of-plane instability. A plot of Equation 7 is shown in Figure 5 as a solid line. Also shown in the figure as a dashed line is the case when the "tension" diagonal is also under compression. This condition may occur when the superimposed dead and live gravity loads are high compared to the wind load. In this case, $k_s$ is given by

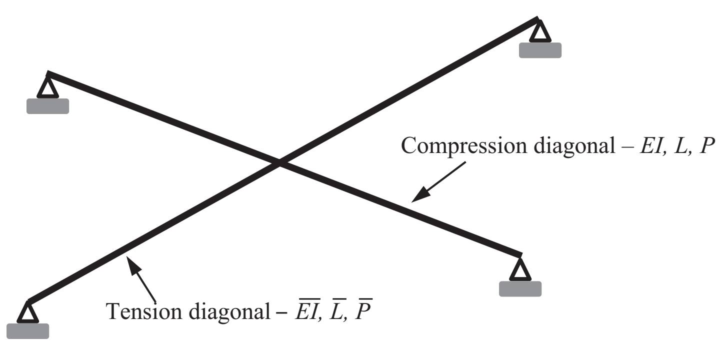


*Fig. 3. Analytical model of a cross-bracing system.*

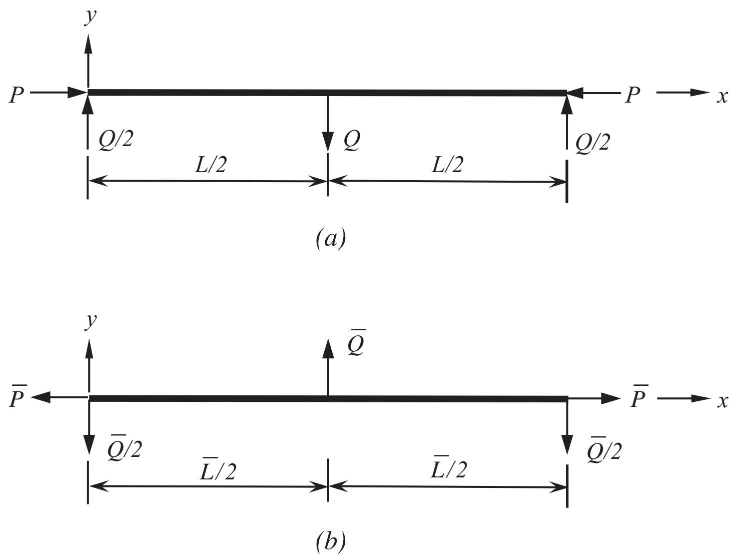


*Fig. 4. Free-body diagrams of (a) compression diagonal and (b) tension diagonal.*

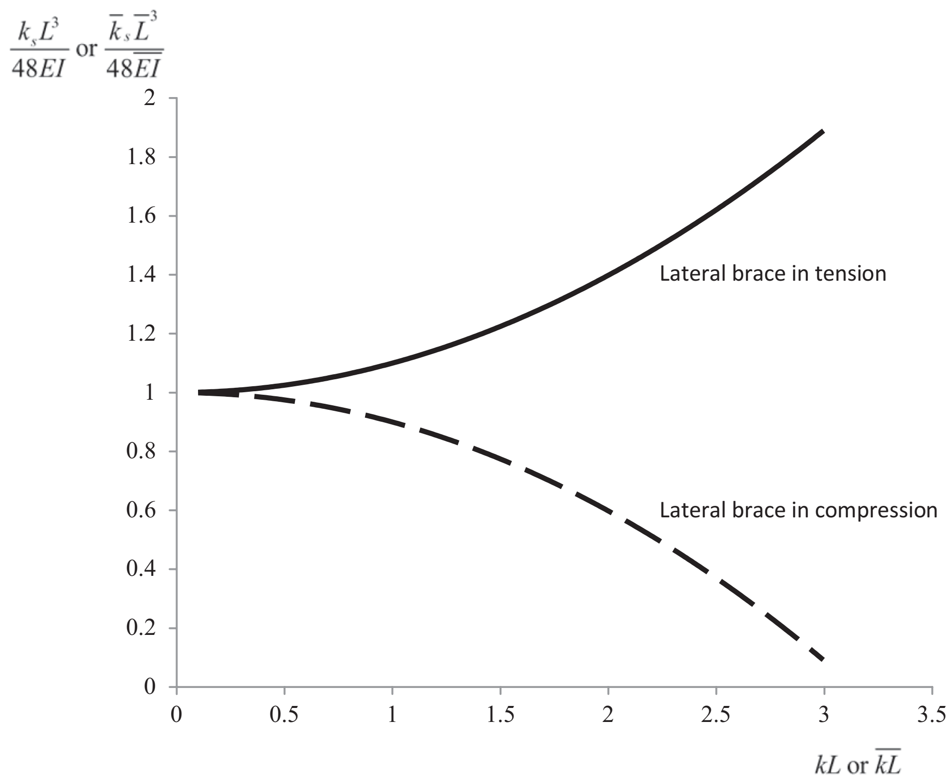


*Fig. 5. Lateral bracing stiffness variation.*

$$k_s = \frac{2\,\overline{EI}\overline{k}^3}{\tan\left(\frac{\overline{kL}}{2}\right) - \frac{\overline{kL}}{2}} = \left[\frac{\left(\overline{kL}\right)^3}{24[\tan\left(\frac{\overline{kL}}{2}\right) - \frac{\overline{kL}}{2}]}\right]\frac{48\overline{EI}}{\overline{L}^3} \tag{8}$$

$$\approx \left[1 - \frac{\left(\overline{kL}\right)^2}{10} - \frac{\left(\overline{kL}\right)^4}{8400}\right]\frac{48\overline{EI}}{\overline{L}^3}$$

