A Cost Evaluation of Space Trusses of Large Span

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THIS STUDY is the first part of a larger economic study dealing with space trusses, domes and cable roofs. The domes and cable roofs considered are characterized by perimeter support around a circular or elliptical plan. This led to the search for a space truss with support at the perimeter.

The particular space truss treated in this work met the above requirement. The mechanics of the structural analysis have been discussed elsewhere by Kato, Takashani, Tsushima, and Hirata in a paper presented at the International Conference on Space Structures,[1] and are presented here in summary form. The methods of analysis and design used here are approximate and give rapid preliminary design solutions. As a consequence of this simplification more than a thousand different problems were solved.

More difficult than the analysis and design of the truss is the estimating of its cost. In endeavoring to solve this problem the author became indebted to two structural steel fabricators who spent considerable time examining the structural details and supplying him with fabrication and erection costs.

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STRUCTURAL ANALYSIS

Kato, et al.,[1] have discussed in detail a space truss called the Takenaka truss, composed of square pyramidal elements connected as shown in Fig. 1. In this truss scheme the top chord members lie on a grid diagonal with respect to the bottom chord. The joints in the two planes are connected by diagonals so that the resultant truss is composed of inverted square pyramidal elements arranged in a checkered pattern. Consequently, this truss has fewer members than other space trusses and it becomes necessary to investigate the stability of the system. Kato has proved that for stability one must, at least:

1. Provide simple vertical supports at the boundary.
2. Prevent horizontal displacements of the structure.
3. Add edge members along the boundary support lines to form triangles at the perimeter of the top plane. (These members prevent the relative rotation of the pyramidal elements.)

When these conditions are satisfied, the truss is stable and, moreover, is statically determinate for vertical loading.

By considering a decreasing grid spacing (center to center distance between the pyramidal elements very small compared to the truss sides) it is possible, in the limit, to reduce the difference equations developed by Kato to differential equations. Letting:

q = transverse load per unit area, and
M = bending moments per unit width in the x and y directions, which are equal, one obtains the equilibrium equation

$$\nabla^2 M = -q \qquad (1)$$

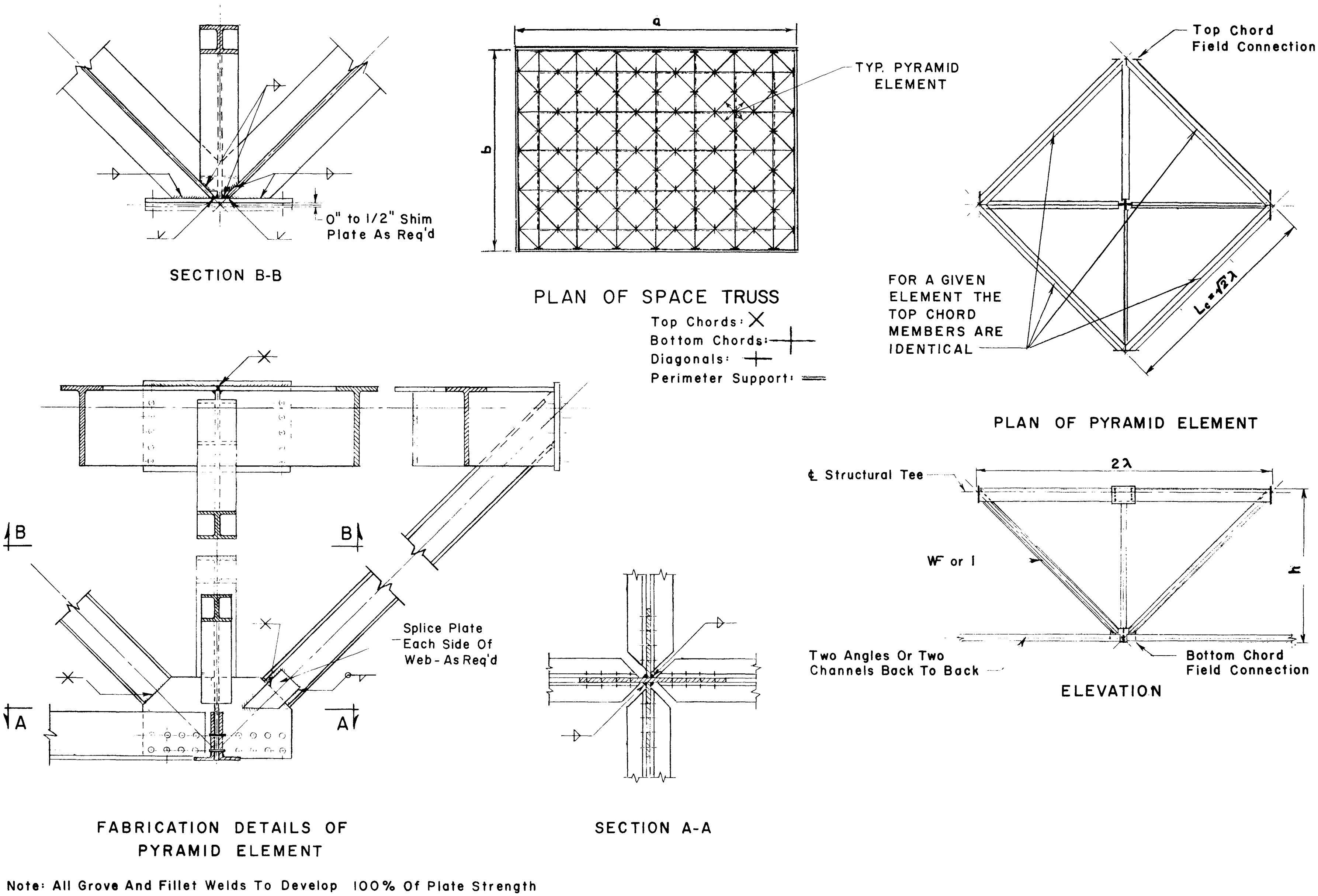


Fig. 1. Space truss of square pyramid elements

Calling h the depth of the truss, L_d the length of the diagonal, and 2λ the lower grid spacing, the bar forces become:

$$N_T = -M\sqrt{2}(\lambda/h) \tag{2}$$

$$N_B = 2M(\lambda/h) \tag{3}$$

$$N_{D,N} = \pm \frac{dM}{dn}\left(\frac{2\lambda}{h}\right)(L_d) \tag{4}$$

where N_T, N_B and $N_{D,N}$ are the direct forces in the top chord, bottom chord and diagonal, respectively, and dM/dn is the derivative of M in the n (x or y) direction.

