A Simplified Approach for Evaluating Second-Order Effects in Low-Rise Steel-Framed Buildings

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ABSTRACT

Consideration of second-order effects is one of the most involved tasks in the analysis and design of steel-framed buildings. More specifically, the evaluation of amplified moments due to frame-level second-order effects is a very time consuming task because, depending on the method used, it may require an additional first-order analysis or the introduction of additional notional loads for each considered load case. Although the use of such methods may not present a problem when the design process is fully automated, hand calculations can be quite complicated for checking the results of a computer-generated design or for designing a small structure in which second-order effects may not even be significant enough to affect the selection of member sizes. This paper presents a simplified approach for the incorporation of second-order effects in the design of low-rise steel-framed structures. The development of this approach involved the design and analysis of a number of systems with various numbers of bays, floors and beam-span to floor-height ratios and included a thorough investigation of the effect of moment amplification on final member sizes. To facilitate this task, a spreadsheet application was developed to automate the design of a typical low-rise steel frame under given loading conditions. A parametric study was then performed to understand the effect of the various design parameters on member-level and frame-level second-order forces and moments. This led to the development of the simplified design approach proposed in this paper.

Keywords: second-order effects, P-delta, load amplification, low-rise buildings, simplified procedure.

INTRODUCTION

Traditional structural analysis methods are based on the assumption that the deformation of a member does not affect its internal load or stress distributions. This is usually referred to as first-order analysis. However, for flexible structural systems, the extra forces or stresses induced by deformations may be quite significant and cannot be ignored. Steel frames are such structures, and the 2010 AISC *Specification for Structural Steel Buildings* states that second-order effects should be considered in the design of frames. The *Specification* provides several different approaches for doing so, including the approximate second-order analysis method presented in Appendix 8 of the 2010 AISC *Specification*.

Nevertheless, there is sufficient evidence from practice that the design of low-rise buildings may not be greatly affected by second-order effects. Carter et al. (2000) note that "most building structures in the United States are not taller than four stories and the actual increase in moments due to deformations of the frame is in many practical cases negligible." Carter et al. also state that there are cases where the consideration of the effect of these deformations can be important. They further make the case for the need for a simplification of specification requirements for the consideration of second-order effects. This paper presents the results of an investigation that was carried out to accomplish this goal.

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REVIEW OF LITERATURE

The analysis and design of steel buildings based on second-order methods can be cumbersome. Ordinary structural analysis methods that do not take the displaced geometry of a member into consideration are identified as first-order methods. Second-order methods are used to find the deflections and secondary moments that could be significant depending on the type of structure and its geometry, loading and support conditions. Several methods have been developed to estimate the second-order effects in steel buildings. Carter and Geschwindner (2008) compared three approaches presented in the 2005 AISC *Specification* and a fourth approach presented in the 13th edition AISC *Steel Construction Manual*:

1. Second-order analysis method (AISC 2005a, Section C2.2a).

2. First-order analysis method (AISC 2005a, Section C2.2b)

3. Direct analysis method (AISC 2005a, Appendix 7)

4. Simplified method (AISC 2005b, p. 2-12 and the AISC *Basic Design Values* cards)

The authors used two simple unbraced frames to compare these methods. The first frame is a one-bay frame that has a rigid roof element spanning between a flagpole column and a leaning column. Drift limitations were not specified for this frame, resulting in a higher ratio of second-order drift to first-order drift, which illustrates the requirements needed for each method to calculate K-factors, notional loads, and required and available strengths. The second frame is a three-bay frame that has rigid roof elements spanning between two flagpole columns and two leaning columns. The drift limit for this frame was specified to be $L/400$ to illustrate the simplification that specifying a drift limit can provide on the analysis of each method. The authors compared these four methods based on the result of the beam-column interaction equation. It was found that the direct analysis method calculates the highest strength, while the simplified method calculates the lowest strength. However, the results for the three-bay frame showed that the values of the interaction formula produced very close results, and there are no significant differences between them. The authors concluded that if conservative assumptions are acceptable, the simplified method would be the simplest approach to use, especially if the drift limit is such that K can be taken equal to 1. Furthermore, second-order effects and leaning columns have a significant impact on strength requirements, but usual drift limits such as $L/400$ sometimes can result in framing that requires no increase in member size for strength.

Eroz et al. (2008) addressed the consideration of partial base fixity using the direct analysis method (DM). The authors developed a refined model of the column base moment-rotation response using an adaptation of the component method (CM) of Eurocode 3 plus a representation of the foundation stiffness. Furthermore, the authors applied an extension of the DM for frames containing web-tapered members. An example clear-span gable frame is checked using several models of its nominally simple four-bolt base detail: ideally pinned, elastic based on $G = 10$ from the AISC sidesway-uninhibited alignment chart, and elastic-perfectly plastic using the component method (CM). It was also shown that the member strength unity checks and the frame deflections obtained by modeling of the column bases as elastic based on $G = 10$ are accurate compared to the refined base model. It is found that the contribution of the significant initial stiffness of typical bases to the overall frame response is limited by the connection strength.

Becker (2008) reviewed a rigid frame that Chief Buildings designed using allowable stress design per the 1989 AISC *Specification* with the effective length method. This design was compared with an analysis of this structure using the direct analysis method from Appendix 7 of the 2005 AISC *Specification* using both ASD and LRFD load combinations. This comparison showed that various load combinations and methods of applying floor live loads can have very dynamic effects on the amplification of moments due to the P-δ effects. Moreover, the author realized that slight variations in the frame configuration and modeling can greatly affect the P-δ moments.

Surovek and Ziemian (2005) presented an overview of known frame analysis methods and described the connection between these methods and specification based approaches for assessment of frame stability. The authors discussed the effectiveness of the direct analysis method in achieving greater simplification in analysis and design checks. The authors discussed three design approaches for assessment of frame stability:

1. The AISC LRFD effective-length-based assessment (AISC, 1999)
2. The direct analysis method
3. An advanced analysis method

The paper highlighted the main differences between the three approaches pertaining to the level of analysis required, the method of accounting for the effects of member inelasticity, and the means for including the effects of geometric imperfections due to fabrication and erection tolerances. Three design approaches were also compared using a design example that included an 11-bay single-story frame. The study concluded that the direct analysis approach produces an accurate representation of the actual behavior of the framing system as it accounts for the effects of member inelasticity and frame imperfections in the assessment of both member and system strengths. This approach is implemented in design offices and is used in the analysis of braced frames, moment frames and mixed systems through the use of commercially available software. This reduces the need for using approximate methods such as those using the effective length factor concept.

Kim and Choi (2005) developed a practical second-order inelastic analysis of three-dimensional steel frames subject to distributed loading. The authors utilized stability functions to account for second-order effects associated with P-δ and P-Δ. The Column Research Council (CRC) tangent modulus concept is used to account for gradual yielding due to residual stresses. The degradation from elastic to zero stiffness associated with development of a hinge was represented by a softening plastic hinge model. In their proposed analysis, the authors represent a member by two elements and three nodal points. A plastic hinge location can be captured in the analysis as the internal nodal point traces the maximum moment location at each load step. Maximum moments and load–displacement relationships predicted by the proposed analysis compared well with those given by other approaches.