In the preceding equations, $\frac{48\,\overline{EI}}{\overline{L}^3}$ is the lateral stiffness of the supporting brace when $\overline{P} = 0$. If the force in the supporting brace is tensile, the terms inside the brackets of Equation 7 represent the magnification effect of tension stiffening, whereas if the force in the supporting brace is compressive, the terms inside the brackets of Equation 8 represent the reduction effect of compression softening. The first three terms of a Taylor series expansion of the bracketed terms are provided in Equations 7 and 8 as well. They are accurate to within 1.2% of the theoretical values in the range ($0 \le \overline{kL} \le 3$) and should be used when $\overline{kL} = 0$ because the theoretical expressions become indeterminate at $\overline{kL} = 0$. If the axial compressive force in the bracing member exceeds $0.4P_y$ where $P_y$ is the yield load, the tangent modulus should be used in place of the elastic modulus in Equation 8.

If we substitute $k = \sqrt{\frac{P}{EI}} = \frac{\pi}{L}\sqrt{\frac{P}{P_e}}$, where $P_e = \frac{\pi^2 EI}{L^2}$, into Equation 6, the equation can be expressed in a nondimensional form (Timoshenko and Gere, 1961) as

$$\frac{2\pi\left(\sqrt{\frac{P}{P_e}}\right)^3}{\frac{\pi}{2}\sqrt{\frac{P}{P_e}} - \tan\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)} = \frac{k_s L}{P_e} \tag{9}$$

In the event that yielding has occurred in the compression diagonal, the concept of tangent modulus can be used, and for design purposes, the nominal compressive strength, $P_n$, can be used in place of $P_e$ in Equation 9. Thus, we have

$$\frac{2\pi\left(\sqrt{\frac{P}{P_n}}\right)^3}{\frac{\pi}{2}\sqrt{\frac{P}{P_n}} - \tan\left(\frac{\pi}{2}\sqrt{\frac{P}{P_n}}\right)} = \frac{k_s L}{P_n} \tag{10}$$

where, according to the AISC 360-10,

$$P_n = \begin{cases} \left(0.658^{\frac{F_y}{F_e}}\right)P_y & \text{when } \frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}} \text{ or } \frac{F_y}{F_e} \le 2.25 \\ \left(0.877\frac{F_e}{F_y}\right)P_y & \text{when } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} \text{ or } \frac{F_y}{F_e} > 2.25 \end{cases} \tag{11}$$

in which $F_y$ is the yield stress, $F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$ with $K = 1$ for a pinned-pinned member, $L$ is the full length of the compression diagonal, $r$ is the radius of gyration, $P_y = AF_y$ is the yield load, and $A$ is the cross-sectional area. Because $P_n$ as expressed in Equation 11 takes into consideration the member out-of-straightness effect, the effect of geometrical imperfections on member strength is implicitly accounted for. Because the compression diagonal is conservatively assumed to be pinned at both ends in this study, its effective length factor $K$ is equal to 1. As a result, $L$ will be used in place of $KL$ in the discussion to follow.

It is important to note that as $k_s$ increases, there comes a point when the compression diagonal becomes fully braced (Winter, 1960; Yura, 1996) in that the compression diagonal will buckle in its second mode as shown in Figure 2c. When this happens, any further increase in $k_s$ will not bring about an increase in its buckling capacity. This represents a limiting condition for the cross-bracing system, and the value of $k_s$ that corresponds to this condition is referred to as the transition brace stiffness $k_{st}$. The computation of this transition brace stiffness will be given in the next section. In the discussion to follow, the system is said to be fully braced when this limiting condition is attained, and it is said to be partially braced when this limiting condition has not been reached.

## TRANSITION BRACE STIFFNESS

As mentioned in the preceding section, as the lateral brace stiffness increases, a limiting condition will be reached in which the capacity of the compression diagonal will remain stationary. The magnitude of $P$ for this limiting state, denoted as $P_{peak}$, can be obtained by evaluating $P_n$ in Equation 11 using the unbraced length (i.e., $L/2$) of the compression diagonal. The transition brace stiffness, $k_{st}$, can then be evaluated from Equation 10 by substituting $P_{peak}$ for $P$ in the equation. If we denote $P_o$ as the capacity of the compression diagonal when the lateral brace stiffness, $k_s$, is 0 (i.e., when $P_n$ in Equation 11 is evaluated using the full length $L$ of the compression diagonal), the following equation for $k_{st}$ can be derived:

$$\frac{k_{st}L}{P_o} = \frac{2\pi\left(\sqrt{\dfrac{P_{peak}}{P_o}}\right)^3}{\dfrac{\pi}{2}\sqrt{\dfrac{P_{peak}}{P_o}} - \tan\left(\dfrac{\pi}{2}\sqrt{\dfrac{P_{peak}}{P_o}}\right)} \tag{12}$$

where $P_{peak}/P_o$, obtained by taking the ratio of $P_n$ evaluated for the unbraced length $L/2$ to $P_n$ evaluated for the full length $L$ of the compression diagonal, is given by

$$\frac{P_{peak}}{P_o} = \begin{cases} 1.369^{\frac{F_y}{F_e}} & \text{when} \quad \dfrac{F_y}{F_e} \le 2.25 & \text{(13a)} \\ \dfrac{0.901^{\frac{F_y}{F_e}}}{0.877\left(\dfrac{F_e}{F_y}\right)} & \text{when} \quad 2.25 < \dfrac{F_y}{F_e} \le 9.00 & \text{(13b)} \\ 4 & \text{when} \quad \dfrac{F_y}{F_e} > 9.00 & \text{(13c)} \end{cases}$$

and $F_y$ and $F_e$ are as defined in Equation 11.

