Notes on the Nodal and Relative Lateral Stability Bracing Requirements of AISC 360

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ABSTRACT

The requirements for stability bracing of columns and beams have been included in AISC specifications since 1999. These requirements are intended to permit properly braced columns and beams to attain the buckling strength of the member as if braced by immovable supports and thus be designed as braced members according to the requirements of the *Specification*. The *Specification* addresses lateral stability bracing of columns and beams and torsional stability bracing of beams. This paper will address only the requirements for lateral stability bracing.

Although these requirements appear to be fairly straightforward, one question is regularly asked by those trying to put them into practice: What is the difference between "nodal" and "relative" bracing? This paper looks at the background of the provisions, describes how they have been obtained from the theoretical equations and then suggests how best to distinguish between these "nodal" and "relative" braces. It also shows that the approach taken by the *Specification* is safe and permits simple rules to be applied to a wide range of bracing problems. In addition, recommendations are made for revision of the requirements.

Keywords: beam bracing, column bracing, brace force, brace stiffness, lateral bracing.

INTRODUCTION

Lateral bracing for columns and beams sufficient to permit them to attain the buckling strength of the member as if braced by immovable supports is called stability bracing. The stability bracing requirements found in Appendix 6 of the AISC *Specification for Structural Steel Buildings* (AISC, 2010a) have remained essentially unchanged since they were first introduced in the 1999 LRFD *Specification for Structural Steel Buildings* (AISC, 2000). George Winter (1958) appears to have been the first to recognize that stability bracing must not only develop sufficient strength but must also have sufficient stiffness to provide the necessary stability bracing of columns and beams. This stability bracing is equivalent in effectiveness to an immovable support and in this paper is also referred to as bracing.

Although the *Specification* requirements appear to be fairly straightforward, practicing engineers often ask about the difference between "nodal" and "relative" bracing. This paper will first look at the background of the provisions and then suggest how best to distinguish between these "nodal" and "relative" braces. It will also show that the approach taken by the *Specification* is safe and permits simple rules to be applied to a wide range of bracing problems.

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Figure 1 shows four ways to provide lateral bracing for a column; three are identified as nodal bracing and one as relative bracing. Case (a) shows braces that are immovable supports. The column is divided into two segments with length L_b. Thus, the Euler buckling load (flexural buckling) for this column is $P_e = \pi^2 EI/L_b^2$. The column of case (b) is braced at the midpoint with a spring. Thus, if the given spring has sufficient stiffness and strength, this column will also buckle at the load P_e as for case (a). Case (c) column is similar to case (b) except that the immovable support at the top has been replaced by a spring. If these springs have sufficient stiffness and strength, this column will also buckle at the same load as in case (a). The last column to consider is that given as case (d). This structure shows a column that is braced by a series of diagonal braces, interconnected in such a way that the column on the right is sufficiently braced to permit it to buckle at the same load as the column of case (a).

COLUMNS WITH NODAL BRACES

Column Case (b)

Timoshenko (1936) used the basic theory of elastic stability to determine the ideal bracing stiffness for the column given as case (b) and similar columns with numerous equally spaced braces. Winter (1958) recognized that it would not be necessary to determine the exact stiffness and strength requirement if a practical and simple method could be developed that would also account for initial imperfections of the column. The column shown in Figure 2 is the column addressed initially by Winter. The column is shown in its perfectly straight position as a solid line. The assumed imperfect column is shown as a dashed line with an initial displacement at the point of the spring, δ_o. As a compressive load is applied to the column, it will deform as shown by the second dashed line with an additional displacement, δ. The corresponding force in the spring is $F = \beta\delta$, where β is the spring stiffness at the brace point. If the spring stiffness is sufficient to permit buckling as defined in case (a), the stiffness will be defined as β_{req}. The column will snap through to the two half-wave modes shown as a thin solid line in Figure 2, and the load will be equal to the Euler buckling load, $P_e = \pi^2 EI/L_b^2$. In this buckled shape, considering small displacements, placing a hinge at the point of the spring causes little loss of accuracy because the moment at this point may be taken as zero. Taking moments about this hinge for either half of the column gives

$$M = \frac{\beta_{req}\delta}{2} L_b - P_e\left(\delta_o + \delta\right) = 0 \tag{1}$$

which leads to

$$\beta_{req} = \frac{2P_e}{L_b}\left(\frac{\delta_o}{\delta} + 1\right) \tag{2}$$

For the perfect or ideal column, $\delta_o = 0$, and the ideal stiffness can be defined as

$$\beta_{ideal} = \frac{2P_e}{L_b} \tag{3}$$

The ideal stiffness given in Equation 3 is the minimum brace stiffness needed for the column to attain the Euler buckling load if the column were perfectly straight. For real braces, this brace stiffness will include the influence of all components that make up the brace between the column being braced and the immovable support. This would include connections and any other structure the brace might be connected to.

It will be shown later that providing the ideal stiffness leads to a very large spring force at buckling. However, given that a real column is not ideal, the required brace stiffness can be determined by combining Equations 2 and 3, thus

$$\beta_{req} = \beta_{ideal}\left(\frac{\delta_o}{\delta} + 1\right) \tag{4}$$

and the brace force is given by

$$F = \beta_{req}\delta = \beta_{ideal}\left(\delta_o + \delta\right) \tag{5}$$

Winter (1958) proceeds to determine the ideal stiffness for columns similar to Figure 1b with two, three and four equally spaced springs while having an immovable support at the top of the column. These values are identical to those presented by Timoshenko (1936) and are given here in Table 1, where

$$\beta_{ideal} = \eta_0 P_e/L_b \tag{6}$$

A value of $n = 1$, which leads to $\eta_0 = 2.0$, corresponds to the condition that led to Equation 3 as shown in Figure 2. The subscript of η refers to the lateral degree of freedom at the top of the column.