Chen and Wang (1999) proposed a new moment amplification factor B_1* for calculating the P-δ effect in the design of steel beam-columns. The main purpose of the new moment amplification factor was to replace the B_1 factor of the 1993 AISC LRFD *Specification* in order to account for inaccuracies in predicting P-δ effects especially when the member is subjected to large magnitudes of axial load. Moreover, the researchers have conducted parametric studies to evaluate the accuracy of the proposed moment amplification factor and compare it with that from theoretical analysis and current design specification. The researchers stated that the new factor is more precise in predicting P-δ effects when compared with the current design specification.

Sohal and Syed (1992) developed an inelastic amplification factor (IAF) to account for the inaccuracies associated with using the elastic amplification factor that was proposed in the literature. The elastic amplification factor provided a good estimation of the amplified moment only for members with very high slenderness that fail by elastic buckling. However, members with intermediate slenderness, especially those with high shape factors, will fail by inelastic buckling. Design interaction equations developed with the LRFD B_1 factor and the inelastic amplification factor proposed by the authors were compared with numerical inelastic strengths of beam columns. The authors concluded that:

1. The inelastic amplification factor (IAF) is higher than the elastic amplification factor for all studied cases.
2. The IAF increases with decreasing slenderness ratios for any given value of applied load to critical load P/P_{cr}.
3. The effects of residual stresses and initial out-of-straightness increase the IAF.
4. The use of the IAF in the present LRFD interaction for beam-columns bending about their weak axes results in an over-conservative prediction of the strength of most beam-columns.

Duan and Chen (1989) developed a unified design interaction equation for braced steel beam columns subjected to compression combined with biaxial bending. The proposed interaction equation satisfies both strength and stability criteria. A comparison between the proposed design equation, as well as the AISC equations, with the exact inelastic solutions of I-shaped beam-columns and with the results obtained from 81 full-scale tests of biaxially loaded I-shaped beam-columns was made. The authors concluded that the proposed formula is simple in format, has a smooth transition from the pure bending case to the pure axial load case, and is valid for long and short members. They also indicated that the proposed formula is only valid for I-sections as formula parameters will be different for other sections.

This literature survey showed that extensive research has been conducted in the design and analysis of steel-framed buildings to account for second-order effects, but none of this research dealt specifically with low-rise buildings or tried to come up with a simplified approach for their design

DEVELOPMENT OF THE PROPOSED SIMPLIFIED DESIGN METHOD

For structures designed on the basis of first-order elastic analysis, the 2010 AISC *Specification* provides an approximate method to account for second-order effects. This method involves the application of moment and axial force amplification factors to results that are obtained using first-order analyses. The amplified effects are given by the following 2010 AISC *Specification* equations:

$$M_r = B_1 M_{nt} + B_2 M_{lt} \qquad \text{(AISC 2010, Eq. A-8-1)}$$

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{(AISC 2010, Eq. A-8-2)}$$

In these equations, M_r and P_r are the second-order moment and axial force to be used in member verification, M_{nt} and P_{nt} are the first-order moment and axial force obtained using LRFD or ASD load combinations assuming no lateral translation of the frame, and M_{lt} and P_{lt} are the first-order moment and axial force that are due to lateral reactions from those same LRFD or ASD load combinations but assuming lateral translation of the frame is possible. The factor B_1 is used to approximate second-order effects at the member level (P-δ effects), and the factor B_2, which is only applicable to unbraced frames, is used to account for second-order effects at the frame level (P-Δ effects). Two approaches can be used to compute B_2: one is based on the elastic story sidesway buckling resistance, and one is based on limiting interstory drift. The procedure being proposed herein is based on the approach that uses the elastic story buckling resistance.

One of the easiest ways the process can be simplified is to eliminate the need for the two analyses that are required to compute the coefficients B_1 and B_2. This would be straightforward for structures with a given set of characteristics if it is known that one of the effects is dominant over the other. This is because, for such structures, amplified load effects may be directly related back to total load effects without having to distinguish between member-level and frame-level amplification since the amplification is mainly coming from one source. Based on this observation, and since the objective is to come up with a simplified procedure for low-rise building structures, it was decided to examine amplification factors for a number of these structures to determine if either of the effects is dominant. This required that a large number of low-rise structures be designed to obtain enough data to analyze and base a possible new procedure upon. These systems were one to four floors high and one to four bays wide. For each system bay, length was set to 22.5, 25, 27.5, 30 or 32.5 ft, whereas floor height was kept at 15 ft. This variety in frame aspect ratios ensures an adequate representation of building geometries. The frame spacing for each system was taken as 20, 24, 30 or 40 ft and a 120-ft total building width was assumed. The unbraced length of roof beams was set to a 5-ft assumed typical roof joist spacing, and floor beams were assumed to be fully braced by supported slabs. Each column was assumed to be pinned at the base and braced at one-third height (i.e., at 5-ft intervals) with respect to its weak axis to ensure that strong axis buckling controls the design of all the columns. This will ensure that any design parameters that are to be obtained from this procedure will be applicable to the column under consideration irrespective of its lateral bracing configuration.

The gravity loads applied to each system consisted of an 80-psf dead load, a 50-psf live load, and a 25- and 20-psf roof dead and live load, respectively. Lateral loads consisted of wind and seismic and were based on input data for the Peoria, Illinois, area (wind speed of 90 miles per hour, a short period mapped spectral response acceleration of 17.5% g, and a long period mapped spectral response acceleration of 7.8% g). A specific location is being mentioned only to have a point of reference for initial design load determination; any other location could have served the purpose of this investigation. Figure 1 shows the elevation drawing of a typical frame and loads. Considering that 320 frames needed to be designed, a spreadsheet application was developed to automate the design process and complete the task in a timely and efficient manner. This application accepts as input the number of floors, number of bays, first floor height, typical beam span-to-floor height ratio, preferred beam and column nominal depths, and typical loads. It then goes on to generate the structural model and loads and then finds the lightest hot-rolled section for each structural member. The development of a special spreadsheet application for this project was judged to be more efficient than using available structural analysis and design software for the following reasons:

1. The structural model generation and load application processes can be automated, leading to a minimum amount of data input being required for each run.

2. A better control of the design process in terms of member sizes and groupings can be achieved.

3. The ability exists to extract any intermediate data that might be deemed useful for the parametric study. Most of these data are impossible to get from currently available commercial software.

Upon completing each design, member section, member level and position, ϕP_n, ϕM_n, P_{nt}, P_{lt}, M_{nt}, M_{lt}, B_1, B_2 and the interaction ratio were stored for each member in the system. The results of the designs of all the systems were then combined in a single worksheet that was used for the parametric analysis. Figure 2 shows a partial screen capture of the results worksheet of the spreadsheet application for a typical two-bay, two-floor system.

PARAMETRIC ANALYSIS

A perusal of the results from the performed analyses showed a clear dominance of frame-level moment and axial force magnification for the types of structures under consideration because the values of B_1 for all members were around 1.0, while values of B_2 varied from 1.01 to 1.29. This means that it may be possible to directly relate amplified moments and forces to those obtained from a first-order analysis using one single amplification coefficient. For a system that was designed using the B_1/B_2 amplification method, single amplification coefficients for axial load and moment can be defined as $B_P = P_u/(P_{nt} + P_{lt})$ and $B_M = M_u/(M_{nt} + M_{lt})$, respectively.