A plot of Equation 12 as a function of $F_y/F_e$, using the expressions for $P_{peak}/P_o$ given in Equations 13a, 13b and 13c, is shown in Figure 6. It can be seen that the nondimensional transition brace stiffness, $k_{st}L/P_o$, varies as a function of $F_y/F_e$ when $F_y/F_e$ is less than or equal to 9, but becomes a constant when $F_y/F_e$ exceeds 9. Because $F_e$ is inversely proportional to the square of $L/r$, it can be concluded that $k_{st}L/P_o$ increases with $L/r$ until it reaches $3\pi\sqrt{E/F_y}$, when $k_{st}L/P_o$ becomes a constant.

## CROSS-BRACING SYSTEM BEHAVIOR

Figure 7 shows how the load-carrying capacity, $P_{max}$, of a typical compression diagonal varies with the lateral brace stiffness, $k_s$, for several slenderness ratios $L/r$. With reference to this figure, the following observations can be made:

1. All the curves consist of an initial nonlinear portion, as described by Equation 10, before the transition brace stiffness is attained followed by a horizontal line that represents the limiting condition when the transition brace stiffness is reached. When the brace stiffness is less than its transition value, the compression diagonal buckles in a symmetric mode (Figure 2b). Once the brace stiffness is equal to or larger than its transition value, the buckling mode of the compression diagonal will become anti-symmetric (Figure 2c).

2. The load capacity of the compression diagonal is a function of $L/r$ only if it is fully braced, but it is a function of both $L/r$ and $k_s$ if it is partially braced.

3. When the slenderness ratio of the compression diagonal is large (e.g., $L/r = 400$), the member behaves elastically, and so the relationship between the load capacity and the lateral brace stiffness follows that of Equation 9 with a limiting value for $P_{max}/P_o = 4$. As $L/r$ decreases, $P_{max}/P_o$ falls below 4. This is because compression

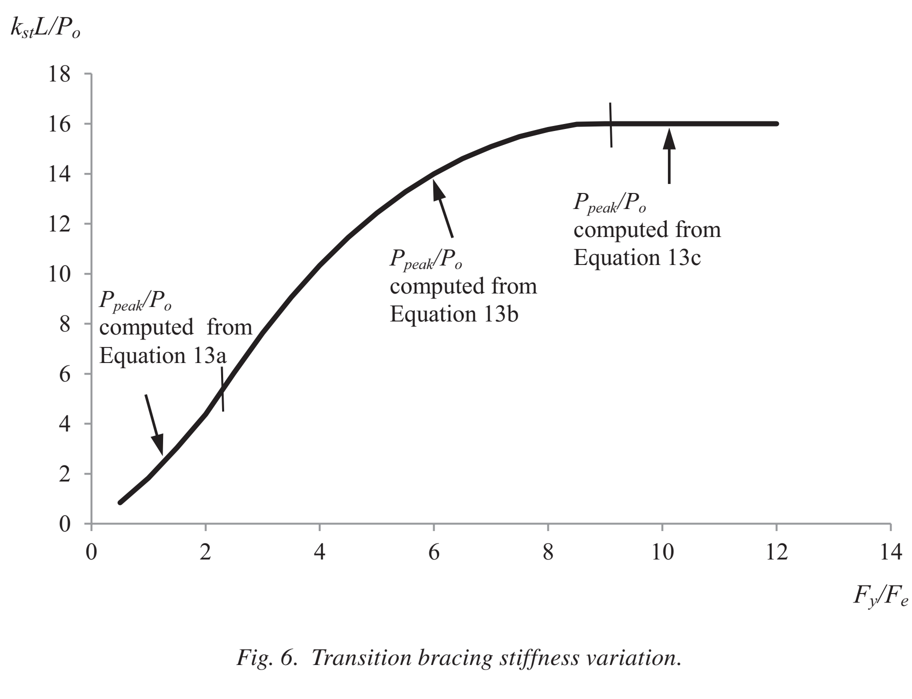


*Fig. 6. Transition bracing stiffness variation.*

members with low slenderness ratios tend to fail in the inelastic range. In the extreme case when the compression diagonal is so short that cross section yielding becomes the limit state, $P_{max}/P_o$ will approach 1, meaning the bracing effect from the cross diagonal will be totally ineffective.

4. Although different slenderness ratios will lead to different limiting values for $P_{max}/P_o$, the slenderness effect on $P_{max}/P_o$ is not apparent when $k_s$ is less than $k_{st}$—that is, the ascending (nonlinear) portion of the curves shows very little change regardless of $L/r$.

## DESIGN RECOMMENDATIONS

In this section, design equations for a cross-bracing system when the compression diagonal is under fully or partial braced condition will be presented.