More recently, Zhang, Beliveau and Huston (1993) published a single equation that provides the ideal stiffness for any number of springs. Their equation is

$$\beta_{ideal} = 4\sin^2\left[\frac{\pi}{2}\left(\frac{n}{n+1}\right)\right]\frac{P_e}{L_b} = \lambda\frac{P_e}{L_b} \tag{7}$$

The coefficient λ in Equation 7 provides identical values to those given by Timoshenko (1936).

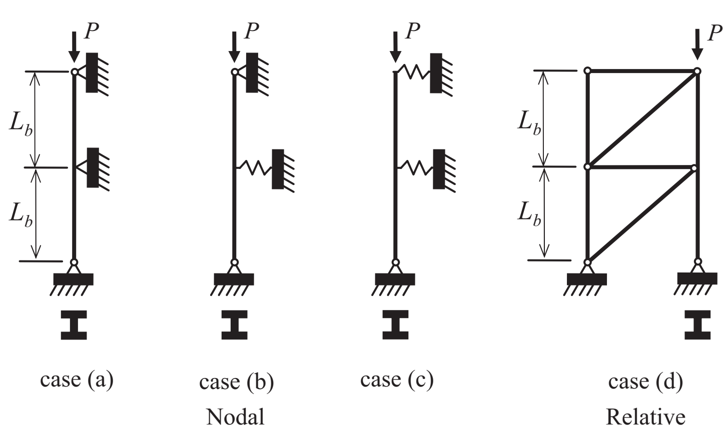

Fig. 1. Lateral bracing for column stability.

Table 1. Coefficient, η_0, for Determination of Ideal Spring Stiffness, Case (b) n = number of springs (Timoshenko, 1936)								
n	1	2	3	4	5	6	10	infinite
η_0	2.00	3.00	3.41	3.62	3.73	3.80	3.92	4.00

Table 2. Coefficient, η_1, for Determination of Ideal Spring Stiffness, Case (c), n = number of springs (Zhang et al., 1993)								
n	1	2	3	4	5	6	10	infinite
η_1	1.00	2.62	3.25	3.53	3.68	3.77	3.91	4.00

Column Case (c)

The column of case (c) behaves differently than the column of case (b) because, in addition to the intermediate braced points, the top of this column is permitted to displace laterally. This column was treated by Timoshenko and Gere (1961) for the case of only one spring, which was located at the top of the column as shown in Figure 3. Taking Winter's (1958) equilibrium approach and using the same definitions as previously, the moment about the column base is

$$M = \beta_{req}\delta L_b - P_e\left(\delta_o + \delta\right) = 0 \tag{8}$$

which gives the required stiffness as

$$\beta_{req} = \frac{P_e}{L_b}\left(\frac{\delta_o}{\delta} + 1\right) \tag{9}$$

For the ideal column with $\delta_o = 0$, the ideal stiffness is

$$\beta_{ideal} = \frac{P_e}{L_b} \tag{10}$$

Although Timoshenko and Gere (1961) do not provide results for more than one spring in the arrangement of case (c), Zhang et al. (1993) provide an equation for the ideal spring stiffness for any number of springs as

$$\beta_{ideal} = 4\sin^2\left[\frac{\pi}{2}\left(\frac{2n-1}{2n+1}\right)\right]\frac{P_e}{L_b} = \eta_1\frac{P_e}{L_b} \tag{11}$$

The ideal stiffness given by Equation 11 is the minimum stiffness if the column were ideal. Note that multiplier η_1 (Table 2) starts at 1.0 for a single spring and increases to 4.0 for an infinite number of springs. The multiplier η_0 (Table 1) starts at 2.0 for a single spring and also increases to 4.0 for an infinite number of springs.

As was the situation with the column of case (b), a real column is not ideal, and the brace stiffness can be determined by combining Equations 9 and 10, thus

$$\beta_{req} = \beta_{ideal}\left(\frac{\delta_o}{\delta} + 1\right) \tag{12}$$

and the brace force is given by

$$F = \beta_{req}\delta = \beta_{ideal}\left(\delta_o + \delta\right) \tag{13}$$

Equation 12 is identical to Equation 4. Thus, regardless of

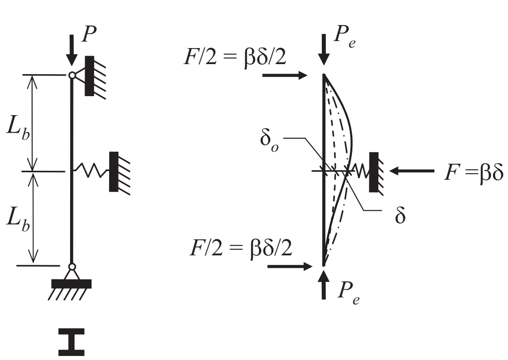

Fig. 2. Column buckling (Winter, 1958).

