Discussion

Plastic Design by Moment Balancing

Paper presented by EDWIN H. GAYLORD (October, 1967, issue)

Discussion by **GERMAN GURFINKEL**

THE AUTHOR has presented a simple method of plastic design of rigid frames using an ingenious technique of moment balancing. Because of its simplicity and use of concepts familiar to structural engineers, the method is particularly attractive and enhances the whole field of plastic design.

Traditional designers, used to elastic analysis of continuous frames, are interested in the behavior of the structure under service load conditions. To them, the author's method provides no assurance that under service load conditions certain critical sections of the frame will not be overstressed or even yielded. To investigate this possibility the writer analyzed the two-story, two-bay frame, designed by the author in Figs. 9 through 14 of the paper, assuming 29,000 ksi for Young's Modulus and a non-yielding elastic structure throughout.

German Gurfinkel is Assistant Professor of Civil Engineering, University of Illinois, Urbana, Ill.

The frame is represented in Fig. 16 using the author's labeling numbers for the members and introducing a numbering system for the joints. The loads to be supported by the frame are shown in Fig. 16 and are due to the effects of: gravity (uniformly distributed load on members **7**, **8**, **9** and **10**) and wind (concentrated load at joints **4** and **7**) acting from either left to right or right to left. Three loading conditions were studied. First, gravity loads alone; second, gravity loads plus wind from the left; and third, gravity loads plus wind from the right. The results are summarized in Table 1.

It can be readily noticed from the observation of Table 1 that most of the structure is stressed to values within the elastic range. However, it can also be seen that members **8** and **10** are subjected to stresses higher than yield (33 ksi for A7 steel), at joints **8** and **5** respectively, for the first and third conditions of loading. This means that initial yielding occurs in these sections under service load conditions. The elastic analysis is theoretically invalidated because of the appearance of initial yield in