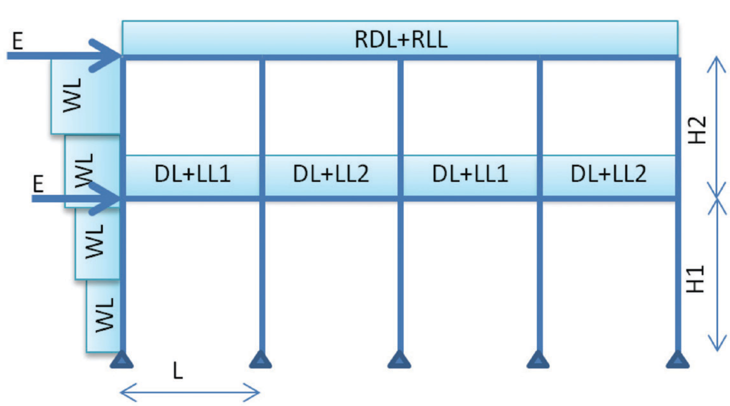

Fig. 1. Typical design frame geometry and loading.

B_P is an axial force magnification coefficient and B_M is a bending moment amplification coefficient. These newly defined coefficients were then computed for all the columns that were designed in the selected systems, and their variation with various design parameters (such as the number of floors, number of bays, bay length, member level and member position) were examined. Member level refers to the floor the column belongs to, and member position refers to the position of the column within the floor, i.e., exterior or interior. In general, the parametric analysis showed that axial force and bending moment magnification was mainly affected by the level of the member and the number of floors in a system and, to a lesser extent, by the number of bays. Member position within the frame seems to have some effect on the moment magnification coefficient B_M but not as much on the axial force magnification coefficient B_P. Values of B_M were observed to be 2.2 to 9.5% higher for interior columns than those for exterior columns depending on the number of floors, number of bays and bay width to frame height ratio.

Figure 3 shows a typical variation of the axial force magnification coefficient B_P, and Figure 4 shows a typical variation of the bending moment magnification coefficient B_M with member level. These coefficients are noticeably higher for first floor columns than they are for upper level columns. Moreover, it is interesting to note that the B_P and B_M coefficients are higher for columns at the third level than they are for those at the second level. This can be explained by the adopted design approach because a change in column size was set to occur after every two floors. As a result, the columns at the first and third levels are the ones governing the design and the columns at the second and fourth levels ended up being overdesigned.

Figure 5 shows a typical variation of the axial force magnification coefficient B_P with the number of floors and the number of bays, and Figure 6 shows a typical variation of the bending moment magnification coefficient B_M with the number of floors and the number of bays. The graphs shown correspond to the same first floor column in all the studied

	A	B	C	D	E	F	G	H	I	J	K	L	AD	AE	AF	AG
1	Member	No of Floors	No of Bays	Floor Height (ft)	Bay Length (ft)	Frame Spacing (ft)	Lb / Ly (ft)	Mem Type	SectNum	Size	Member Level	Member Position	B_1	B_2	R	Load Combination
2	1	2	2	15	15	20	5	Column	232	W12X16	1	1	1.0000	1.1454	0.827	15
3	2	2	2	15	15	20	5	Column	231	W12X19	1	2	1.0000	1.1454	0.882	14
4	3	2	2	15	15	20	5	Column	232	W12X16	1	3	1.0000	1.1454	0.827	14
5	4	2	2	15	15	20	5	Column	232	W12X16	2	1	1.0000	1.0152	0.347	3
6	5	2	2	15	15	20	5	Column	231	W12X19	2	2	1.0000	1.0152	0.186	11
7	6	2	2	15	15	20	5	Column	232	W12X16	2	3	1.0000	1.0152	0.347	2
8	7	2	2	15	15	20	0	Beam	231	W12X19	1	1	1.0025	1.0000	0.904	14
9	8	2	2	15	15	20	0	Beam	231	W12X19	1	2	1.0025	1.0000	0.904	15
10	9	2	2	15	15	20	5	Beam	233	W12X14	2	1	1.0038	1.0000	0.460	8
11	10	2	2	15	15	20	5	Beam	233	W12X14	2	2	1.0038	1.0000	0.460	9
12																

Fig. 2. Partial screen capture of results worksheet for a two-bay, two-floor system.