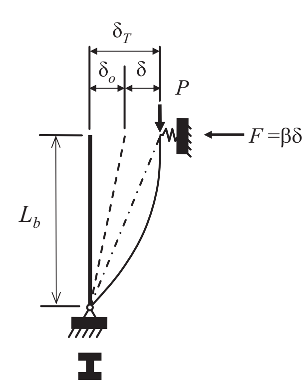

Fig. 3. Brace at top of column (Timoshenko and Gere, 1961).

which spring arrangement is used, the relationship between the required stiffness and the ideal stiffness is the same and is a function of the relationship between the initial displacement and the final displacement at buckling.

To investigate the relationship between spring stiffness and column buckling strength, the column of Figure 3 will again be used. Taking moments about the pin at the base of the column gives

$$P\delta_T = \beta L_b \left(\delta_T - \delta_o\right) \tag{14}$$

Using the ideal stiffness from Equation 10 and rearranging yields

$$\frac{P}{P_e} = \frac{\beta}{\beta_{ideal}}\left[\frac{\left(\delta_T - \delta_o\right)}{\delta_T}\right] = \frac{\beta}{\beta_{ideal}}\left[1 - \frac{1}{\delta_T/\delta_o}\right] \tag{15}$$

From Equation 15 it can be seen that, for a brace with ideal stiffness, δ_T/δ_o must approach infinity in order for the column buckling load to approach P_e. This is illustrated in Figure 4 for the column with a spring of ideal stiffness through the curve labeled β_{ideal}. With the deflection approaching infinity, the brace force will also approach infinity. This, of course, is an untenable solution. However, if the spring stiffness is taken as $1.5\beta_{ideal}$, it can be shown that the column will attain the buckling load with a displacement of $\delta_T/\delta_o = 3$. If the spring stiffness is $2\beta_{ideal}$, the column will attain the buckling load when $\delta_T/\delta_o = 2$, and when the spring stiffness is $3\beta_{ideal}$, the column will attain the buckling load when $\delta_T/\delta_o = 1.5$. These curves are illustrated in Figure 4.

For a column to attain $P = P_e$, Equation 15 leads to a brace (spring) stiffness of

$$\beta = \frac{\delta_T/\delta_o}{\left(\delta_T/\delta_o - 1\right)}\beta_{ideal} \tag{16}$$

Combining Equations 10 and 16, the brace force, $F = \beta(\delta_T - \delta_o)$, simplifies to

$$F = \frac{\delta_T}{\delta_o}\left(\frac{\delta_o}{L_b}P_e\right) \tag{17}$$

Both Equations 16 and 17 are plotted in Figure 5 as a function of $\delta_T/\delta_o = 0$. The figure shows that for $\delta_T/\delta_o = 2$, the combination of required brace stiffness and brace force is optimal for design.

The case of $\delta_T/\delta_o = 2$ means that the displacement at buckling will be equal to the initial deflection. Thus, $\delta = \delta_o$, for which Equation 12 is simplified to

$$\beta_{req} = \beta_{ideal}\left(1+1\right) = 2\beta_{ideal} \tag{18}$$

Equations 5 and 13, which give the required brace force, are also identical for the two arrangements of bracing. Thus, regardless of arrangement of nodal braces, the required brace force is a function of the ideal stiffness and the displacement at the brace point. To determine the brace force, it is not sufficient to establish the relationship between the initial imperfection and the final deflection; actual numerical values must be established. With the column length defined as in these cases, the permitted out-of-plumbness tolerance is usually taken as $0.002L_b$, based on the AISC *Code of Standard Practice* (AISC, 2010b). For the assumption of $\delta = \delta_o$, the brace force is

$$\begin{aligned} F &= \beta_{ideal}\left(\delta_o + \delta\right) \\ &= \beta_{ideal}\left(0.002L_b + 0.002L_b\right) = 0.004L_b\beta_{ideal} \end{aligned} \tag{19}$$

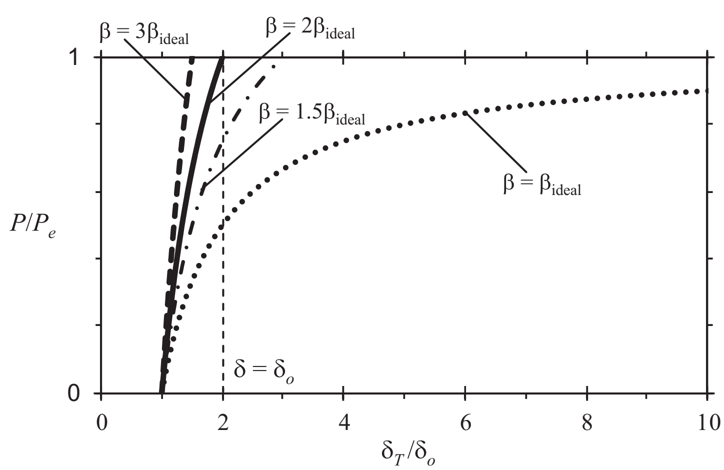

Fig. 4. Influence of brace stiffness, β*, on column buckling load,* P.

This coincides with the value obtained using Equation 17, with $\delta_T/\delta_o = 2$ and $\delta_o/L_b = 0.002$.