Neglecting the displacements due to shear, the differential equation for the vertical displacements w may be written:

$$\nabla^2 w = -2M/D \tag{5}$$

where

$$D = \frac{1}{2\lambda} EI = \text{truss rigidity per unit width}$$

$$I = \frac{h^2\sqrt{2}A_T A_B}{A_B + 2A_T}$$

A_T = area of the top chord

A_B = area of the bottom chord

Since the truss is simply supported around a rectangular perimeter a by b, one may take the first term of the Fourier series expansion of q:

$$q = \frac{16}{\pi^2} q_0 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \tag{6}$$

and determine M and w approximately from Equations (1) and (5) as:

$$M = \frac{16}{\pi^4} \frac{q_0 a^2}{[1 + (a^2/b^2)]} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \tag{7}$$

and

$$w = \frac{32}{\pi^6} \frac{q_0 a^4}{D[1 + (a^2/b^2)]} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \tag{8}$$

With $b = a$ and $b = 2a$ the maximum bar forces and maximum displacement are:

1. With $b = a$:

$$N_B = 0.082 q_0 a^2\left(\frac{2\lambda}{h}\right) \tag{9}$$

$$N_T = -0.058 q_0 a^2\left(\frac{2\lambda}{h}\right) \tag{10}$$

$$N_D = 0.26 q_0 a\left(\frac{2\lambda}{h} L_d\right) \tag{11}$$

$$w = 0.0083 q_0 (a^4/D) \tag{12}$$

Table 1. Comparison between Experimental Results and Approximate and Exact Methods of Analysis

	Experimental[a]	Theoretical	
		Exact[a]	Approximate
Displacement	30.0 mm	34 mm	32 mm
Top chord (#6)	−3,040 kg	−4,710 kg	−4,700 kg
Bot chord (#10)	6,200 kg	7,020 kg	7,420 kg
Diagonal (#11)	2,940 kg	2,770 kg	2,440 kg

[a] *Tables 3 and 4 of paper by Kato, et al., Ref. 1.*

2. With $b = 2a$

$$N_B = 0.132 q_0 a^2\left(\frac{2\lambda}{h}\right) \tag{13}$$

$$N_T = -0.094 q_0 a^2\left(\frac{2\lambda}{h}\right) \tag{14}$$

$$N_D = 0.416 q_0 a\left(\frac{2\lambda}{h} L_d\right) \tag{15}$$

$$w = 0.0213 q_0 (a^4/D) \tag{16}$$

In order to check the validity of these equations, Table 1 compares the results with those obtained by the more exact difference equations of Kato.

STRUCTURAL DESIGN

From Equations (4) and (7) one obtains the following expressions for the maximum diagonal bar forces in the x and y direction, respectively:

$$N_{D,X/\max} = \pm \frac{32}{\pi^3} \frac{q_0 a}{[1 + (a^2/b^2)]} \lambda\left(\frac{L_d}{h}\right) \tag{17}$$

$$N_{D,Y/\max} = N_{D,X/\max}(a/b) \tag{18}$$

The average diagonal bar force may be determined by a summation which, in the limit as 2λ goes to zero, reduces to an integration:

$$N_{D,X/\text{avg}} = (4/\pi^2)\, N_{D,X/\max} \tag{19}$$

$$N_{D,Y/\text{avg}} = (4/\pi^2)\, N_{D,Y/\max} \tag{20}$$

Letting the allowable tensile stress F_a equal $0.6F_y$, the maximum and average areas for the tension diagonals are:

$$A_{X,T/\text{avg}} = \frac{214}{\pi^5} \alpha \frac{q}{F_y} \lambda^2 \frac{a}{h} \frac{[1 + (h^2/\lambda^2)]^{1/2}}{[1 + (a^2/b^2)]} \tag{21}$$

$$A_{Y,T/\text{avg}} = A_{X,T/\text{avg}}(a/b) \tag{22}$$

$$A_{X,T/\max} = A_{X,T/\text{avg}}\, \pi^2/4 \tag{23}$$

$$A_{Y,T/\max} = A_{Y,T/\text{avg}}\, \pi^2/4 \tag{24}$$

Table 2. Cross-Sectional Area Properties

Compression Diagonal		
Member Designation	Area (in.^{2})	$\beta = \frac{A}{r_y^2}$
4WF13	**3.82**	**3.90**
6WF25	7.37	3.20
8WF48	14.11	3.27
8WF67	19.70	4.39
10WF89	26.19	3.79
10WF112	32.92	4.62
12WF161	47.38	4.62
14WF237	69.69	4.15
		Avg β = **3.99**

Tension Diagonal		
Member Designation	Area (in.^{2})	$\beta = \frac{A}{r_y^2}$
3 I 5.7	**1.64**	5.8
4 I 7.7	2.21	6.2
		Avg β = 6.0

Bottom Chord		
Member Designation	Area (in.^{2})	$\beta = \frac{A}{r_y^2}$
2—2½ × 2 × 3/16 ∠	**1.62**	2.61
2—5 × 3 × 3/8 ∠	5.72	3.29
2—7 × 4 × 1/2 ∠	7.96	2.96
2—8 × 4 × 1/2 ∠	11.50	4.50
		Avg β = **3.34**

Top Chord				
Member Designation	Area A (in.^{2})	$\gamma = \frac{r_x}{d_1}$	$k = \frac{y_1}{d_1}$	$\eta = \frac{d_1}{A^{1/2}}$
ST **4—8.5**	**2.50**	0.283	0.210	2.53
ST 5—14.5	4.27	0.274	0.206	2.50
ST 8—32	9.40	0.284	0.216	2.60
ST 9—52.5	15.43	0.270	0.199	2.35
ST 10—71	20.88	0.271	0.202	2.36
ST 13—88.5	26.05	0.280	0.214	2.56
ST 16—110	32.37	0.282	0.213	2.60
ST 18—130	38.28	0.282	0.215	2.61
ST 18—150	44.09	0.285	0.224	2.70
		Avg γ = 0.279	Avg k = 0.212	Avg η = 2.54

For definition of r_x, d_1 and y_1 see Ref. 65.