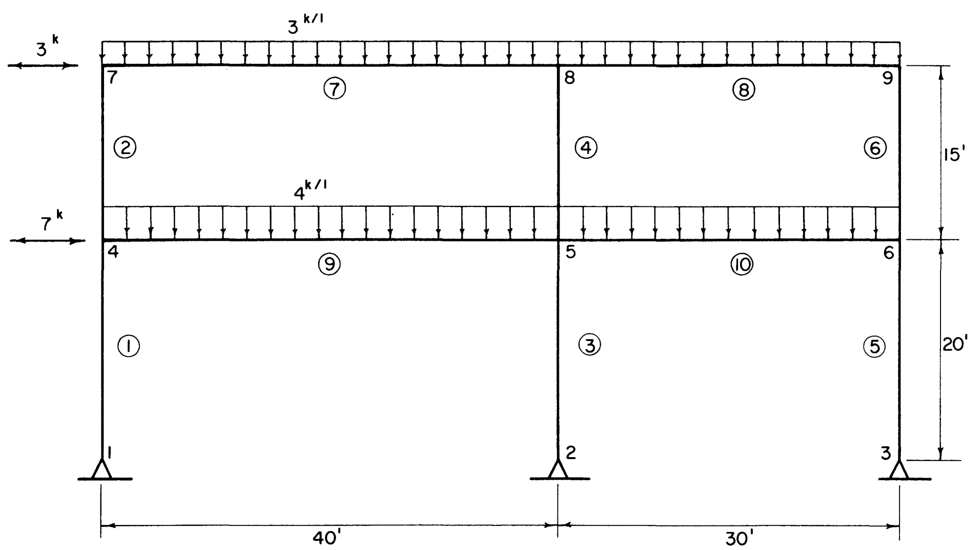

Figure 16

Table 1. Elastic Analysis of Original Frame

Member	Section	Area (in.²)	Section Mod. (in.³)	Joint	Gravity Load Only			Gravity Load Plus Wind From Left			Gravity Load Plus Wind From Right		
					Axial Force (kip)	Bending Moment (kip-in.)	Max. Stress (ksi)	Axial Force (kip)	Bending Moment (kip-in.)	Max. Stress (ksi)	Axial Force (kip)	Bending Moment (kip-in.)	Max. Stress (ksi)
1	18WF64	18.8	117.0	1	130.6	0	7.0	126.6	0	6.8	134.6	0	7.1
				4		−1110	16.4		−99	7.6		−2117	25.4
2	18WF77	22.63	141.7	4	56.0	−3055	24.1	55.4	−3174	24.9	56.6	−2936	23.2
				7		−3131	24.6		−2968	23.5		−3295	25.8
3	14WF78	22.94	121.1	2	267.0	0	11.6	268.1	0	11.7	265.9	0	11.6
				5		578	16.4		1532	24.3		−376	14.7
4	14WF68	20.0	103.0	5	114.4	1487	20.1	114.5	1641	21.7	114.3	1332	18.6
				8		1503	20.3		1732	22.5		1274	18.1
5	12WF53	15.59	70.7	3	92.4	0	5.9	95.3	0	6.1	89.6	0	5.8
				6		530	13.4		967	19.8		93	7.1
6	12WF58	17.06	78.1	6	39.6	1600	22.8	40.1	1616	23.1	39.1	1585	22.6
				9		1596	22.8		1692	24.1		1500	21.5
7	24WF84	24.71	196.3	7	34.4	3131	17.4	37.1	2968	16.6	31.6	3295	17.8
				8		−5047	27.1		−5188	27.9		−4905	25.3
8	21WF55	16.18	109.7	8	17.8	3544	33.4[a]	18.4	3456	32.6	17.1	3631	34.2[a]
				9		−1596	15.6		−1692	16.5		−1500	14.8
9	27WF94	27.65	242.8	4	−29.8	4163	−18.3	−26.7	3273	−14.4	−32.8	5053	−22.0
				5		−6773	−29.0		−7490	−31.8		−6056	−26.1
10	21WF68	20.02	139.9	5	−15.5	4709	−34.5[a]	−14.4	4317	−31.6	−16.8	5100	−38.3[a]
				6		−2130	−16.0		−2583	−19.2		−1678	−12.8

[a] *Initial yielding of the section.*

Note: *Maximum stresses due to positive moments in beams 7, 8, 9 and 10 are 16, 14, 17 and 15 ksi, respectively.*
Maximum deflections at Midspan in beams 7, 8, 9 and 10 are 0.8, 0.07, 0.8 and 0.08 in. respectively.
A positive sign indicates compression for axial force and stress, and an anticlockwise rotation for bending moment at the end of a member.

Table 2. Elastic Analysis of Modified Frame

Member	Section	Area (in.²)	Section Mod. (in.³)	Joint	Gravity Load Only			Gravity Load Plus Wind From Left			Gravity Load Plus Wind from Right		
					Axial Force (kip)	Bending Moment (kip-in.)	Max. Stress (ksi)	Axial Force (kip)	Bending Moment (kip-in.)	Max. Stress (ksi)	Axial Force (kip)	Bending Moment (kip-in.)	Max. Stress (ksi)
1	18WF64	18.80	117.0	1	129.9	0	6.9	126.1	0	6.7	133.7	0	7.1
				4		−1082	16.2		−83	7.4		−2081	24.9
2	18WF77	22.63	141.7	4	55.8	−2995	23.6	55.2	−3128	24.5	56.3	−2862	22.7
				7		−3073	24.2		−2922	23.1		−3225	25.3
3	14WF78	22.94	121.1	2	268.5	0	11.7	269.3	0	11.7	267.8	0	11.7
				5		584	16.5		1537	24.4		−371	14.8
4	14WF68	20.00	103.0	5	115.0	1495	20.3	115.0	1652	21.8	114.9	1339	18.8
				8		1517	20.5		1742	22.7		1292	18.4
5	12WF53	15.59	70.7	3	91.6	0	5.9	94.6	0	6.1	88.5	0	5.7
				6		498	13.0		946	19.5		51	6.4
6	12WF58	17.06	78.1	6	39.3	1523	21.8	39.8	1559	22.3	38.7	1486	21.3
				9		1534	22.0		1637	23.2		1430	20.6
7	24WF84	24.71	196.3	7	33.7	3073	17.0	36.6	2922	16.4	30.8	3225	17.6
				8		−5111	27.4		−5242	28.2		−4980	26.5
8	21WF62[a]	18.23	126.4	8	17.0	3594	29.4	17.8	3501	28.7	16.2	3688	30.1
				9		−1534	13.0		−1637	14.0		−1430	12.2
9	21WF94	27.65	242.8	4	−29.2	4077	−17.9	−26.3	3211	−14.2	−32.1	4943	−21.5
				5		−6879	−29.4		−7562	−32.1		−6196	−26.7
10	24WF68[a]	20.00	153.1	5	−14.9	4800	−32.0	−13.8	4373	−29.3	−16.0	5227	−34.9[b]
				6		−2021	−13.9		−2505	−17.0		−1537	−10.8