Specification Provisions for Nodal Braces

The required brace stiffness and force for column nodal braces are given in Appendix 6.2.2 of the *Specification*. Winter (1958) noted that "the small magnitudes of both rigidity and strength of bracing which are sufficient to provide extremely large effects…suggest that it is not necessary to compute these two characteristics with great accuracy." The largest required brace stiffness occurs as the number of braces approaches infinity for the columns of cases (b) and (c). In both cases, the coefficient, η_0 or η_1, is equal to 4.0. Thus, the *Specification* adopted this condition as a simple and conservative requirement and from Equation 18, with $\beta_{ideal} = 4P_e/L_b$,

$$\beta_{req} = \beta_{ideal}\left(1+1\right) = 2\beta_{ideal} = \frac{8P_e}{L_b} \tag{20}$$

In the elastic buckling region of column behavior, the nominal strength is given by *Specification* Equation E3-3. In terms of force this can be written as

$$P_n = 0.877P_e \tag{21}$$

Because the designer will be starting with the column required strength, P_r, it will be useful to establish a relationship between P_r and P_e. If the required strength of the column is exactly equal to the available strength, then

$$P_r = \phi P_n = 0.9\left(0.877P_e\right) = 0.789P_e \tag{22}$$

Solving Equation 22 for P_e and defining a resistance factor for design of bracing stiffness, $\phi = 0.789$, yields

$$P_e = \frac{P_r}{0.789} = \frac{P_r}{\phi} \tag{23}$$

Substituting Equation 23 into Equation 20 gives

$$\beta_{req} = \frac{8P_r}{\phi L_b} \tag{24}$$

Conservatively defining $\phi = 0.75$ yields *Specification* Equation A-6-4 for LRFD. A similar substitution with Ω will give the ASD equation. The same requirements apply to columns controlled by inelastic buckling. The *Specification* Commentary provides a way to reduce the required stiffness to account for the actual number of intermediate braces but does not distinguish between the columns of Figures 1b and 1c.

Although the *Specification* establishes the required brace stiffness based on an infinite number of equally spaced braces, the required brace force is based on the two-story column shown in Figure 2. Using Equation 19 and substituting the ideal stiffness for a single spring at mid-height, from Table 1, which is $\eta_0 = 2.0$, yields

$$F = 0.004L_b\beta_{ideal} = 0.004L_b\left(\frac{2P_e}{L_b}\right) = 0.008P_e \tag{25}$$

Because the column will only be called upon to provide the required strength, P_r, this required strength will be directly substituted in place of P_e. Design for brace strength will utilize the safety and resistance factors associated with design of the specific bracing members.

Winter (1958) assumed that the shape of the initial

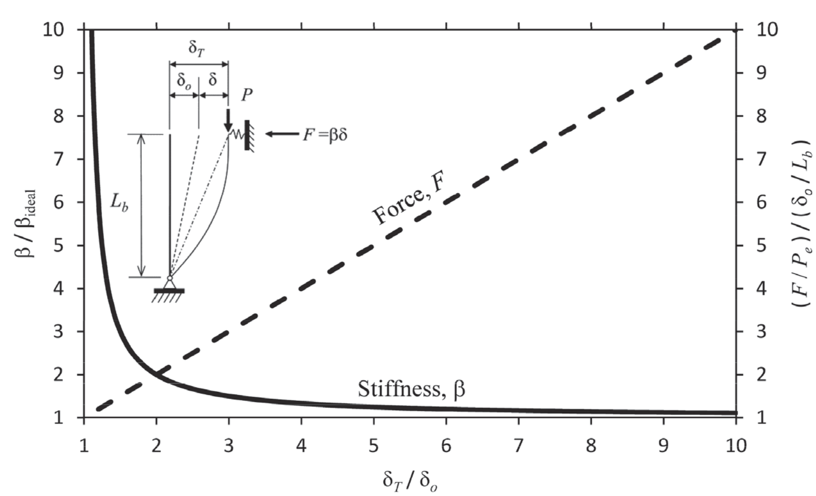

Fig. 5. Required brace stiffness, β, *and brace force,* F, *for column load* $P = P_e$.

imperfections followed the same sine wave as the buckled column. Plaut (1993) showed that Equation 25, based on Winter's approach, can be unconservative for assumed displacements with shapes other than that assumed by Winter. To account for this, *Specification* Equation A-6-3 for the required brace force uses the multiplier of 0.01 rather than 0.008:

$$P_{rb} = 0.01P_r \tag{26}$$

COLUMNS WITH RELATIVE BRACES

Column Case (d)

The approach to development of the requirements for relative braces usually starts with the structure shown in Figure 6. A review of that structure shows that the diagonal bracing member connects the top of the column to an immovable support. Thus, this structure can be modeled like the structure in Figure 3. The only difference is that the stiffness and force requirements relate to the horizontal direction, and they must be converted for design to the longitudinal direction of the brace. Accepting that Figure 3 is a simplification of the structure in Figure 6, the required stiffness for the structure of Figure 6 is derived from equilibrium:

$$\beta_{req} = \frac{P_e}{L_b}\left(\frac{\delta_o}{\delta} + 1\right) \tag{27}$$

which is identical to Equation 9. For the perfect or ideal column, $\delta_o = 0$ and the ideal stiffness is

$$\beta_{ideal} = \frac{\rho_1 P_e}{L_b} \tag{28}$$

where $\rho_1 = 1.0$. The required brace force is then given by

$$F = \beta_{ideal}\left(\delta_o + \delta\right) \tag{29}$$

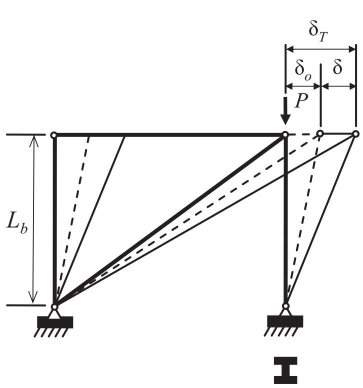

Fig. 6. Relative column brace.