The ratio

$$\alpha = \frac{\text{actual bar area}}{\text{theoretical bar area}} \qquad (25)$$

is introduced in Equation (21) to allow for the fact that the actual cross-sectional area is usually greater than the theoretical one required. For rolled shapes $\alpha = 1.1$. The average areas given above do not reflect the fact that the minimum area is limited by the maximum allowable slenderness ratio L/r. A further adjustment in this value is made in Equations (51) through (54).

In the design of the compression diagonals it is necessary to consider buckling. An examination of a sample of wide-flange shapes which may be used as compression diagonals reveals that:

$$r_y^2 \cong A/\beta \qquad (26)$$

with β approximately constant (Table 2). Furthermore, for this problem it can be shown that for the compression diagonals:

$$\frac{KL_d}{r} > C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \qquad (27)$$

with $K = 0.7$. It follows that the allowable stress is given by:

$$F_a = \frac{1}{2K^2} \frac{\pi^2 E}{(L_d/r_y)^2} \qquad (28)$$

Using the maximum and average diagonal forces previously considered, one obtains the corresponding areas:

$$A_{X,C/\text{avg}} = \frac{16\sqrt{2}}{\pi^3} \frac{K\lambda^2}{\sqrt{\pi}} \left(1 + \frac{h^2}{\lambda^2}\right) \times \left[\alpha\beta_1 \frac{q_0}{E} \frac{a}{h} \frac{1}{[1 + (h^2/\lambda^2)]^{1/2}} \frac{1}{[1 + (a^2/b^2)]}\right]^{1/2} \qquad (29)$$

$$A_{Y,C/\text{avg}} = A_{X,C/\text{avg}}(a/b)^{1/2} \qquad (30)$$

$$A_{X,C/\max} = \frac{\pi}{2} A_{X,C/\mathrm{avg}} \tag{31}$$

$$A_{Y,C/\max} = \frac{\pi}{2} A_{Y,C/\mathrm{avg}} \tag{32}$$

The maximum and average chord forces are derived in an analogous manner, with the result that for the compression (top) chords:

$$N_{T,C/\mathrm{avg}} = \frac{128}{\sqrt{2}\pi^6} \frac{\lambda}{h} \frac{q_0 a^2}{[1 + (a^2/b^2)]} \tag{33}$$

$$N_{T,C/\max} = \frac{\pi^2}{4} N_{C,T/\mathrm{avg}} \tag{34}$$

For the tension (bottom) chords:

$$N_{B,T/\mathrm{avg}} = \sqrt{2} N_{T,C/\mathrm{avg}} \tag{35}$$

$$N_{B,T/\max} = \sqrt{2} N_{T,C/\max} \tag{36}$$

For the bottom chords, which are in tension, the allowable stress is $F_a = 0.6F_y$ and the areas are:

$$A_{B,T/\mathrm{avg}} = \frac{\alpha}{\epsilon} \frac{213}{\pi^6} \frac{\lambda}{h} \frac{a^2}{[1 + (a^2/b^2)]} \frac{q}{F_y} \tag{37}$$

$$A_{B,T/\max} = \frac{\pi^2}{4} A_{C,T/\mathrm{avg}} \tag{38}$$

where

$$\epsilon = \frac{\text{effective cross-sectional area}}{\text{net cross-sectional area}} \tag{39}$$

$$= 0.90 \text{ for bolted connections}$$

For the top chord, besides the compressive force, it is also necessary to consider the bending moments that result from the fact that the metal deck spans between the top chord members.* It is assumed that this deck spans in one direction for the pyramidal elements and in the orthogonal direction for the bays in between. This results in a uniform reaction $q_f \times L_c/2$ from the metal deck where

q_f = the uniform load consisting of the full live load plus the weight of the deck and roofing

L_c = length of the top chord bars

It should be noted that the uniform load q carried by the space truss is the full dead load, including the weight of the truss, plus the live load which may be reduced by a live load reduction factor RF:

$$\mathrm{RF} = \frac{\text{uniform live load carried by the truss}}{\text{uniform live load carried by the top chord}} \tag{40}$$

* *For a uniform load it was assumed that because of partial restraint $M = wl^2/10$. With the top chord longer than 16 ft an intermediate purlin was used and the moment increase was accounted for.*

Considering a tee section, it is necessary to define the following constants based on cross-sectional area properties (Table 2):

$$\gamma = r_x/d_1 = 0.279 \tag{41}$$

$$k = y_1/d_1 = 0.212 \tag{42}$$

$$\eta = d_1/A^{1/2} = 2.54 \tag{43}$$

It is assumed that the metal deck offers lateral restraint;[47] consequently, buckling of the compressed members occurs only about their major axis. Using the interaction formula**

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \tag{44}$$

with

$$F_b = 0.6F_y \tag{45}$$

and with

$$F_a = \left[1 - \left(\frac{KL_c}{r_x}\right)^2 \frac{1}{4\pi^2} \frac{F_y}{E}\right] \frac{F_y}{\mathrm{F.S.}} \tag{46}$$

since $(KL_c/r) < C_c$, where K is equal to 0.7.† The Factor of Safety (F.S.) varies‡ and has a maximum value of 1.92 when $KL_c/r = C_c$. With $L_c = \sqrt{2}\lambda$:

$$A_{T,C} = \lambda^2 \frac{q}{F_y} \left\{\frac{K}{6\gamma^2} \frac{q_f}{q} \frac{L_c}{d_1} + \frac{C(256/\sqrt{2}\pi^2)(\lambda/h)\{(a^2/\lambda^2)/[1 + (a^2/b^2)]\}}{[1 - (K^2/\gamma^2)(L_c/d_1)(1/4\pi^2)(F_y/E)]}\right\} \tag{47}$$

where $A_{T,C} = A_{T,C/\mathrm{avg}}$ with $C = 1$

$A_{T,C} = A_{T,C/\max}$ with $C = 2/\pi$

Equation (47) is solved by iteration (with the initial value of the length to depth ratio, L_c/d_1, taken as 20), since d_1 is a function of A.