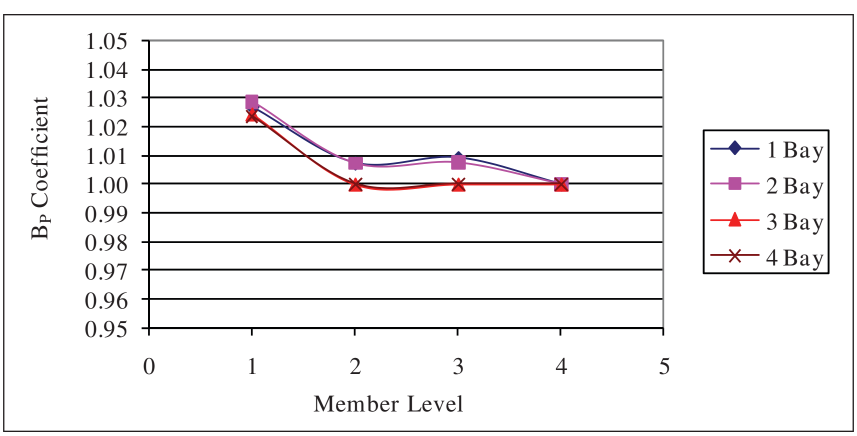

Fig. 3. Typical variation of the coefficient Bp *based on member level.*

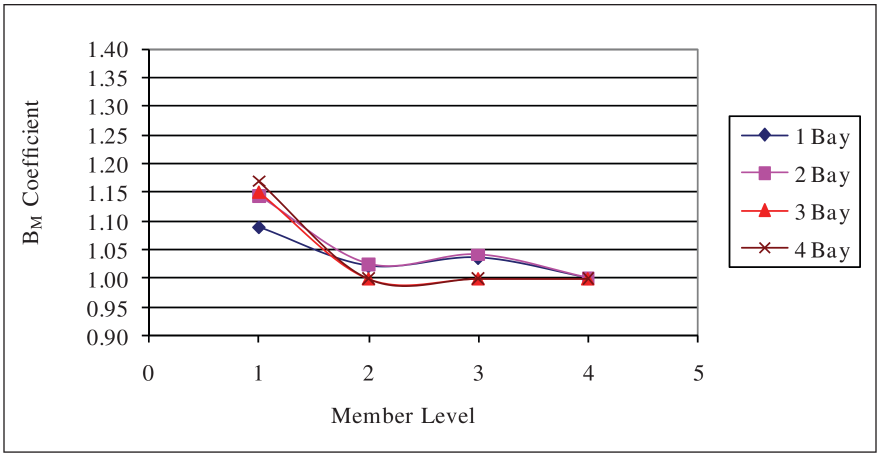

Fig. 4. Typical variation of the coefficient B_M *based on member level.*

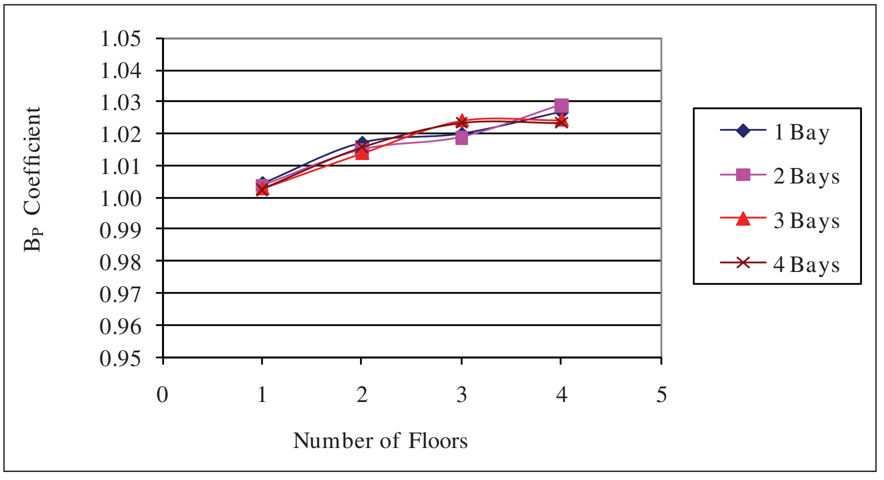

Fig. 5. Typical variation of the coefficient B_P *with the number of floors.*

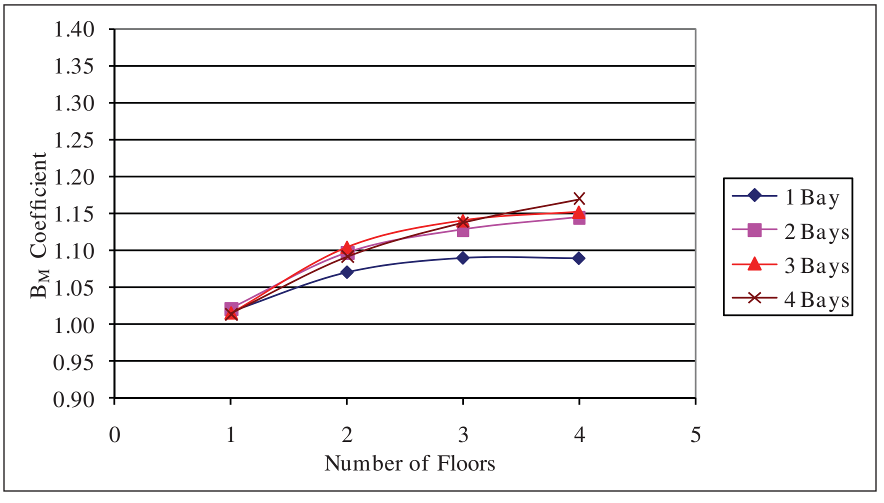

Fig. 6. Typical variation of the coefficient B_M *with the number of floors.*

systems and show a clear increase in the values of the magnification coefficients with an increase in the number of floors of the system. However, the number of bays seems to have a bigger effect on the moment magnification coefficient B_M than it has on the axial force magnification B_P, and this effect gets larger with a larger number of floors.

Because the objective of the study is to come up with a simplified axial force and bending moment magnification method, it was imperative to come up with a single variable to base these coefficients on; using member level, number of levels, and the number of bays in a structure will not produce a practical approach. The results from the aforementioned analyses indicate that there may be a correlation between the amount of axial load a column is subjected to and the axial force and bending moment amplification coefficients. To investigate this further, the variation of B_P and B_M with $(P_{nt} + P_{lt})/\phi P_n$ was examined. This ratio was selected because it incorporates not only applied loads but also geometric and material properties of the affected column. An initial study of the data revealed that a range of values of B_P and B_M can be associated with any single value of $(P_{nt}+P_{lt})/\phi P_n$. This is because the data represent a variety of columns that may be affected by P-delta effects to different degrees. The conservative approach was then to try to correlate maximum B_P and B_M values to $(P_{nt} + P_{lt})/\phi P_n$. To do this, the ratios of $(P_{nt} + P_{lt})/\phi P_n$ were divided into intervals of 0.05, and the maximum values of B_P and B_M were obtained for each interval and plotted. Figure 7 shows the variation of the maximum B_P coefficient with $(P_{nt} + P_{lt})/\phi P_n$ with a linear trend line (R^2 = 0.9229) and a quadratic trend line (R^2 = 0.9468). It is clear from this graph that a linear representation of the variation of B_P with $(P_{nt} + P_{lt})/\phi P_n$ is adequate and can be used as a basis for a simplified evaluation of this coefficient. Although the quadratic equation has a slightly higher R^2 value, the simpler format of the linear equation makes it a more practical choice.

Figure 8 shows the variation of the maximum B_M coefficient with $(P_{nt} + P_{lt})/\phi P_n$ with a linear trend line (R^2 = 0.8689) and a quadratic trend line (R^2 = 0.8994). Here again, the graph indicates that a linear representation of the variation of B_M with $(P_{nt} + P_{lt})/\phi P_n$ is adequate and can be used as a basis for a simplified evaluation of this coefficient.

PROPOSED DESIGN PROCEDURE

The following procedure is being proposed to account for second-order effects in the design of low-rise steel framed buildings. The methodology is based on using Equation C2-6a (AISC, 2005a), which does not depend on using drift as a parameter in calculating the amplification coefficient B_2. This procedure is applicable to pinned-base rigid-frame systems that have up to four floors, that are one to four bays wide and that have a beam length to frame height ratio between 1.5 and 2.2. Furthermore, all the limitations of the second-order analysis by amplified first-order analysis method that are stipulated in Appendix 8 of the 2010 AISC *Specification* apply to this proposed method as well. The method is presented as follows:

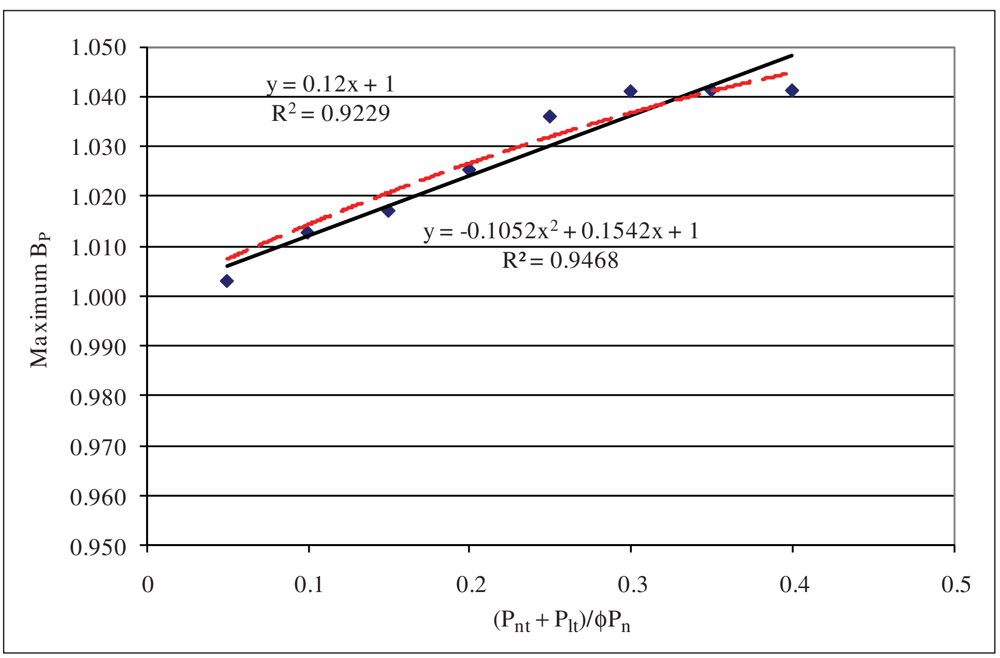

Fig. 7. Variation of maximum B_P *with* $(P_{nt} + P_{lt})/\phi P_n$.

- Analyze the structure for the simultaneous action of gravity and lateral loads, and obtain the total combined axial load (P_t) and the total combined bending moment (M_t) for the member under consideration using LRFD or ASD load combinations as appropriate.