The extension of the one-story structure of Figure 6 to a multistory structure similar to that in Figure 1d is rarely presented in the literature. It is that extension that causes difficulty in understanding exactly what a relative brace is because, clearly, the two cases just discussed could both be called nodal braces.

Zhang et al. (1993) presented a study of column bracing stiffness that addressed both nodal and relative braces through an energy analysis. They first showed that for a single-degree-of-freedom system, as shown here in Figures 3 and 6, the ideal stiffness is as given in Equation 28. They then proceed to address multi-degree-of-freedom systems and found, again through an energy analysis, that for systems similar to that shown in Figure 1d, ρ_1 is always 1.0. In addition to the multi-degree-of-freedom system shown in Figure 1d, they also studied a system similar to that in Figure 1d but with an immovable support at the top of the structure. They found that the coefficient for this case, taken here as ρ_0, was also always 1.0, regardless of the number of brace points. Thus, the ideal stiffness (case with $\delta_0 = 0$) for all these relative bracing systems is given as

$$\beta_{ideal} = \frac{\rho P_e}{L_b} \tag{30}$$

with $\rho = \rho_0 = \rho_1 = 1.0$.

The condensed stiffness matrix for the multistory nodal brace system in Figure 1c (with identical braces at each story) is the constant brace stiffness times the identity matrix. Thus, there is no interaction between the displacement at one brace point and the displacement at any other brace point. The stiffness matrix for the relative brace system shown in Figure 1d is the constant brace stiffness times a diagonal matrix with a bandwidth of 3. Thus, there is interaction between adjacent braced points. It is this interaction that separates a relative brace system from a nodal brace system.

Specification Provisions for Relative Braces

Using Equations 27, 28 and 29, the equations for relative braces found in the *Specification* can be developed. If the final deflection is again taken equal to the initial imperfection, as was done for nodal braces, $\delta = \delta_o$ and from Equation 27 the required brace stiffness is

$$\beta_{req} = \frac{2P_e}{L_b} = 2\beta_{ideal} \tag{31}$$

Substituting for P_e, as was done earlier for nodal braces, yields the required brace stiffness as given by *Specification* Equation A-6-2:

$$\beta_{req} = \frac{2P_r}{\phi L_b} \tag{32}$$

As with the derivation of Equation 25 for nodal braces, if a tolerable value of displacement at buckling, δ, is assumed equal to the initial out-of-plumbness, $\delta = \delta_o = 0.002L_b$, and P_e is replaced by P_r, the required brace force becomes

$$P_{rb} = \beta_{ideal}\left(\delta_o + \delta\right) = 0.004P_r \tag{33}$$

which is *Specification* Equation A-6-1.

NUMERICAL EXAMPLES

A three-story column will be used to confirm that the ideal spring stiffness results in the column buckling at the Euler buckling load. Figure 7 shows four possibilities for bracing of the column, similar to the four columns shown in Figure 1. For the column, $I = 18.3$ in.[4] (I_y for a W8×24) and L_b = 10.0 ft, SAP2000 (CSI, 2011) is used to determine the buckling load for each column considering only flexural deformations of the column.

Case (a): Immovable Supports

The Euler buckling load for this column with a length of 10.0 ft is

$$P_e = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(29{,}000)(18.3)}{\left[10(12)\right]^2} = 363.7 \text{ kips}$$

With immovable supports, the buckling load for the three-story column, as determined by SAP2000, is $P_{critical}$ = 363.8 kips.

Case (b): Two Intermediate Spring Supports, $n = 2$, $\eta_0 = 3.00$

Again, with the Euler buckling load at 363.7 kips, the ideal stiffness is

$$\beta_{ideal} = \frac{\eta_0 P_e}{L_b} = \frac{3.00P_e}{L_b} = \frac{3.00(363.7)}{10(12)} = 9.09 \text{ kips/in.}$$

The minimum spring stiffness for these two intermediate springs resulting in a buckling load of 363.7 kips, as determined by SAP2000, is β = 9.09 kips/in.

Case (c): Two Intermediate Spring Supports Plus Top Spring Support, $n = 3$, $\eta_1 = 3.25$

Again, with the Euler buckling load at 363.7 kips, the ideal stiffness is

$$\beta_{ideal} = \frac{\eta_1 P_e}{L_b} = \frac{3.25P_e}{L_b} = \frac{3.25(363.7)}{10(12)} = 9.85 \text{ kips/in.}$$

The minimum spring stiffness for these three springs resulting in a buckling load of 363.7 kips, as determined by SAP2000, is β = 9.84 kips/in.