### Fully Braced Condition

When a compression diagonal is fully braced (i.e., when $k_s \geq k_{st}$), its capacity is a function of its slenderness ratio, $L/r$, only and is independent of the brace stiffness, $k_s$. Under this condition, the capacity denoted as $P_{peak}$ can be calculated from Equation 11 by replacing $KL$ with $l = L/2$—that is,

$$P_{peak} = \begin{cases} \left(0.658^{\frac{F_y}{F_e}}\right) P_y & \text{when } \dfrac{l}{r} \leq 4.71\sqrt{\dfrac{E}{F_y}} \text{ or } \dfrac{F_y}{F_e} \leq 2.25 \\ \left(0.877\dfrac{F_e}{F_y}\right) P_y & \text{when } \dfrac{l}{r} > 4.71\sqrt{\dfrac{E}{F_y}} \text{ or } \dfrac{F_y}{F_e} > 2.25 \end{cases} \tag{14}$$

where $F_e = \dfrac{\pi^2 E}{\left(\dfrac{l}{r}\right)^2}$, and $l = L/2$ is the unbraced length (i.e., half the full length) of the compression diagonal.

### Partially Braced Condition

When the fully braced condition has not been reached (i.e., when $0 \leq k_s < k_{st}$), the compression diagonal is said to be partially braced. For the partially braced condition, $P_{max}$ is a function of both $L/r$ and $k_s$. To simplify matters, it is assumed that $P_{max}/P_n$ of the compression diagonal varies linearly with $k_sL/P_n$—that is, a linearized form of Equation 10 will be used. This assumption will result in a conservative estimate for $P_{max}$. The proposed linearization involves determining the intercept and slope of a straight line that can approximate the curve given by Equation 10 and plotted in Figure 7 for a given $L/r$.

*Intercepts*

Because the intercept represents the capacity of the compression diagonal when $k_s = 0$, its value can be obtained directly from Equation 11 by using the full-length $L$ of the compression diagonal. Therefore, if we denote $P_o$ as the intercept, we have

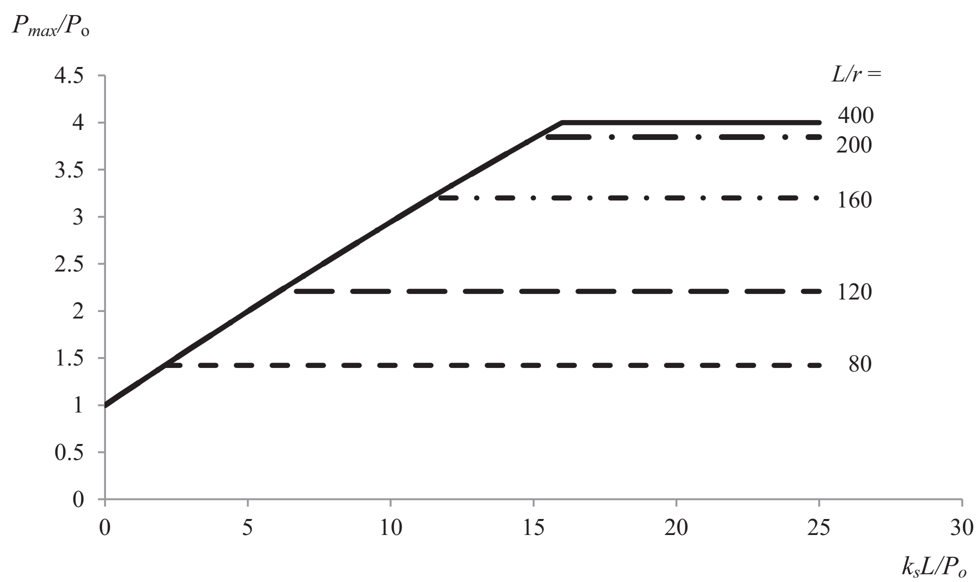


*Fig. 7. Plot of* $P_{max}/P_o$ *versus* $k_sL/P_o$.

$$P_o = \begin{cases} \left(0.658^{\frac{F_y}{F_e}}\right) P_y & \text{when } \frac{L}{r} \le 4.71\sqrt{\frac{E}{F_y}} \text{ or } \frac{F_y}{F_e} \le 2.25 \\ \left(0.877\frac{F_e}{F_y}\right) P_y & \text{when } \frac{L}{r} > 4.71\sqrt{\frac{E}{F_y}} \text{ or } \frac{F_y}{F_e} > 2.25 \end{cases} \tag{15}$$

where $F_y$ and $F_e$ are as defined in Equation 11.

*Slopes*

Slopes represent the ratio of the increase in $P_{max}$ as $k_s$ increases in the range $0 \le k_s < k_{st}$. If we denote $S_o$ as the slopes, we have from Figure 7 (with $P_n = P_o$ at $k_s = 0$),

$$S_o = \frac{\dfrac{P_{peak}}{P_o} - 1}{\dfrac{k_{st} L}{P_o}} \tag{16}$$

Upon substitution of Equation 12 and Equation 13 into Equation 16, $S_o$ can be expressed as a function of $F_y/F_e$. A plot of $S_o$ versus $F_y/F_e$ is given in Figure 8. Because the variation (from 0.202 to 0.188) is not significant, it is recommended that a constant value of 0.188 be used for design. Note that this value can also be obtained by approximating the full length of the curved inclined line in Figure 7 by a straight line that shares the same end points as the curved line and taking its slope—that is, $S_o = (4 - 1)/16 = 0.188$.