- Compute the coefficient B_P:

$$\text{LRFD} \quad B_P = \frac{1}{8}\left(\frac{P_t}{P_{cx}}\right)+1$$

$$\text{ASD} \quad B_P = \left(\frac{2}{15}\right)\left(\frac{P_t}{P_{cx}}\right)+1$$

- Compute the coefficient B_M:

$$\text{LRFD} \quad B_M = \frac{1}{2}\left(\frac{P_t}{P_{cx}}\right)+1$$

$$\text{ASD} \quad B_M = \left(\frac{8}{15}\right)\left(\frac{P_t}{P_{cx}}\right)+1$$

- Compute the required axial strength:

$$\text{LRFD} \quad P_u = B_P P_t$$

$$\text{ASD} \quad P_a = B_P P_t$$

- Compute the required bending moment:

$$\text{LRFD} \quad M_u = B_M M_t$$

$$\text{ASD} \quad M_a = B_M M_t$$

where P_t is the total combined axial load, M_t is the total combined bending moment using LRFD or ASD load combinations as applicable, and P_c is ϕP_n for LRFD and P_n/Ω for ASD.

For structures that are up to two stories tall, and based on the discussion in the parametric analysis section of this paper, the following values may conservatively be used to amplify forces and moments obtained by a first order analysis:

- Use $B_P = 1.03$
- Use $B_M = 1.10$ for single-bay systems
- Use $B_M = 1.15$ for two-bay systems
- Use $B_M = 1.20$ for three-bay systems
- Use $B_M = 1.25$ for four-bay systems

The existence of gravity-only (leaning) columns can be accounted for by including their load in P_t (in the equations to compute B_P and B_M only) or by adjusting K_x. Several approaches are available in the AISC *Specification* Commentary (AISC, 2010) to include this effect, but the approach

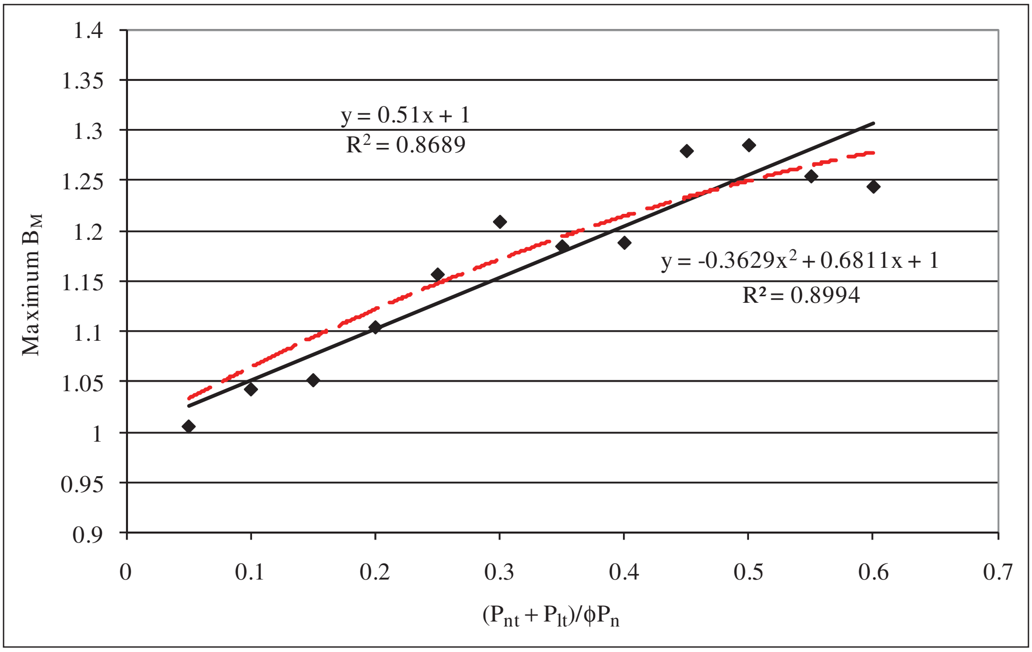

Fig. 8. Variation of maximum B_M *with* $(P_{nt} + P_{lt})/\phi P_n$.

Table 1. Comparison between Stability Analysis Methods (AISC, 2005) and Proposed Method for One-Bay Frame	
Method	**Interaction Equation**
Second-order	0.84
First-order	0.811
Direct analysis	0.796
Simplified	0.966
Proposed method	0.817

that was used by Carter and Geschwindner (2008) in their examples, which had been provided in previous commentaries (Lim and McNamara, 1972), is also used in this paper.

COMPARISON WITH EXISTING METHODS

Carter and Geschwindner (2008) used two simple frames to compare four methods of stability analysis and design:

1. Second-order analysis method (AISC 2005a, Section C2.2a)
2. First-order analysis method (AISC 2005a, Section C2.2b)
3. Direct analysis method (AISC 2005a, Appendix 7)
4. Simplified method (AISC 2005b, p. 2-12 and the AISC *Basic Design Values* cards).

A comparison between the proposed method presented in this paper and the results obtained by Carter and Geschwindner (2008) for a one-bay unbraced frame and a three-bay unbraced frame with leaning columns is presented.

One-Bay Frame

The trial shape (W14×90) used by Carter and Geschwindner for Column A in Figure 9 will be used for the comparison of the proposed method and the four methods of stability presented in the 2005 AISC *Steel Construction Manual*.

Based upon the loading shown in Figure 9, the first-order axial force, strong-axis moment and design parameters for Column A are:

$P_t = 200$ kips $\quad M_t = (20 \text{ kips})\,(15 \text{ ft}) = 300$ kip-ft
$L_x = L_y = 15$ ft $\quad K_x = 2.0 \quad K_y = 1.0$
$L_b = 15$ ft $\quad C_b = 1.67$

To be consistent with the comparison presented by Carter and Geschwindner (2008), the value of $K_x = 2.0$, the theoretical value for a column with a fixed base and a top that is free to rotate and translate, is used rather than the value of 2.1 recommended for design in the 2005 AISC *Specification* Commentary Table C-C2.2.

The effect of the leaning-column is calculated as

$$\Sigma P_{leaning} / \Sigma P_{stability} = (200 \text{ kips})/(200 \text{ kips}) = 1$$

$$K_x{}^* = K_x(1 + \Sigma P_{leaning}/\Sigma P_{stability})^{1/2} = 2.0(1+1)^{1/2} = 2.83$$

Based upon these design parameters, the axial and strong axis available flexural strengths of the ASTM A992 W14×90 are:

$$P_c = \phi P_{nx} = 721 \text{ kips} \qquad M_{cx} = \phi_b M_{nx} = 573 \text{ kip-ft}$$

The new proposed amplification factors are:

$$B_p = \frac{1}{8}\left(\frac{P_t}{\phi P_{nx}}\right) + 1 = 1.035 \qquad B_M = \frac{1}{2}\left(\frac{P_t}{\phi P_{nx}}\right) + 1 = 1.139$$

$$P_r = B_p\, P_t = (1.035)(200 \text{ kips}) = 207 \text{ kips}$$

$$M_{rx} = B_M\, M_t = (1.139)(300 \text{ kip-ft}) = 341.7 \text{ kip-ft}$$

$P_r/P_c = 207/721 = 0.287$. Because $P_r/P_c \geq 0.2$, Equation H1-1a is applicable.