Case (d): Three-Story Column with Truss Type Bracing, ρ = 1.0

For this structure, the horizontal and vertical members are assumed axially rigid. Thus, the bracing stiffness will all be

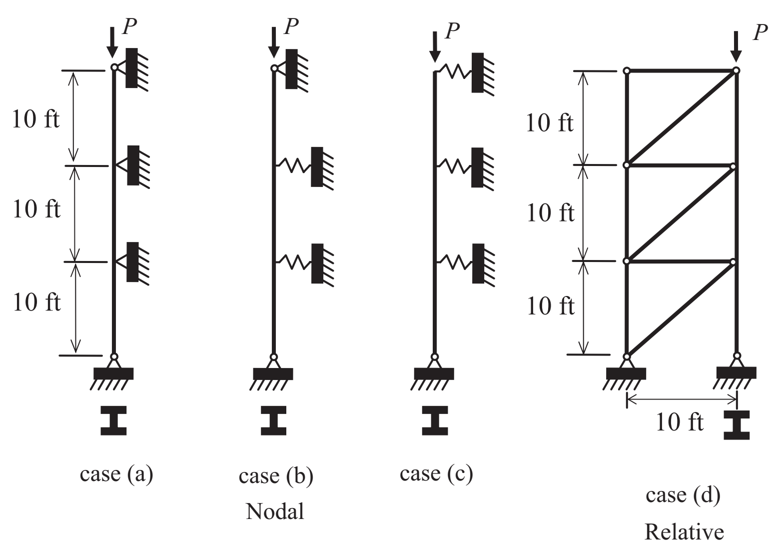

Fig. 7. Structures for lateral bracing for column stability examples.

contributed by the diagonal braces. Again, with the Euler buckling load at 363.7 kips, the ideal stiffness is

$$\beta_{ideal} = \frac{\rho P_e}{L_b} = \frac{1.00 P_e}{L_b} = \frac{1.00(363.7)}{10(12)} = 3.03 \text{ kips/in.}$$

The minimum spring stiffness for the three diagonals resulting in a buckling load of 363.7 kips, as determined by SAP2000, is $\beta = 3.03$ kips/in., which corresponds to a brace area of 0.0355 in.^{2}

RECOMMENDED REVISIONS TO APPENDIX 6 REQUIREMENTS FOR COLUMN BRACES

Based on the development presented here, there appear to be two inconsistencies in the *Specification* requirements. The first has to do with determination of the required brace force for nodal braces and the second the brace force for relative braces.

The required brace stiffness given in Equation 24 (Equation A-6-4) is based on the assumption of an infinite number of nodal braces, $\eta_0 = 4.0$, while the required brace force given in Equation 25 is based on a single intermediate nodal brace, $\eta_0 = 2.0$. It would seem to be a more reasonable assumption to base both the required stiffness and strength on the same structure. Because the assumption of an infinite number of braces is conservative for all cases, use of that same assumption for required brace force would mean that Equation 25 should be

$$F = 0.004 L_b \beta_{ideal} = 0.004 L_b \left(\frac{4P_e}{L_b}\right) = 0.016 P_e \tag{34}$$

Using the increase for a variation in the shape of the initial imperfections based on the work of Plaut (1993) and how that work was used to get from Equation 25 to Equation 26 (Equation A-6-3), the required brace force should be

$$P_{rb} = 0.02 P_r \tag{35}$$

It is interesting to note that this recommended requirement is the same as what had been used historically (before 1999); brace force equals 2% of the force in the column.

The *Specification* Commentary provides an approach to reduce the required brace stiffness to account for the actual number of braces. However, because the required brace force is already based on the presence of a single brace, there can be no reduction of the brace force to account for the actual number of braces.

Although no studies have been found to illustrate the impact of the variation in shape of the initial imperfections on the relative bracing system, it would seem logical that there is an influence and that it would be similar to that on a nodal brace system. That being the case, Equation 33 (Equation A-6-1) should be

$$P_{rb} = 0.005 P_r \tag{36}$$

WHICH IS IT, NODAL OR RELATIVE?

The *Specification* makes the distinction between nodal and relative braces in order to provide simplified equations for design of braces. Using the definitions of nodal and relative braces found in the *Specification* Commentary, the brace of Figure 3 is a nodal brace, and the brace of Figure 6 is a relative brace. Yet, as was shown here, for the case of this one-story structure, the brace points are identical in how they behave and, thus, have the same theoretical strength and stiffness requirements. For a nodal bracing system, all braces are nodal. For a relative bracing system, all diagonal braces are relative braces, while all other members are axially rigid. There are many other ways to provide bracing for columns. However, based on the derivations illustrated here, the *Specification* requirements do not directly apply to those situations.

As the number of brace points increases, the ideal stiffness coefficient for nodal braces approaches 4, while the ideal stiffness coefficient for relative braces remains at 1. This difference is also reflected in the required brace strength. Thus, it is desirable to distinguish between the two types of bracing systems. A nodal brace connects a column to an immovable support. The condensed stiffness matrix for a multistory nodal brace system (Figures 1a, 1b and 1c) with identical braces at each story is the constant brace stiffness times the identity matrix. Thus, there is no interaction between the displacement at one brace point and the displacement at any other brace point. A relative brace system, however, braces a column in such a way that there is interaction between the displacements at each end of the column unbraced length. In this system (Figure 1d), the stiffness matrix is the constant brace stiffness times a diagonal matrix with a bandwidth of 3, showing the interaction between adjacent brace points. It is this interaction that defines a relative brace system and distinguishes it from a nodal brace system.

Another way to identify the type of bracing is by examining the braced member and assuming that it is hinged at the brace points. These hinges lead to a structural mechanism when any single brace point is considered laterally unsupported. If the mechanism accommodates a deflected shape involving the displacement of more than one brace point, then the bracing is relative. When investigating possible mechanisms, only diagonal braces or spring supports may be removed. This is consistent with the assumption that these members are the only source of axial deformations.