One notes that the average forces derived above are the average of forces that vary either as the function

$$f(x,y) = \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$$

** *The interaction formula from the 1947 AISC Specification was used to simplify the program for preliminary design.*

† *Naslund (Ref. 46) states: "It is the author's opinion that the joint rigidity created by the welded intersection of web and chord members is sufficient to permit the use of 70 percent of the distance between panel points for the chord member length."*

‡ *The actual factor of safety given in the 1963 AISC Specification varies: F.S. = $\frac{5}{3} + \frac{3}{8}\frac{(Kl/r)}{C_c} - \frac{(Kl/r)^3}{8C_c^3}$*

or as the first derivative of $f(x,y)$ [Equations (2), (3), (4) and (7)]. When the cross-sectional area is derived from the force, the area varies in the same manner as the force for the tension members, and in approximately this manner for the other members. This, however, is not the actual variation at all points x and y, since the cross-sectional area is also limited by the following code requirements, some of which reflect buckling:

For the compression diagonals ($K = 0.7$):

$$K \frac{L}{r_y} \leq 200 \tag{48}$$

For tension members (bottom chord and tension diagonals, $K = 1.0$)*:

$$K \frac{L}{r_y} \leq 240 \tag{49}$$

For the top chord, since it supports a flat roof:

$$\frac{d_1}{L_c} \geq \frac{f_b}{600{,}000 \text{ psi}} \tag{50}$$

From these restrictions the following *minimum* areas are derived:

Compression diagonal:

$$A_{X,C/\min} = A_{Y,C/\min} = \frac{\alpha\beta K^2\lambda^2[1 + (h^2/\lambda^2)]}{4 \times 10^4} \tag{51}$$

Tension diagonal:

$$A_{X,T/\min} = A_{Y,T/\min} = \frac{\alpha\beta K^2\lambda^2[1 + (h^2/\lambda^2)]}{5.76 \times 10^4} \tag{52}$$

Bottom chord:

$$A_{B,T/\min} = \frac{\alpha\beta K^2 4\lambda^2}{5.76 \times 10^4} \tag{53}$$

Top chord:

$$A_{T,C/\min} = \frac{2\alpha\lambda^2}{\gamma\eta} \sqrt{\frac{q_f(K)}{12{,}000 \text{ psi}}} \tag{54}$$

where the values of β, from Table 2, are:

Compression diagonal: $\beta = 3.90$ (55)

Tension diagonal: $\beta = 6.0$ (56)

Bottom chord: $\beta = 3.34$ (57)

The minimum area is further limited by the minimum area of rolled shapes available; these are shown boldface in Table 2.

* *The bottom chord is bolted; consequently, it is considered as pin-connected.*

The resulting variation in the cross-sectional area over the plan area of the truss is shown in Fig. 2. The minimum area occurs over region **P**. The point of maximum area occurs at the midpoint of the space truss for the top and bottom chords and at the center of opposite sides for the diagonals spanning in that direction. Let us now define an adjusted average area $\bar{A}$ which is the average cross-section area of the members in region **Q** of Fig. 3. With either $A_{\min}$ and the area of region **P** known, or $\bar{A}$ and the area of region **Q** known, the weight of the structural member in the region under question can be determined from the equation:

$$\text{Weight} = (A_{\min} \text{ or } \bar{A}) \frac{(\text{Area of region})}{(2\lambda)^2} L_m \times w_s \tag{58}$$

where L_m is the total length of the member in a single pyramid element and w_s is the unit weight of steel. From these weights for the top chords, bottom chords and diagonals, the total weight carried by the truss may be calculated. If this weight is not within 5 percent of the assumed weight, the analysis and design is repeated. In addition, the weights of the members in these regions are used directly to determine the cost of the space truss.

Let us next determine the length to depth ratios for which the maximum allowable deflection is not exceeded. With the live load deflection limited by

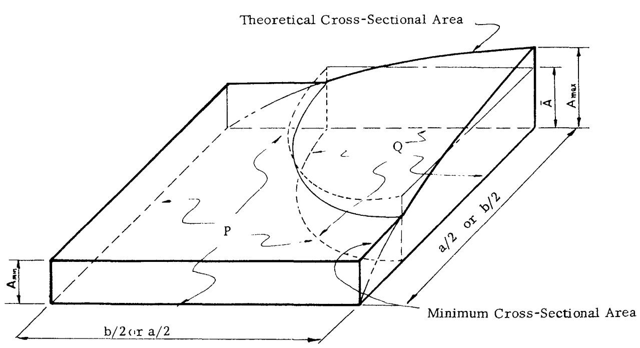


Fig. 2. Variation of cross-sectional area plotted over ¼-span of the space truss

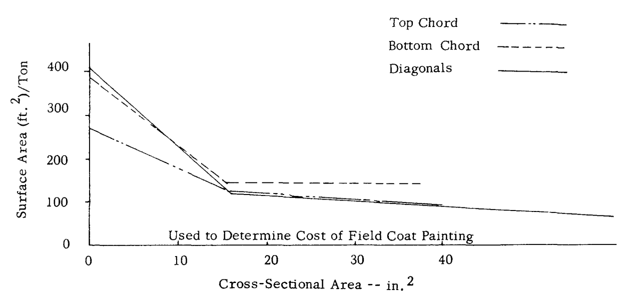


Fig. 3. Surface area per ton vs. cross-sectional area

$$\Delta_{LL} \leq \frac{b}{m}$$

$$m = 360 \text{ for floors}$$

$$n = 240 \text{ for roofs}$$

one obtains from Equations (8) and (38):

$$\frac{b}{h} \leq \frac{\pi^2 q E \sqrt{2}(A_T/A_B)}{1.2 m q_{LL} F_y [1 + \sqrt{2}(A_T/A_B)]} \left(\frac{b}{a} + \frac{a}{b}\right) \quad (59)$$

For the problems considered in this work $A_T/A_B = 1.35$, $F_y = 50$ ksi, $E = 30 \times 10^3$ ksi, $m = 240$, and

$$\frac{b}{h} \leq 13.5 \frac{q}{q_{LL}} \left(\frac{b}{a} + \frac{a}{b}\right) \quad (60)$$

When this inequality is satisfied, stress and not deflection governs for design.

COST ANALYSIS

From the formulas of the previous section, the areas and quantities of material for each of the members of the truss may be determined. Since the ultimate goal of this study is the evaluation of minimum cost for various spans and live loads, it is advantageous to program these formulas for the digital computer. With this information the optimum span to depth ratio and span to grid spacing ratio can be determined by varying these parameters until the minimum cost is obtained.