[a] *Modified section.*
[b] *Initial yielding of the section.*
Note: *A positive sign indicates compression for axial force and stress, and an anticlockwise rotation for bending moment at the end of a member.*

certain portions of the structure. It is likely, however, that the actual moments in the rest of the structure would be quite similar to those shown in Table 1. Assuming then, that the elastic analysis reflects the conditions under service loads, the important fact remains that some permanent deformation occurs in the structure. These deformations will take place in addition to those which may normally occur during fabrication, erection and welding of the structure. It may seem advisable, therefore, even when the original frame is capable of supporting the loads, to increase the size of members **8** and **10** in order to prevent the structure from yielding under service load conditions. The required section modulus may be obtained approximately as the product of the original section modulus times the ratio of maximum stress to yield stress. Using this rule, members **8** and **10** would require 114 in.^{3} and 162 in.^{3} of section modulus, respectively. The new sections selected for members **8** and **10** are 21WF62 (126.4 in.^{3}) and 24WF68 (153.1 in.^{3}), respectively. The latter is somewhat short of section modulus but was selected, nevertheless, because of the small difference and the fact that it actually weighs the same as the original section.

The results of the elastic analysis on the modified structure are shown in Table 2. It can be seen that the structure remains elastic under all conditions of loading except for small overstressing in member **10** at joint **5** under full gravity load and wind from the right. If the next larger section, 24WF76, were selected for member **10**, instead of the chosen 24WF68, no yielding would occur at joint **5**. With this final change, the modified frame remains fully elastic under service load conditions. Maximum deflections of the beams are well below the allowable limits. Also the modified frame provides a larger factor of safety, as compared to the original frame, against ultimate load conditions. In other words, heavier loads would be required to transform the modified frame into a mechanism, because of the fact that it is made of stronger members. It is true that it will cost about 2 percent more than the author's original design, 18,100 lbs vs. 17,700 lbs per frame, but the change to a fully elastic behavior under service loads may well be worth the difference.

It can be concluded, therefore, that most steel designers may use the author's method to design a continuous frame with minimum effort. To designers who are satisfied with strength only, the job is finished after moments have been plastically balanced for the various possible conditions of loading. To traditional designers who wish an elastic behavior of the frame under service load conditions in addition to meeting strength requirements, the author's method may be quite useful in determining the initial sections of the members. As has been shown before, it is likely that only minor modifications in the frame may be required to satisfy an elastic design.

Discussion by **EDWIN H. GAYLORD**

PROFESSOR GURFINKEL's elastic analysis of the plastically designed two-bay, two-story frame is an interesting contribution. It is true, as he observes, that the designer who holds to tradition may be concerned about the localized yielding which is predicted at the maximum service load, and may wish to eliminate it by reproportioning the structure. He has shown that this is easily done once the frame has been proportioned according to plastic methods. However, as in this case, the result is likely to be a frame which satisfies neither the elastic-design requirement that the stress nowhere exceed a prescribed allowable value, nor the plastic-design requirement that the collapse strength of the structure need only equal the service load multiplied by a prescribed factor of safety (load factor), whether or not localized yielding develops at service loads.

Professor Gurfinkel mentions the deformations which normally develop during fabrication and erection. It should be noted that the residual stresses which these operations produce will, in general, invalidate the results of an elastic analysis. Thus, a frame built according to the author's design will undoubtedly show early local yielding at more than the two cross sections which are predicted by elastic analysis based on a stress-free condition at no load.

Except for their influence on various types of buckling, residual stresses do not alter the collapse load of a structure, although they do affect the load-deflection path to the collapse load. Furthermore, the collapse load can be predicted quite closely independently of residual stresses, provided they are accounted for in evaluating buckling phenomena. On the other hand, stresses at loads less than the collapse load cannot be predicted independently of the initial residual stresses. In the author's opinion, therefore, too much significance is attached to calculated stresses which exceed the yield stress if they are used only to justify resizing the members of a plastically designed frame.

This frame was proportioned in A7 steel, even though it is no longer produced, to facilitate comparison of design by moment balancing with design of the same frame by the mechanism method of analysis as given on pp. 358–372 of Ref. 4.

REFERENCE

4. *Beedle, L. S.* Plastic Design of Steel Frames, *John Wiley & Sons, New York, 1958.*

Edwin H. Gaylord is Professor of Civil Engineering, University of Illinois, Urbana, Ill.