This can be illustrated by considering the column of

Figure 7d as if it were hinged at each brace point. The removal of the bottom diagonal brace triggers a mechanism where all three brace points above the base displace laterally as a rigid body. This is an indication that the bottom diagonal provides relative bracing. If the diagonal brace at the second level is removed, a mechanism is formed where the two brace points above this brace are displaced laterally. Thus, this too is a relative brace. If this same approach is applied to any of the examples of nodal braces, it will be seen that the only brace point to displace is the point actually braced, thus confirming it is a nodal brace.

Once the required force at the brace points is determined, it is generally sufficient to consider that this force acts nonconcurrently at each of the brace points. This is consistent with the assumption that the braced member acts as if it were hinged at brace points and that instability occurs with buckling of a single segment between hinges.

To simplify design, the *Specification* provisions for a nodal brace use the worst-case stiffness requirement of an infinite number of braces. This means that when using the requirements of *Specification* Appendix 6, the required stiffness of a nodal brace is 4 times that of a relative brace (comparing Equations 20 and 31) and, using the approach detailed here, the required strength of the nodal brace is 2.5 times that of a relative brace (comparing Equations 26 and 33). Because the option is available in the *Specification*, it is desirable to design braces like those of Figures 1d and 6 as relative braces. However, for any column bracing system, if there is a question as to how to classify the brace, a nodal bracing solution will generally be conservative. Because the required stiffness and strength are usually small in magnitude, this extra conservatism is not likely to be a burden on the design.

The recommended revisions presented in the previous section make both the stiffness and strength requirements of nodal braces equal to four times that of relative braces.

Beams with Lateral Braces

Two types of lateral braces are defined for beams, nodal braces and relative braces as shown in Figure 8. The *Specification* provisions are derived from the same models previously discussed for columns. However, there are a number of factors that affect the requirements for beam braces that were not a part of the discussion for columns. These include the conversion of the beam moment to an equivalent axial force, the presence of load applied to the top flange of the beam and the possibility of double curvature bending. The *Specification* Commentary shows how these factors are incorporated as Equation C-A-6-5, based on the presentation by Yura (2001):

$$\beta_{br} = \frac{2N_i C_t \left(C_b P_f\right) C_d}{\phi L_b} \tag{37}$$

In this equation, the 2 represents the relationship between the required and ideal brace stiffness as shown for columns by Equations 18 and 31; N_i is equivalent to η_0 given in Table 1 for nodal braces of columns and is given in the *Specification* Commentary by the approximate equation, $N_i = (4 - 2/n)$; C_t accounts for top flange loading and is taken as 1.0 when the beam is loaded at its centroid and $1 + (1.2/n)$ otherwise; and C_d accounts for reverse curvature bending. The term C_bP_f uses the flexural moment gradient factor, C_b, *Specification* Equation F1-1, to increase the possible flange force, $P_f = \pi^2 EI_{yc} \big/ L_b^2$, due to a moment diagram less severe than uniform moment, where $I_{yc} = t_f b^3_f/12$, the out-of-plane moment of inertia of the compression flange.

For the *Specification* requirements, the term C_bP_f in Equation 37 is replaced by an equivalent flange force, M_r/h_o, where M_r is the maximum required flexural strength of the beam as if it were under a uniform moment and h_o is the distance between beam flange centroids. For nodal braces, N_i varies from 2 to 4 based on the number of braces. The upper limit of 4 was selected for columns. However, for beams, C_t also varies with the number of braces. Therefore it is helpful to look at the range of the product of these terms. Table 3 shows the multipliers η_0, N_i, C_t and N_iC_t.

Recognizing that the range of the product N_iC_t is from 4.00 to 4.80 and remembering Winter's original goal was to find a simple yet conservative approach, for the *Specification* requirement this product is conservatively taken as 5.

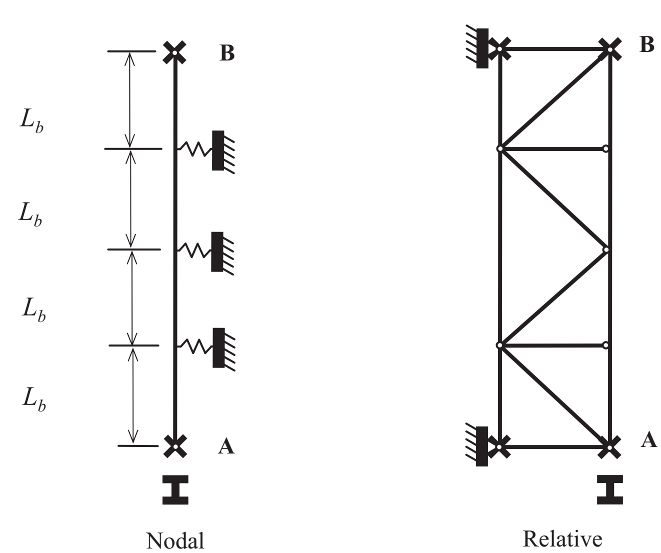

Fig. 8. Plan view of beam AB *with compression flange lateral brace.*

Table 3. Coefficients for Nodal Beam Braces n = number of springs (Yura, 2001)								
n	1	2	3	4	5	6	10	infinite
η_0	2.00	3.00	3.41	3.62	3.73	3.80	3.92	4.00
N_i	2.00	3.00	3.33	3.50	3.60	3.67	3.80	4.00
C_t	2.20	1.60	1.40	1.30	1.24	1.20	1.12	1.00
N_iC_t	4.40	4.80	4.67	4.55	4.46	4.40	4.26	4.00