The total cost of the truss was determined by considering the cost of the metal deck and the cost of the truss separately with the latter divided into cost of materials, fabrication, erection and painting.

By considering various open beam long span steel deck sections with and without intermediate purlins, the conditions for minimum weight of the metal deck was found. This resulted in the following equation for total unit weight of the steel deck and purlins (w_{rs}) in psf.:

$$w_{rs} = 2 + 0.0785 L_c + 0.025(q_{LL} - 20) \quad (61)$$

where

L_c = length of the top chord (ft)

q_{LL} = live load (psf)

The cost of the roofing surface was taken as \$520.00 per ton. The weight of the built up roof and insulation was taken as 7 psf.

The material cost for members of the truss were obtained from a supplier for 36 ksi and 50 ksi steel. This data is summarized in Table 3 and in Fig. 4.

Fabrication costs were obtained from two fabricators, according to the details of Fig. 1 and to the following assumptions:

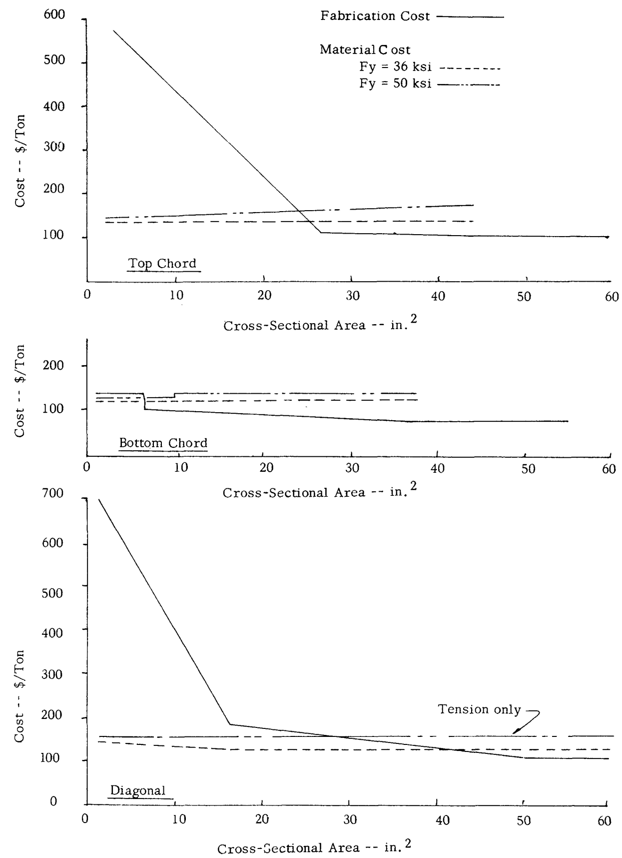


Fig. 4. Cost per ton vs. cross-sectional area

1. The pyramid elements would be either shop assembled or welded in a jig at the site.
2. The bottom elevation of the truss was assumed to be 60 ft above grade. Scaffolding would be built to this elevation or the elements would be assembled on the ground and lifted to the top of the columns.
3. The individual elements would be joined together by field bolting.

The resultant fabrication costs are summarized in Table 3 and Fig. 4.

The erection costs were taken as \$212.00 per ton for elements weighing less than 1.15 ton and \$162.00 per ton for elements weighing more than this.

Painting costs were figured at \$10.00 per ton for the shop coat, while the field coat was based on the surface area to be covered. The relationship between the surface area per ton and cross-sectional area for rolled shapes is given in Table 3 and Fig. 3. Field painting costs were then calculated on the assumption that a structural steel painter costing \$7.00 per hour could cover 1200 sq ft in an eight hour day, while one gallon of paint costing \$8.25 can cover 450 sq ft.[41]

Table 3. Cost vs. Cross-Sectional Area

		Member Designation	Area (in.²)	Material Cost ($/Ton)		Fabrication Cost ($/ton)	Painting Field Coat Area(ft²)/ton
				F_y = 36 ksi	F_y = 50 ksi		
Diagonals	Com-pression	4WF13	3.82	141	—	624	307
		6WF25	7.37	135	—	480	242
		8WF48	14.11	128	—	300	169
		8WF67	19.70	128	—	180	124
	Tens. & Comp.	10WF89	26.19	127	154	170	115
		10WF112	32.92	127	154	110	93
		12WF161	47.38	126	153	102	78
		14WF237	69.69	126	168	68	66
	Tension	4I7.7	2.21	143	150	624	404
		6I12.5	3.61	139	146	624	337
		8I23	6.71	137	154	480	232
		12I50	14.57	133	154	182	149
Bottom Chord		2—5 × 3 × 3/8 ∠	5.72	131	138	148	274
		2—6 × 3½ × 3/8 ∠	6.84	131	138	108	271
		2—7 × 4 × 3/8 ∠	7.96	130	137	104	270
		2—8 × 4 × 1/2 ∠	11.50	130	147	102	204
		2—9 × 4 × 9/16 ∠	14.00	129	146	102	181
		2—9 × 4 × 3/4 ∠	18.38	129	146	98	141
		2—9 × 4 × 1 ∠	24.00	129	156	94	106
		2—15 [50	29.28	131	148	90	145
		2—18 [58	33.96	131	148	86	148
Top Chord		ST 5—14.5	4.27	148	165	560	250
		ST 8—32	9.40	138	155	450	172
		ST 9—52.5	15.43	138	155	296	133
		ST 10—71	20.88	142	159	176	112
		ST 13—88.5	26.05	140	157	170	104
		ST 16—110	32.37	141	158	110	98
		ST 18—130	38.28	141	168	106	89
		ST 18—150	44.09	141	168[a]	104	78

[a] *Available with a maximum yield stress of 46 ksi*

With the above information included in the program, the computer results were checked against those from a simply supported space truss built in 1963. This comparison appears in Table 4. One should first note that, for the same live load, the dead load of the truss from the computer design is approximately two-thirds the weight of the 1963 space truss. This is a consequence of the different framing system which results in half as many diagonals as well as shorter compression elements. Since the Takenaka truss was unknown in 1963, such a saving in weight could not have been anticipated. As a consequence of the different truss weight, a comparison between total cost in dollars per sq ft would be meaningless. If, on the other hand, one compares cost in dollars per pound of steel, it is found that when these trusses carry the same live load the computer results give $0.30 per lb, while the 1963 truss (allowing for a 10 per cent increase for inflation) gives $0.28 per lb. This difference can be explained by the fact that the members of the computer truss are lighter and, consequently, the unit cost in dollars per pound of steel is greater. If, on the other hand, one compares trusses that have approximately the same weight, one finds that the costs of the trusses are identical (figures in boldface).