Using the intercept and slope equations presented earlier, the load capacity of a partially braced compression diagonal can be written as

$$P_{max} = S_o \times k_s L + P_o \tag{17}$$

In summary, the load carrying capacity of the compression diagonal of a cross-brace system is

$$P_{max} = \begin{cases} P_{peak} & \text{for the fully braced condition} \\ S_o \times k_s L + P_o & \text{for the partially braced condition} \end{cases} \tag{18}$$

where $S_o$ = 0.188; $L$ is the full length of the compression diagonal; $P_{peak}$ and $P_o$ are to be computed from Equations 14 and 15, respectively; and $k_s$ is calculated from Equation 7 or 8, depending on whether the supporting brace is in tension or compression.

## BRACE FORCE

The brace force, denoted as $Q$ in Figure 4, is a function of $k_s$ and the amount of out-of-plane deformation of the cross-bracing system when $P_{max}$ is reached. Based on a parametric study (Lui and Khanse, 2008) in which a series of pinned-end compression members having different slenderness ratios, supported by a spring with different stiffness placed at different locations and considering inelasticity and initial geometric imperfections, were analyzed numerically, the upper- and lower-bound envelope curves for $Q/P_{max}$ plotted as a function of $k_s L/P_e$ are shown in Figure 9. From the figure, a conservative $Q/P_{max}$ value of 4% was recommended. This is different from the 1% value recommended in the AISC specification (2010) for a nodal bracing system. This is because the 1% value is applicable only if the lateral stiffness provided is twice that of the critical brace stiffness defined

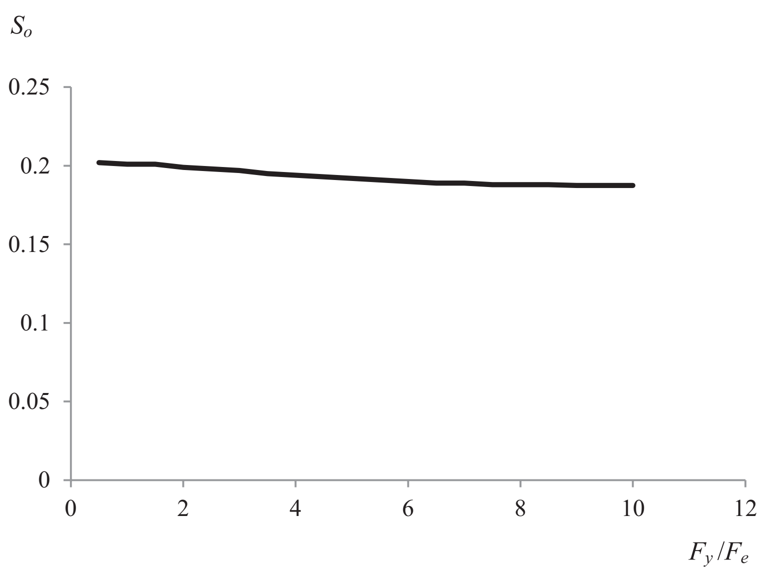


*Fig. 8. Variation of* $S_o$ *with* $F_y/F_e$.

as the stiffness needed to develop $P_e = \pi^2 EI/l^2$, where $l$ is the unbraced length (i.e., length between adjacent braced points) of the compression member. In the current context, this condition cannot be guaranteed because the same size section has to be used for both the tension and compressional diagonals because wind direction can reverse. In other words, unlike a typical nodal bracing problem when the design of the brace can be separated from the design of the member it braces, the design of the tension and compression diagonals is dependent on each other.

## DESIGN PROCEDURE

In this section, design guidelines that take into consideration the interaction effect of the two diagonals of a cross-bracing system will be proposed. Design examples will then be given to demonstrate how the proposed procedure can be used for the design of cross-bracing systems for concentrically braced frames. In some applications, the cross-bracing members can be prestrained or prestressed during installation to enhance their stiffness. The pretensioned stress is usually in the range of 1 to 5% of the material yield stress. If such prestress is present, it should be accounted for in the analysis in obtaining the internal axial forces in the cross diagonals.

The following procedure is recommended for the design of a cross-bracing system:

1. Determine the required axial strength, $P_u$, for both diagonals.

2. Select a trial section based on the compression, $P_u$, and if both diagonals are in compression, select a trial section based on the larger of the two compressive, $P_u$. Because the load capacity of the compression diagonal, $P_{max}$, is not a constant but varies with $L/r$ for the fully braced condition, and with $L/r$ and $k_s$ for the partially braced condition as shown in Figure 7, an assumed increase of the unbraced (i.e., first mode) capacity of the compression diagonal $P_o$ by a certain percentage should be used. In terms of design, this means a reduction in $P_u$ can be used. In the example problems, a trial section is selected based on a reduced required compressive strength of $P_u/1.25$, but depending on the expected $L/r$ and $k_s$ values, other reduction factor for $P_u$ can be used as well.

3. Using the trial section properties, calculate $L/r$ for the compression diagonal, and determine $k_s$ from Figure 5 or, alternatively, from Equation 7 if the supporting diagonal is in tension and from Equation 8 if the supporting diagonal is in compression.