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}\right) = 0.287 + \frac{8}{9}\left(\frac{341.7}{573}\right) = 0.817$$

Table 1 shows that the proposed method of predicting the required strength of the stability column in the one-bay frame shown in Figure 9 is comparable to the other stability methods studied by Carter and Geschwindner (2008).

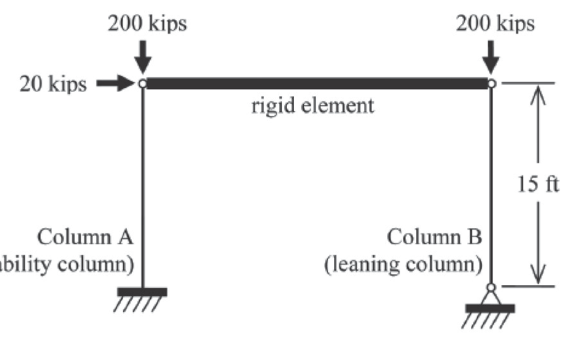

Fig. 9. One-bay unbraced frame used in examples by Carter and Geschwindner (2008).

Table 2. Comparison between Stability Analysis Methods (AISC, 2005) and Proposed Method for Three-Bay Frame	
Method	**Interaction Equation**
Second-order	0.232
First-order	0.239
Direct analysis	0.235
Simplified	0.240
Proposed method	0.245

Three-Bay Frame

The trial shape (W14×109) used by Carter and Geschwindner for Column D will be used for the comparison of the proposed method and the four methods of stability presented in the 2005 AISC *Steel Construction Manual*.

Based upon the loading shown in Figure 10, the first-order axial force, strong-axis moment and design parameters for Column A are:

$P_t = 150$ kips $\quad M_t = (15 \text{ kips})(15 \text{ ft})/2 = 113$ kip-ft

$L_x = L_y = 15$ ft $\quad K_x = 2.0 \quad K_y = 1.0$

$L_b = 15$ ft $\quad C_b = 1.67$

The effect of leaning-column is calculated as:

$$\Sigma P_{leaning} / \Sigma P_{stability} = 2(75 \text{ kips})/[2(150 \text{ kips})] = ½$$

$$\begin{aligned} K_x^* &= K_x (1 + \Sigma P_{leaning} / \Sigma P_{stability})^{½} \\ &= 2.0\,(1 + 0.5)^{½} \\ &= 2.45 \end{aligned}$$

The y-axis effective length is calculated as:

$$K_y L_y = 15 \text{ ft}$$

The equivalent x-axis effective length is calculated as:

$$\frac{k_x^* L_x}{r_x / r_y} = \frac{(2.45)(15)}{1.67} = 22 \text{ ft (governs)}$$

Based upon these design parameters, the axial and strong axis available flexural strengths of the ASTM A992 W14×109 are:

$$P_c = \phi P_n = 1000 \text{ kips}$$

$$M_{cx} = \phi_b M_{nx} = 720 \text{ kip-ft}$$

The new proposed amplification coefficients are:

$$B_p = \frac{1}{8}\left(\frac{P_t}{\phi P_{nx}}\right) + 1 = 1.01 \quad B_M = \frac{1}{2}\left(\frac{P_t}{\phi P_{nx}}\right) + 1 = 1.075$$

$$P_r = B_p\, P_t = (1.019)(150 \text{ kips}) = 152.85 \text{ kips}$$

$$M_{rx} = B_M\, M_t = (1.075)(113 \text{ kip-ft}) = 121.48 \text{ kip-ft}$$

$P_r / P_c = 152.85/1000 = 0.153$. Because $P_r / P_c < 0.2$, Equation H1-1b is applicable.

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}}\right) = \frac{0.153}{2} + \left(\frac{121.48}{720}\right) = 0.245$$

Table 2 shows that the proposed method prediction of the interaction equation is approximately the same as that predicted by the other stability method studied by Carter and Geschwindner (2008), and there is no significant change among all the methods studied for the three-bay frame.

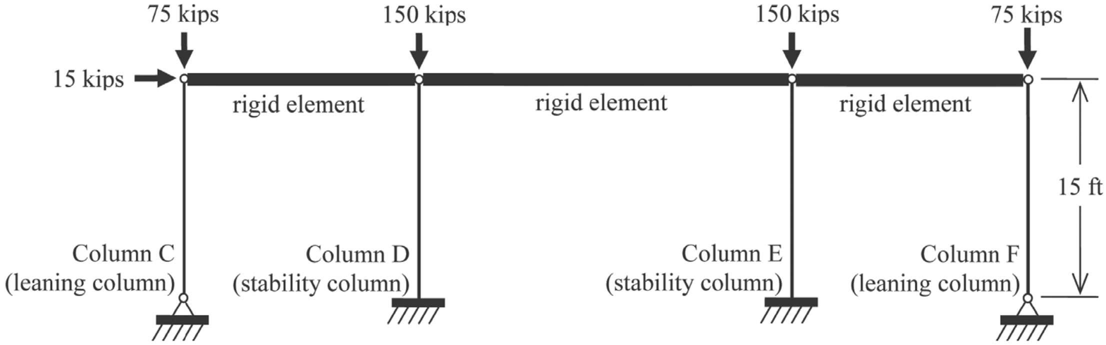

Fig. 10. Three-bay unbraced frame used in examples Carter and Geschwindner (2008).

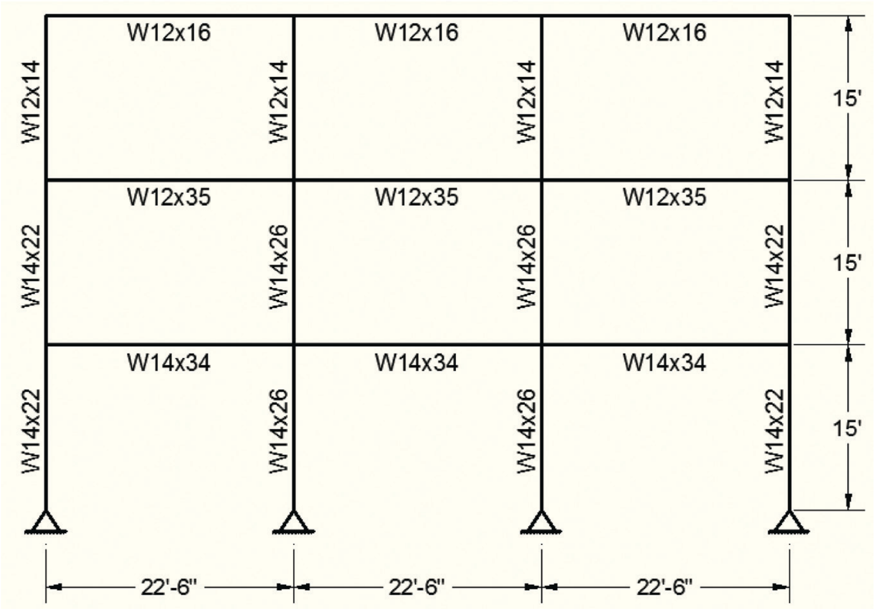

Fig. 11. Three-bay, three-floor system design example.