It must be recognized that the costs in this study will vary with time and geographic location. In order to make adjustments possible, these costs are broken down into two parts: one tied to general costs, which will increase with time as general construction costs increase, and one tied to a specific base which is dependent upon both geographic location and time. This division is given in Table 5. It must be recognized that this division is in part arbitrary and any adjustments made can only be approximate.

Table 4. Comparison between the Computer Design and a Space Truss Built in 1963.

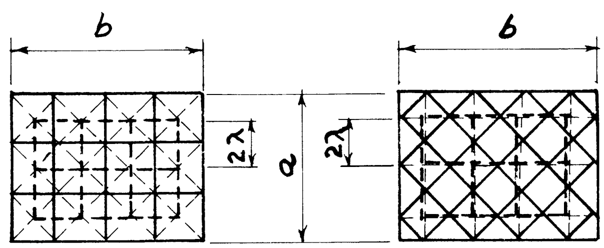
Item	Space Truss Built in 1963	Computer Design	
		Same Live Ld.	Diff. Live Ld.
Dimensions: *a* *b* *h* λ	300 ft 400 ft 17 ft to 30 ft 16.67 ft	300 ft 400 ft 15 ft 16.67 ft	300 ft 400 ft 25 ft 16.67 ft
Weight of Roofing: Roofing Material Insulation Acoustical Deck Lighting Joists	 2.5 psf 1 4 1 2		
Total	10.5 psf	9.8 psf	10.3 psf
Weight of Truss	15.0 psf	9.3 psf	13.5 psf
Total Dead Load	25.5 psf	19.1 psf	23.9 psf
Live Load	20 psf	20 psf	40 psf
Live Load Reduction Factor	0.6	0.6	1.0
Cost of Truss: $/sq ft (1963) $/sq ft (1967) $/lb (1967)	 $3.75/sq ft $4.13/sq ft **$0.28/lb**	 — $2.85/sq ft $0.30/lb	 — $3.82 sq/ft **$0.28/lb**

Framing Systems:

Top Chord ————

Bottom Chord — — — —

Diagonal - - - - -



1963 Space Truss   Computer Design

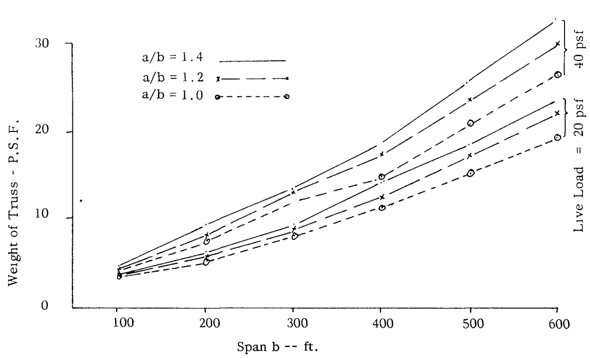


Fig. 5. Weight of truss vs. span

Table 5

Item	Tied to General Costs (%)	Tied to Specific Costs	
		%	Rates Used in This Study
Metal deck	—	100	Price @ $520.00/ton
Material	—	100	Basic price @ $140.00/ton structural shapes
Fabrication	40	60	Welder @ $6.50/hr
Erection	40	60	Steel erector @ $6.50/hr

Table 6. 2λ and $b/2\lambda$ Used in Study
(Max. value of 2λ limited by $2\lambda = 0.2b$ and $\sqrt{2}\,\lambda \leq 30.0$ ft)

b	2λ max (ft)	$(b/2\lambda)$ min	Other values of $b/2\lambda$ used
100	20	5	10
200	40	5	10, 15
300	30	10	15
400	40	10	15, 20
500	33.3	15	20
600	40	15	20

RANGE OF GEOMETRIC PARAMETERS a/b, b, $b/2\lambda$ AND b/h

In the cost analysis, aspect ratios (a/b) of 1.0, 1.2 and 1.4 were considered with the span b varying from 100 to 600 ft in increments of 100 ft. With an aspect ratio of 1.2, the largest possible grid spacing (2λ) is $0.2b$. The grid spacing is further limited by the maximum span of the metal deck on the top chord $(\sqrt{2}\lambda)$ taken as 30 ft. These restrictions taken together result in the maximum values of 2λ indicated in Table 6. Any smaller grid spacing must be a subdivision of this maximum value. Much greater flexibility exists in the choice of $b/2\lambda$ for aspect ratios of 1.0 and 1.4; however, for purposes of comparison the same values were used for each of the aspect ratios.

Alternatively, the span to depth ratio b/h may be varied at will. Naslund in dealing with a different type of space truss has suggested a value of 20,[46] while Gensert in discussing one way trusses has suggested a value of 10.[14] For the purposes of this study values of b/h from 10 to 30 were considered in increments of 2.5.

DISCUSSION AND CONCLUSIONS

The computer program was used to solve space trusses defined by the aforementioned parameters and loaded with live loads of 20 and 40 psf. With these loads and the range of parameters noted above, a total of 756 different trusses were solved.

An attempt was made to abstract from the resulting data general information independent of live load and aspect ratio. It was possible, for example, to relate the minimum total cost in dollars per sq ft to the weight of the truss (Fig. 6). The weight of the truss in turn is related to the span, loads and aspect ratios through Fig. 5. From Figs. 7 and 8* the cost of the metal deck can be determined. Subtracting this from the total cost one finds the cost of the truss. With this cost and the weight of the truss known, the fabrication, erection, material and painting costs can be found from Fig. 9. By the method previously discussed, which uses the information from Table 5, these costs can be adjusted to account for different labor costs and inflation.