4. Calculate the transition brace stiffness, $k_{st}$, from Equation 12.

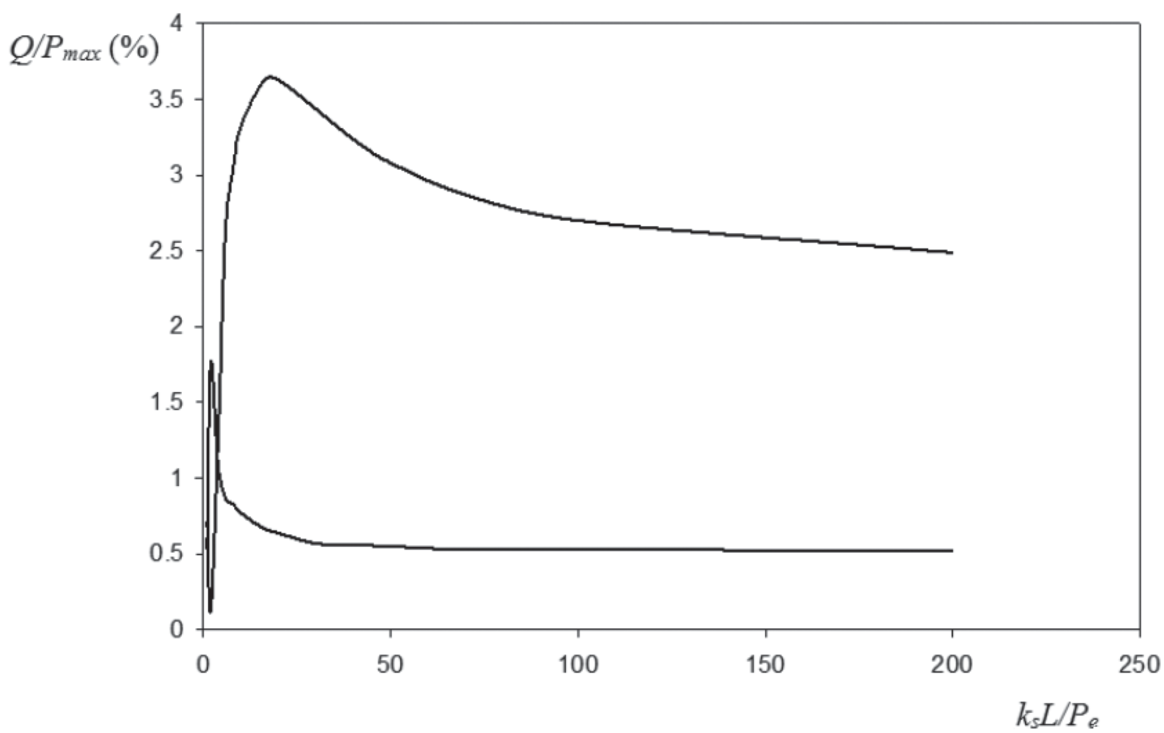


*Fig. 9. Upper- and lower-bound curves for the braced force.*

5. If $k_s \geq k_{st}$, the compression diagonal is fully braced and so $P_{max}$ (= $P_{peak}$) is to be computed from Equation 14. However, if $k_s < k_{st}$, the compression diagonal is only partially braced, so $P_{max}$ is to be computed from Equation 17.
6. Check the adequacy of both the compression and tension diagonals using the axial force–flexure interaction equation by subjecting the members to their respective $P_u$ and to a lateral force equal to 4% of the compressive $P_u$. If the interaction equation is not satisfied, select a new trial section and repeat steps 3 through 6.

## DESIGN EXAMPLES

### Example 1

A square hollow structural section (HSS) is to be used for the cross braces of a diagonal bracing system of an industrial building to resist wind load. If the length of the members is 20 ft and the required axial strengths in the tension and compression diagonals are computed to be 10 and 35 kips, respectively, select an appropriate HSS. Assume the members are pinned at both ends and welded together at their intersection point. Use ASTM A500 Grade B steel.

*Solution:*

As a first trial, use a reduced required axial compressive strength of:

$$P_{u,reduced} = \left(\frac{35}{1.25}\right) = 28 \text{ kips}$$

Using the AISC Compression Member Selection Tables with $KL$ = (1)(20) = 20 ft, select HSS 4×4×¼ as a trial section for the compression diagonal. Because the wind can blow in either direction, the same section is to be used for the tension diagonal.

Material properties: ASTM A500 Grade B steel: $F_y = 46$ ksi, $E = 29{,}000$ ksi.

Geometric properties: $A = 3.37$ in.$^2$, $I = 7.80$ in.$^4$, $r = 1.52$ in., $Z = 4.69$ in.$^3$, $L = 240$ in.

Determine the lateral stiffness of the tension diagonal from Equation 7:

$$k_s = \left[\frac{\left(\overline{kL}\right)^3}{24\left[\frac{\overline{kL}}{2} - \tanh\left(\frac{\overline{kL}}{2}\right)\right]}\right]\frac{48\overline{EI}}{\overline{L}^3} = 0.985 \text{ kip/in.}$$

Calculate the transition lateral stiffness from Equation 12.

Because $F_y/F_e = 4.01$, $P_{peak}/P_o = 3.01$ from Equation 13b and $P_o = 33.9$ kips from Equation 15. Hence, using Equation 12 we have

$$\frac{k_{st}L}{P_o} = \frac{2\pi\left(\sqrt{\frac{P_{peak}}{P_o}}\right)^3}{\frac{\pi}{2}\sqrt{\frac{P_{peak}}{P_o}} - \tan\left(\frac{\pi}{2}\sqrt{\frac{P_{peak}}{P_o}}\right)} = 10.4, \text{ or } k_{st} = 10.4\frac{P_o}{L} = 1.47 \text{ kip / in.}$$

Because $k_s < k_{st}$, the compression diagonal is only partially braced.