With these two substitutions, Equation 37 for nodal beam braces becomes *Specification* Equation A-6-8:

$$\beta_{br} = \frac{1}{\phi}\left(\frac{10M_rC_d}{L_bh_o}\right) \tag{38}$$

For a column relative brace, it was shown in Equation 30 that $\beta_{ideal} = \rho P_e/L_b$ and $\rho = 1.0$. Thus, for a relative beam brace, N_i will be taken as 1.0. From Table 3 it is seen that the maximum value of C_t is 2.2 for a single brace with top flange loading. For simplicity, C_t is taken as 2.0 for relative bracing according to Yura (2001), so Equation 37 becomes, for a relative beam brace, *Specification* Equation A-6-6:

$$\beta_{br} = \frac{1}{\phi}\left(\frac{4M_rC_d}{L_bh_o}\right) \tag{39}$$

A similar approach can be followed to obtain the *Specification* equations for ASD.

The brace force requirements are the same as they were for columns, with the addition of the influence of location of load on the cross section and reverse curvature bending, if applicable. From Equations 38 and 39, recognizing that $\beta_{br} = 2\beta_{ideal}$ and using $\delta = \delta_o = 0.002L_b$ as was done for columns, the required nodal brace force is obtained using $F = \beta_{ideal}(\delta_o + \delta)$, which gives *Specification* Equation A-6-7:

$$P_{rb} = 0.02\,M_rC_d\,/\,h_o \tag{40}$$

For a relative brace, *Specification* Equation A-6-5 is

$$P_{rb} = 0.008\,M_rC_d\,/\,h_o \tag{41}$$

In each of these required stiffness and strength equations, according to the *Specification*, $C_d = 2$ for the brace closest to the inflection point and $C_d = 1$ for all other braces on a beam in double curvature and for all braces of a beam in single curvature. As was the case for column bracing design, the resistance and safety factors, ϕ and Ω, will be applied in the brace strength design.

The distinction between nodal braces and relative braces for beams is the same as it was for columns. If there is an interaction between braced points, then the braces can be treated as relative braces. However, treating all cases of bracing as nodal will always be conservative and, as was the case for columns, will not be a burdensome requirement.

It is important to note that for beams loaded at their centroid, $C_t = 1$ and the stiffness and strength requirements of nodal beam braces would be equal to four times that of relative beam braces.

CONCLUSIONS

The intent of the lateral stability bracing requirements of *Specification* Appendix 6 is to provide a simple yet conservative approach for sizing braces. This paper has shown how these requirements were developed, has described the distinction between nodal and relative braces, and has pointed out two apparent inconsistencies. Recommendations have been offered for changes in two of the *Specification* equations. It was recommended that *Specification* Equation A-6-3 be changed to

$$P_{rb} = 0.02P_r \tag{35}$$

and *Specification* Equation A-6-1 be changed to

$$P_{rb} = 0.005P_r \tag{36}$$

One additional requirement should be discussed. If the bracing is included in a second-order analysis that incorporates the initial out-of-straightness of the member, the results of that analysis may be used in lieu of the lateral stability bracing requirements of *Specification* Appendix 6. Because *Specification* Chapter C requires that a second-order analysis, including initial out-of-straightness, be carried out for the lateral-load-resisting system and because column bracing will be included in that analysis, application of the requirements of *Specification* Appendix 6 for column bracing can often be avoided.

Because beam bracing is normally not included in a second-order analysis, the beam bracing provisions usually cannot be avoided. In addition to the lateral bracing requirements for beams discussed in this paper, *Specification* Appendix 6 includes provisions for torsional bracing.

SYMBOLS

C_b Lateral-torsional buckling modification factor for nonuniform moment diagrams

C_d Coefficient accounting for increased required bracing stiffness at inflection point

C_t Coefficient to account for load location relative to centroidal axis

E Modulus of elasticity of steel

F Force in brace or spring representing brace

I Moment of inertia for axis about which buckling is considered

L_b Unbraced length for flexural buckling

M Moment of forces about a point

M_r Required moment strength

N_i Coefficient to account for number of nodal braces or presence of relative braces

P Axial force on a column

P_e Column elastic buckling strength known as the Euler buckling strength

P_f Beam compressive flange force

P_n Nominal compressive strength

P_r Required compressive strength

P_{rb} Required brace strength

n Number of springs or braces

β Spring or brace stiffness

β_{br} Brace stiffness

β_{ideal} Ideal brace stiffness

β_{req} Required brace stiffness

δ Additional deflection at buckling

δ_o Initial displacement due to imperfection

δ_T Total deflection at buckling

η_0 Coefficient for determination of ideal spring stiffness with only intermediate nodal braces

η_1 Coefficient for determination of ideal spring stiffness with intermediate and column end nodal braces

λ Coefficient for determination of ideal spring stiffness based on Zhang et al. (1993)

ρ Coefficient for determination of ideal spring stiffness for relative braces

ρ_0 Coefficient for determination of ideal spring stiffness with only intermediate relative braces

ρ_1 Coefficient for determination of ideal spring stiffness with intermediate and column end relative braces

ϕ Resistance factor

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