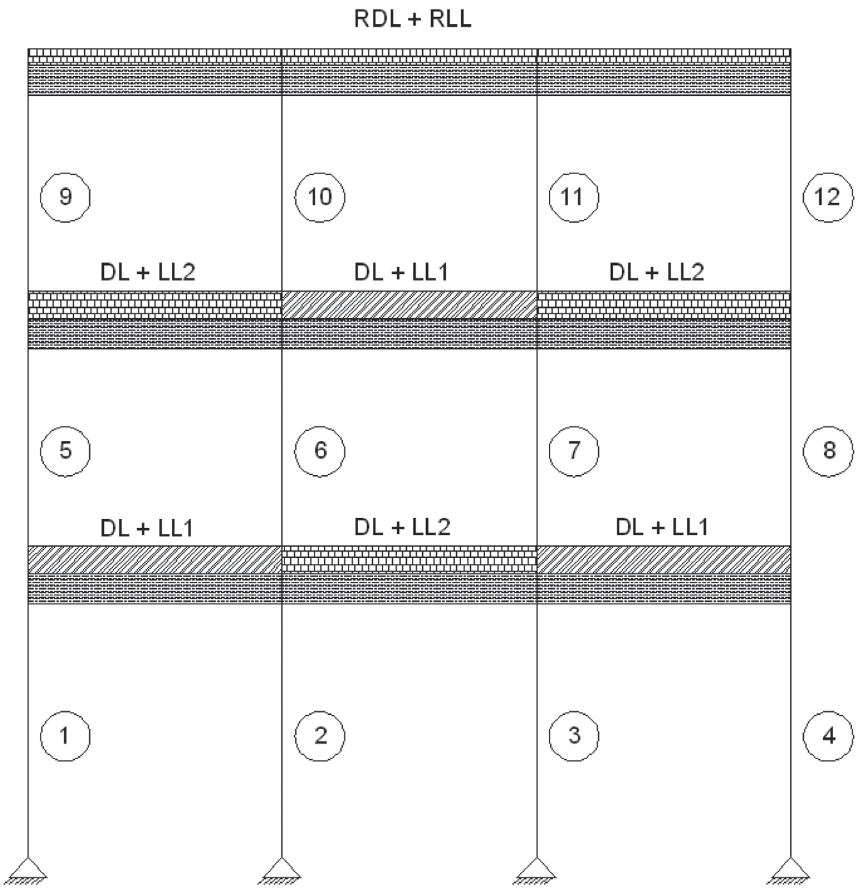

Fig. 12. Three-bay, three-floor system under gravity loading.

DESIGN EXAMPLE

The three-bay, three-floor frame system shown in Figure 11 was used to illustrate the use of the proposed method. The frame is subjected to a floor dead load of 80 psf, a floor live load of 50 psf, and roof dead and live loads of 25 psf and 20 psf, respectively. Frames were assumed to be spaced at 20 ft, and two alternate live loading cases were created to obtain the maximum possible efforts in each member. The total seismic base shear was 12.2 kips, and the total wind force was 10 kips. No consideration was taken for practicality or member connectivity to ensure that strength requirements control the design.

The frame is designed twice: once using the 2005 AISC *Specification* amplified first-order elastic analysis method, and second using the proposed design procedure. The results of the analyses are then compared. First the system is analyzed under gravity loads (as shown in Figure 12), and the first-order axial force (P_{nt}) and the first-order moment (M_{nt}) are computed for each frame member, assuming there is no lateral translation of the frame. The frame is then analyzed for lateral loads (as shown in Figure 13), and the first-order axial force (P_{lt}) and the first-order moment (M_{lt}) caused by lateral translation of the frame are computed for each frame member. The moment amplification coefficients B_1 and B_2 are then computed, and the required axial and flexural strengths of all frame members are evaluated.

According to the proposed design procedure, the frame is only analyzed once under the simultaneous action of gravity and lateral loads (as shown in Figure 14), and the total factored axial load (P_t) and total factored moment (M_t) are computed for each frame member. The coefficients B_P and B_M are then computed using the proposed equations. The required axial and flexural strengths of each member are then computed by multiplying the total factored axial load by the coefficient B_P and the total factored bending moment by the coefficient B_M, respectively. The results of the design of this three-bay, three-floor system are shown in Table 3.

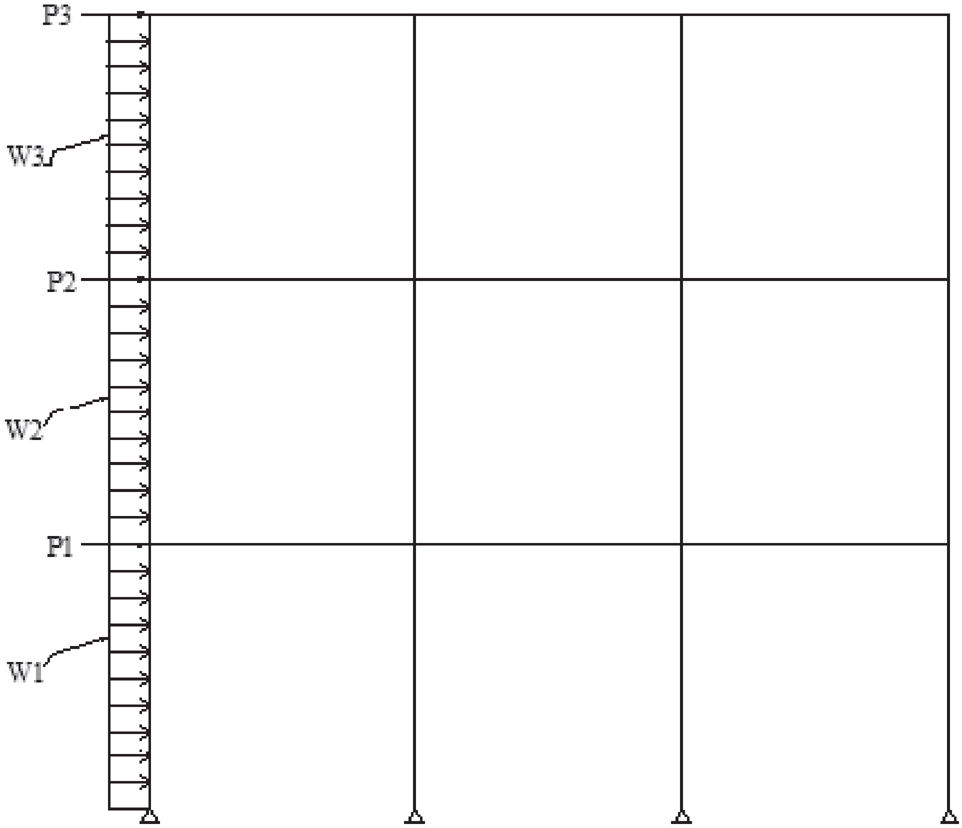

Fig. 13. Three-bay, three-floor system under lateral loading.