Compute $P_{max}$ using Equation 17:

$$P_{max} = S_0 \times k_s L + P_o = 78.3\,\text{kips}$$

The design compressive strength is therefore

$$\phi_c P_{max} = (0.90)(78.3) = 70.5 \text{ kips}$$

Because the lateral interaction force, $F_s$, between the compression and tension diagonals is assumed to be 4% of $P_u$, $F_s = (0.04)(35) = 1.4$ kips.

Now, check the adequacy of the compression and tension diagonals for combined axial force and flexure.

The AISC interaction equation that needs to be checked is:

$$\text{If } \frac{P_r}{P_c} \geq 0.2, \quad \frac{P_r}{P_c} + \frac{8}{9}\frac{M_r}{M_c} \leq 1.0 \tag{19a}$$

$$\text{If } \frac{P_r}{P_c} < 0.2, \quad \frac{1}{2}\frac{P_r}{P_c} + \frac{M_r}{M_c} \leq 1.0 \tag{19b}$$

For the compression diagonal,

$$\begin{aligned} P_r &= 35 \text{ kips} \\ P_c &= \phi_c P_{max} = 70.5 \text{ kips} \\ M_r &= \frac{1}{4} F_s L = 84 \text{ kip-in.} \\ M_c &= \phi_b F_y Z = 0.90 F_y Z = 194 \text{ kip-in.} \end{aligned}$$

Thus,

$$\frac{P_r}{P_c} + \frac{8}{9}\frac{M_r}{M_c} = \frac{35}{70.5} + \frac{8}{9}\left(\frac{84}{194}\right) = 0.881 < 1.0$$

The compression diagonal is OK.

For the tension diagonal,

$$\begin{aligned} P_r &= 10 \text{ kips} \\ P_c &= \phi P_y = 139.5 \text{ kips} \\ M_r &= \frac{1}{4} F_s L = 84 \text{ kip-in.} \\ M_c &= \phi F_y Z_y = 0.90 F_y Z = 194 \text{ kip-in.} \end{aligned}$$

Thus,

$$\frac{1}{2}\frac{P_r}{P_c} + \frac{M_r}{M_c} = \frac{10}{2(139.5)} + \frac{84}{194} = 0.47 < 1.0$$

The tension diagonal is OK.

Therefore, use the HSS4×4×¼ section for both the compression and tension diagonals of the cross-bracing system.

**Example 2**

A cross-bracing system is to be designed using a W-section and ASTM A992 steel for the frame shown in Figure 10. The frame is designed to support a dead load of 20 kips, a live (or roof live) load of 60 kips, and a wind load of 10 kips. All loads are to be applied to the top joints as concentrated loads as shown in the figure. The column and beam sections used for the frame are W8×31 and W6×20, respectively. They are oriented so that their webs are parallel to the plane of the frame. The two diagonals of the bracing system are assumed to be pin connected to the frame and are joined at their mid-points using a welded connection. The diagonals are 32 ft in length and are oriented in such a way that their webs are perpendicular to the plane of the frame.

*Solution:*

Because the frame is statically indeterminate, the analysis results will depend on the relative sizes of the members. Assuming the members used for the cross-bracing system are both W4×13, the axial forces calculated for these members from two controlling load combinations are summarized here.

| **Load Combination** | **Diagonal *AC*** | **Diagonal *BD*** |
|---|---|---|
| $1.2D + 1.6L$ | 15.6 kips (compression) | 15.6 kips (compression) |
| $1.2D + 1.6L_r + 0.5W$ | 12.8 kips (compression) | 19.2 kips (compression) |

Note that there is no "tension" diagonal for this frame. However, for the gravity plus wind load case, because the two diagonals are not subjected to the same axial force, diagonal *AC* can still provide lateral bracing to diagonal *BD*.

For the W4×13 section,

Material properties: ASTM A992 steel: $F_y = 50$ ksi, $E = 29{,}000$ ksi.

Geometric properties: $A = 3.83$ in.$^2$, $I_x = 11.3$ in.$^4$, $r_x = 1.72$ in., $I_y = 3.86$ in.$^4$, $r_y = 1.00$ in., $L = 384$ in.

For the gravity load case, both diagonals are subjected to the same axial force. As a result, neither diagonal can provide out-of-plane lateral restraint to the other diagonal, so $k_s = 0$ for both diagonals. However, they do provide in-plane translational restraint to each other as shown in Figure 2a.

Using Equation 11, and with $(KL/r)_x = (1)(384)/1.72 = 223$ and $(KL/r)_y = (1)(192)/1.00 = 192$, the design compressive strength is determined to be $\phi_c P_n = (\phi_c P_n)_x = 17.4$ kips, which is larger than the required compressive strength of $P_u = 15.6$ kips.

For the gravity plus wind load case, the lateral stiffness that diagonal *AC* can provide to diagonal *BD* can be determined from Equation 8 as

$$k_s = \left[\frac{\left(\overline{kL}\right)^3}{24\left[\tan\left(\frac{\overline{kL}}{2}\right) - \frac{\overline{kL}}{2}\right]}\right]\frac{48\overline{EI}}{\overline{L}^3} = 0.117 \text{ kip/in.}$$

and because $F_y/F_e = 8.68$, $P_{peak}/P_o \approx 4$ from Equation 13b, and $P_o = 19.3$ kips from Equation 15. Hence, by using Equation 12,

$$\frac{k_{st}L}{P_o} = \frac{2\pi\left(\sqrt{\frac{P_{peak}}{P_o}}\right)^3}{\frac{\pi}{2}\sqrt{\frac{P_{peak}}{P_o}} - \tan\left(\frac{\pi}{2}\sqrt{\frac{P_{peak}}{P_o}}\right)} = 16, \text{ or } k_{st} = 16\frac{P_o}{L} = 0.804 \text{ kip / in.}$$

Because $k_s < k_{st}$, diagonal *BD* is only partially braced, so its capacity is to be calculated from Equation 17 as:

$$P_{max} = S_o \times k_s L + P_o = 27.7 \text{ kips}$$

The design compressive strength is therefore

$$\phi_c P_{max} = (0.90)(27.7) = 24.9 \text{ kips}$$

The lateral interaction force, $F_s$, between the two diagonals is assumed to be 4% of $P_u$, so $F_s = (0.04)(19.2) = 0.768$ kips.