* *The dip in the curve at b = 500 ft is due to the fact that the maximum value of 2λ considered for the 500-ft span is less than that considered for either the 400 or 600-ft span. This is a consequence of the restriction placed on 2λ as noted in the previous section and in Table 6.*

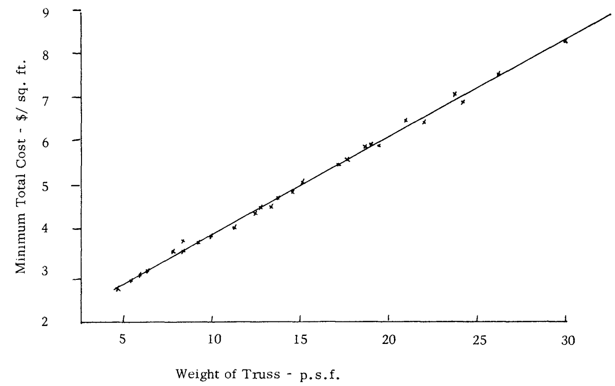


Fig. 6. Minimum total cost per sq ft (including cost of metal deck and purlins) vs. truss weight per sq ft

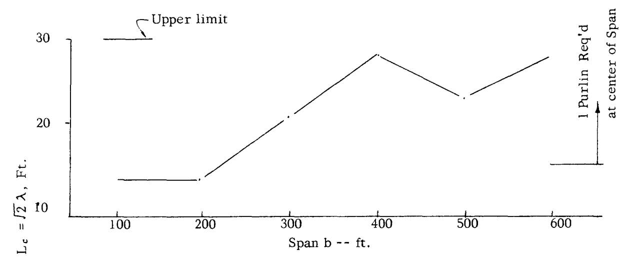


Fig. 7. Length of top chord member per minimum cost

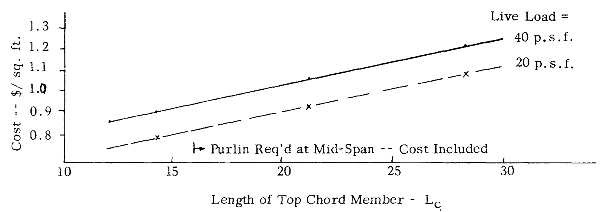


Fig. 8. Cost of metal deck (@ $520/ton)

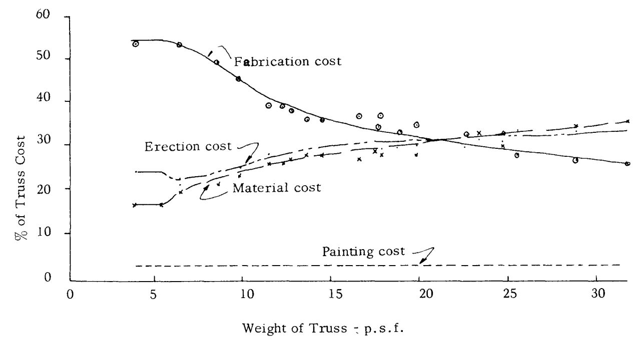


Fig. 9. Costs as a percent of truss cost

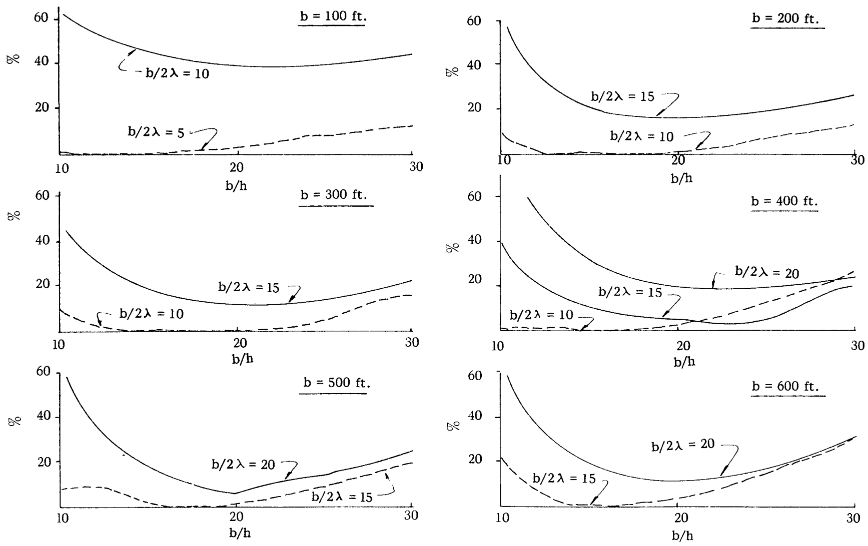


Fig. 10. Percent increase in total cost as a function of b/2 and b/h.
(w = 40 psf, a/b = 1.0)

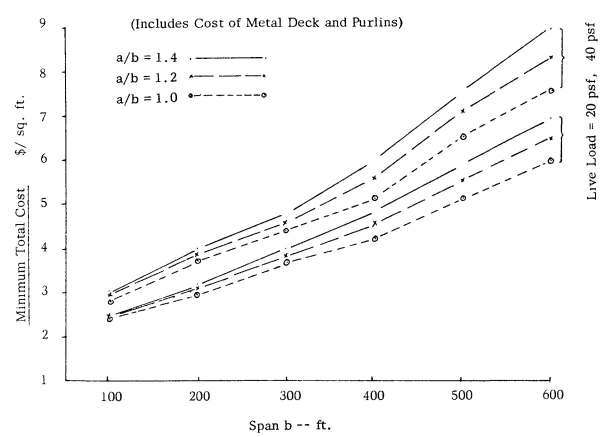


Fig. 11. Minimum total cost per sq ft vs. span

It was observed that for trusses weighing from 4 to 6 psf, the minimum area of rolled shapes instead of stress governed the size of members. As a result the weight of the truss and, consequently, the cost were less dependent on the live load and aspect ratio (Fig. 5). In this range of weight per sq ft the relationship between fabrication, erection and material costs remained constant (Fig. 9), since generally the members of the different trusses were identical. In order to maximize the use of the cross-sectional area available, the most economical design resulted in fewer elements and larger slenderness ratios ($b/2\lambda = 5$ and b/h from 10 to 15, Fig. 10). It would seem that for spans up to 100 ft one should not consider this type of truss except for much heavier loads, and that for the lighter loads considered in this work a truss built up from tubular sections or specially rolled shapes should be considered instead.