Table 3. Design Results of Three-Bay, Three-Floor Frame							
Column	**Based on B_1 and B_2**			**Based on B_P and B_M**			**% increase in *R***
	P_u, kip	**M_u, kip-ft**	***R***	**P_u, kip**	**M_u, kip-ft**	***R***	
1	66.77	−69.95	0.833	67.84	−71.34	0.885	6.24
2	131.56	68.78	0.923	139.95	70.78	0.944	2.27
3	131.56	−68.78	0.923	139.95	−70.78	0.944	2.27
4	66.71	69.91	0.832	67.80	71.30	0.884	6.24
5	45.57	−74.75	0.746	46.78	−82.69	0.912	22.23
6	65.99	44.20	0.510	68.03	48.02	0.544	6.77
7	65.96	−44.14	0.509	67.99	−47.96	0.544	6.78
8	45.57	74.75	0.746	46.78	82.69	0.912	22.23
9	12.97	−35.11	0.595	13.15	−37.05	0.739	24.18
10	14.41	−20.11	0.372	14.64	−21.04	0.414	11.22
11	14.40	19.97	0.370	14.62	20.89	0.411	11.13
12	12.97	35.11	0.595	13.15	37.05	0.739	24.18

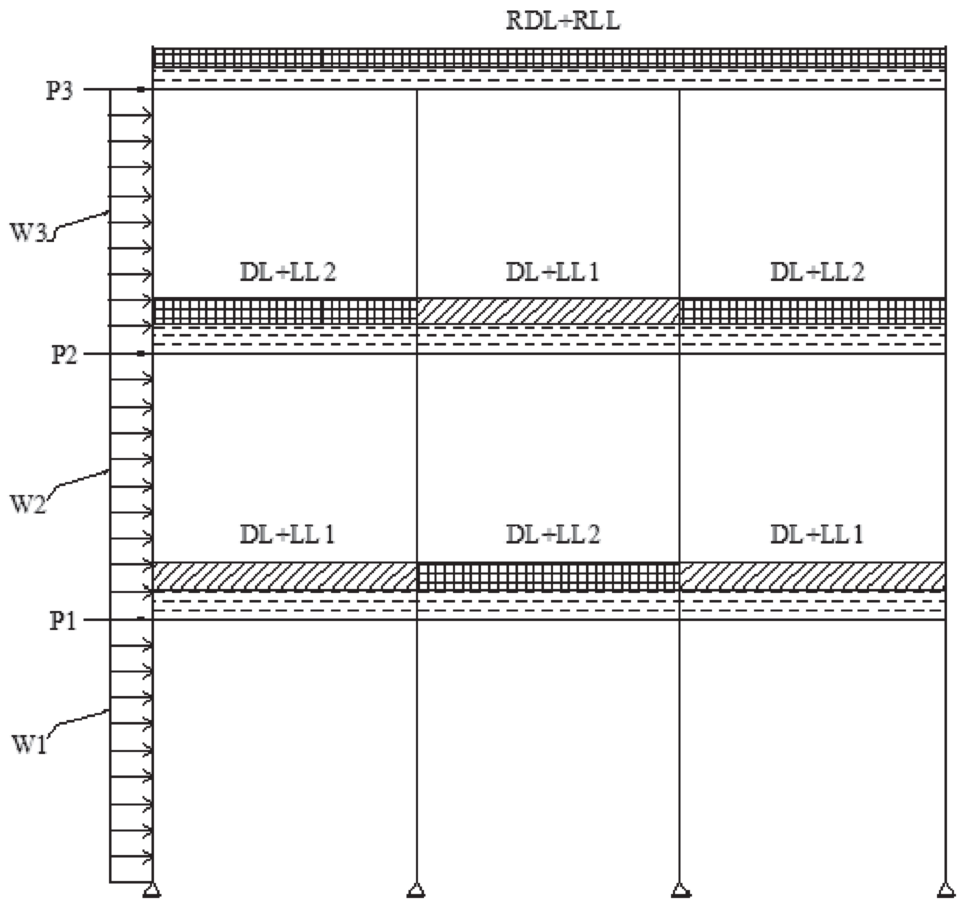

Fig. 14. Three-bay, three-floor system under combined loading.

CONCLUSIONS

A simplified procedure for the evaluation of second-order effects by amplified first-order analysis is proposed in this paper. The procedure applies to the design of low-rise steel-framed buildings. This procedure was compared to four other established design procedures and was found to produce similar results. Moreover, a typical three-bay, three-floor frame was designed using the 2010 AISC approximate second-order analysis, and the proposed method and the results were compared. It is interesting to see in this example that the columns that show the largest increase in the values of the design ratio are in the upper two floors of the structure. It was shown earlier in this study that the design of these columns is usually not affected by the inclusion of second-order effects. They typically carry a light load, causing the member size selection to be controlled by practical rather than strength considerations. They typically carry a light load; consequently, a small increase in that load will translate in a more significant increase in terms of percentage.

The procedure being proposed herein has the advantage of being very straightforward and simple to apply because it only requires a single elastic analysis using existing real loads. Amplified design forces are then obtained by multiplying the first-order analysis results by simple magnification coefficients that are based on the ratio of the applied load to the computed column design strength.

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REFERENCES

AISC (1989), *Specification for Structural Steel Buildings, Allowable Stress Design and Plastic Design*, American Institute of Steel Construction, Chicago, IL.

AISC (1993), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL.

AISC (1999), *Load and Resistance Factor Design Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL.

AISC (2005a), *Specification for Structural Steel Buildings*, ANSI/AISC 360-05, American Institute of Steel Construction, Chicago, IL.

AISC (2005b), *Steel Construction Manual,* 13th ed., American Institute of Steel Construction, Chicago, IL.

AISC (2010), *Specification for Structural Steel Buildings*, ANSI/AISC 360-10, American Institute of Steel Construction, Chicago, IL.

Becker, D. (2008), "Consideration of the Effects of Loading and Frame Configuration Using the Direct Analysis Method," Crossing Borders, *Proceedings, 2008 Structures Congress,* April 24–26, Vancouver, BC, Canada.

Carter, C.J. and Geschwindner, L.F. (2008), "A Comparison of Frame Stability Analysis Methods in ANSI/AISC 360-05," *Engineering Journal*, AISC, Third Quarter, pp. 159–170.

Carter, C.J., Murray, T.M. and Thornton, W.A. (2000), "Cost Effective Steel Building Design—The U.S. Approach," *Progress in Structural Engineering and Materials*, Vol. 2, No. 1, John Wiley and Sons, Ltd., U.K., January–March, pp. 16–25.

Chen, S.J. and Wang, W.C. (1999), "Moment Amplification Factor for P-Δ Effect of Steel Beam-Column," *Journal of Structural Engineering*, ASCE, Vol. 125, No. 2, July, pp. 219–233.

Duan, L. and Chen, W.-F. (1989), "Design Interaction Equation for Steel Beam-Columns," *Journal of Structural Engineering*, ASCE, Vol. 115, No. 5, May, pp. 1225–1243.

Eroz, M., White, D. and DesRoches, R. (2008), "Direct Analysis and Design of Steel Frames Accounting for Partially Restrained Column Base Condition," *Journal of Structural Engineering*, ASCE, Vol. 134, No. 9, pp. 1508–1517.

Kim, S.-E. and Choi, S.-H. (2005), "Practical Second-Order Inelastic Analysis for Three-Dimensional Steel Frames Subjected to Distributed Load," *Thin Walled Structures*, Elsevier, Vol. 43, No. 1, January, pp. 135–160.

Lim, L.C. and McNamara, R.J. (1972), "Stability of Novel Building System," Structural Design of Tall Buildings, Vol. II-16, *Proceedings*, ASCE-IABSE International Conference on the Planning and Design of Tall Buildings, Bethlehem, PA, pp. 499–524.

Sohal, I.S. and Syed, N.A. (1992), "Inelastic Amplification Factor for Design of Steel Beam-Columns," *Journal of Structural Engineering*, ASCE, Vol. 118, No. 7, July, pp. 1822–1840.

Surovek, A. and Ziemian, R. (2005), "The Direct Analysis Method: Bridging the Gap from linear Elastic Analysis to Advanced Analysis in Steel Frame Design," *Proceedings, 2005 Structures Congress and the 2005 Forensic Engineering Symposium.*