Now, check the adequacy of diagonal *BD* for the combined axial force and flexure under the gravity plus wind load case. Because $P_r/P_c = P_u/\phi_c P_{max} = 19.2/24.9 = 0.771$, use Equation 19a with

$$\begin{aligned} P_r &= 19.2 \text{ kips} \\ P_c &= \phi_c P_{max} = 24.9 \text{ kips} \\ M_r &= \frac{1}{4} F_s L = 73.7 \text{ kip-in.} \\ M_c &= \phi_b \times \text{AISC Eq. (F2-2)} = 283 \text{ kip-in.} \end{aligned}$$

In calculating $M_c$, $C_b$ is taken as 1.67, so $\phi_b M_p$ controls, and

$$\frac{P_r}{P_c} + \frac{8}{9}\frac{M_r}{M_c} = \frac{19.2}{24.9} + \frac{8}{9}\left(\frac{73.7}{283}\right) \approx 1.0$$

The design is therefore is OK. Note that diagonal *AC* need not be checked because it has a lower $P_u$ than diagonal *BD*.

Use W4×13 for the diagonals of the cross-bracing system. It should be noted that the W4×13 would have been considered inadequate for the gravity plus wind load case if the lateral bracing effect of the cross-bracing system had not been considered.

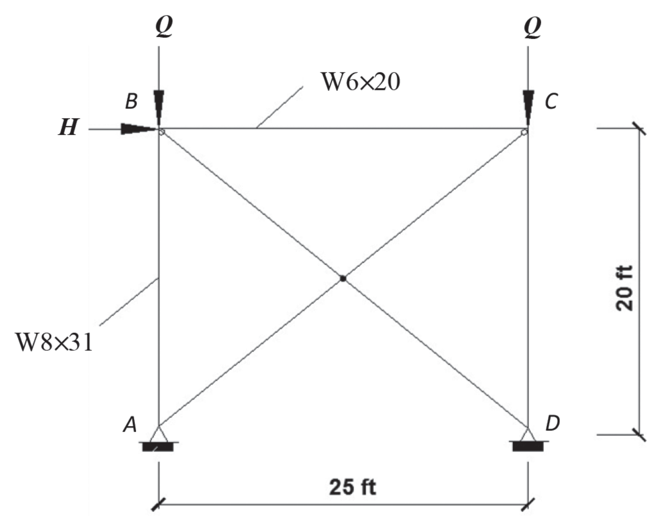


*Fig. 10. Example frame.*

## SUMMARY AND CONCLUSIONS

In a conventional design of cross-bracing systems for braced frames, two relatively simple but somewhat unrealistic approaches are often used. One is a conservative approach in which only the tension diagonal is assumed to be active in controlling frame sway. The contribution of the compression diagonal is totally ignored. On the other extreme, the tension diagonal is assumed to have sufficient stiffness to brace the compression diagonal to allow it to attain a compressive strength that corresponds to its second buckling mode. In reality, the behavior of the system often falls somewhere between these two extreme cases. In this study, the partially braced strength of a typical cross-bracing system is investigated and design guidelines are proposed. Examples are then given to demonstrate how the proposed procedure can be applied for the design of cross-bracing systems. Based on the current study, the following conclusions can be drawn:

1. The compression diagonal is fully braced and can develop a compressive strength corresponding to its second buckling mode only if the tension diagonal possesses sufficient stiffness referred to as the transition stiffness, $k_{st}$, given by Equation 12.

2. The transition brace stiffness increases with $F_y/F_e$ or $L/r$, and for large slenderness (when the compression diagonal remains elastic at incipient instability) becomes asymptotic at $16P_o/L$, where $P_o$ is the axially capacity of the compression diagonal when the lateral bracing stiffness $k_s = 0$, and $L$ is the full length of the member.

3. If $k_s \geq k_{st}$, the compression diagonal is said to be fully braced. The compressive strength of a fully braced compression diagonal, given by Equation 14, is a function its slenderness ratio, $L/r$, only.

4. If $k_s < k_{st}$, the compression diagonal is said to be partially braced. The compressive strength of a partially braced diagonal, given by Equation 17, is a function of both its slenderness ratio, $L/r$, and the lateral bracing stiffness, $k_s$.

5. In cases when both diagonals are in compression, the diagonal with the lower compressive force can still provide bracing to the diagonal with the higher compressive force (see Figure 5) as long as the axial forces in the diagonals are not the same.

6. The internal out-of-plane force developed at the intersection point of the two diagonals when instability occurs can be conservatively taken as 4% of the required compressive strength, $P_u$, of the compression diagonal. This force is assumed to act on both diagonals and their adequacy is checked using the AISC interaction equation for combined axial force and flexure.

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