One notes that with the weight of the truss varying from 6 to 20 psf, the fabrication cost decreases rapidly in comparison with the erection and material costs (Fig. 9). This corresponds to the relationship shown in Fig. 4 for cross-sectional areas less than 30 sq in. This fact, plus the actual observation of the computer output for these problems, confirmed that the members required fall within the range of those available for rolled shapes and reflect design requirements based on stress. For the span and live load for which these truss weights are applicable, i.e., for spans of 200 to 400 ft and for a live load of 40 psf (Fig. 5) for the square truss,* it is seen that a variation of the span to depth ratio from 12 to 22 results in less than a 4 per cent increase in the minimum cost (Fig. 10). This may be explained by the interplay between the weight of the chords and the weight of the compression diagonals, where the chord forces and, hence, the chord area decrease linearly with increase in depth, while the area of the compression diagonals increase as the square of the depth because of elastic buckling. The increase in length of the diagonals also adds to their increase in weight.

For trusses weighing more than 20 psf one notes that material costs exceed erection costs while for lighter trusses they did not (Fig. 9). This is a reflection of the fact that for heavier trusses many members must be built up from plates and an increment increase in cost (15 percent) was added to the usual material, i.e., member cost. Otherwise the material and erection costs in dollars per ton are approximately constant.

* *The curves and conclusions for other aspect ratios and loads are similar. The number of curves in each family (i.e., the variation of $b/2\lambda$) is limited by the restrictions noted previously.*

The above comments based on the weight of the truss may be related to span, live load and aspect ratio through the use of Fig. 5. The actual relationship between these parameters and minimum cost is summarized in Fig. 11.

The above relationships might possibly be used to estimate the cost of trusses involving other framing systems once the weight of the truss is known. For example, using the 1963 truss of Table IV as a case in point, one obtains a total cost of $5.12 per sq ft (Fig. 6). From Figs. 7 and 8 the cost of the metal deck and purlin is found to be $0.96 per sq ft. This leaves a cost of $4.17 per sq ft for the truss which compares with an adjusted 1967 price of $4.13 per sq ft.

For all the problems considered in this work, steel with a yield strength of 36 ksi was used for the compression diagonals which are designed for elastic buckling. On the other hand, high strength steel (50 ksi) generally proved most economical for the top chords, bottom chords and tension diagonals. An exception occurred for trusses weighing less than 6 psf, for then minimum area instead of stress governed the size of members.

The grid spacing and span to depth ratios considered have been noted. Altogether a total of 756 different trusses were solved. In general the optimum grid spacing proved to be the largest permitted (see Table VI). The cost was much more sensitive to grid spacing than to the span to depth ratio. The optimum span to depth ratio varied from 10 to 20 with less than a 10 percent variation in cost in this range.

BIBLIOGRAPHY

1. *Davies, R. M.* Space Structures: Proceedings of the International Conference on Space Structures, London, 1966 *John Wiley and Sons, Inc., New York, N. Y. 1968.*
2. *Baer, O. A.* Analysis of Framed Space Structures, *Space Forms in Steel, Selected Articles, AISC Engineering Journal, New York, 1965, pp. 56–60.*
3. *Becket, W.* UCLA's Steel Space Frame: Beauty on a Budget, *Modern Steel Construction, Vol. VII, No. 2, Second Quarter, 1967, pp. 3–5.*
4. *Britvec, S. J.* Overall Stability of Pin Jointed Frameworks after the Onset of Elastic Buckling, *Ingenieur Archiv., 32 Band Ce Heft, Berlin Springer-Verlag, 1963, pp. 443–452.*
5. *Britvec, S. J.* The Theory of Elastica in the Non Linear Behavior of Plane Frameworks, *Int. J. Mech. Sci. 7, 1965, pp. 661–684.*
6. *Brooks, D. F., and Brotton, D. M.* Formulation of a General Space Frame Programmed for a Large Computer, *Struct. Engr., 44 (5) 1966, pp. 161–167.*
7. *Bruinette, K. E.* A General Formulation of the Elastic-Plastic Analysis of Space Frameworks, *Unpublished Ph.D. Thesis, University of Illinois, Urbana, Ill., Feb. 1966.*
8. *Coblentz, R. E.* Allen County War Memorial Museum, *Welding J., Vol. 32, No. 4, April 1953, pp. 320–323.*
9. *Coy, Paul H.* Structural Analysis of "Unistrut" Space-Frame Roofs, *Ann Arbor, Mich., University of Michigan Press, 1959.*
10. *Dotter, E. V.* Analysis and Design of a Three-Dimensional Tubular Space Frame, *Amer. Soc. of Civil Engr.'s Structural Engineering Conference, 1966, Miami Beach, Fla.*
11. *Dotter, E. V.* On the Design and Construction of Large Space Frame Structures, *Technical Bulletin, No. 122, July–August 1967, pp. 42–46.*
12. *Eisemann, et al.* Space Frame Analysis by Matrices and Computer, *Proc. Amer. Soc. Civ. Engrs., Vol. 88 (ST6), Dec. 1962, pp. 245–277.*
13. *Fortney, J. W., and Krahl, N. W.* Stress Analysis of Space Frames, *Civil Engineering, Amer. Soc. Civ. Engrs., Feb. 1961.*
14. *Gensert, R. M.* Introduction to Cable Structures, *Cable Construction in Contemporary Architecture, Bethlehem Steel Corporation, Bethlehem, Pa.*
15. *Gere, J. M. and Weaver, W.* Analysis of Framed Structures, *D. Van Nostrand Co., Inc., 1965*, pp. 428–431.
16. *Heyman, J.* The Limit Design of Space Frames, *J. Appl. Mech., Vol. 18, 1951.*
17. *Horne, M. R.* The Effect of Finite Deformations in the Elastic Stability of Plane Frames, *Proc. Roy. Soc. A., 226 (1324), 1962, pp. 46–47.*
18. *Hussey, M. J. L.* The General Theory of Cyclically Symmetric Frames (*To be published.*)
19. *Jennings, A., and Majid, K. I.* Computer Specification for Linear and Non-Linear Elastic Space Frame Analysis, *Manchester University Civil Engineering Department Report, 1964.*
20. *Kitamura, H.* An Analysis of the Statically Indeterminate Space Truss Flat Plate by Difference Equation, *Rep. Archit. Inst. Japan, No. 96, 97.*
21. *Kitamura, H., and Yamanda, M.* An Analysis of the Statically Indeterminate Space Truss Flat Plate by Difference Equations, *Rep. Archit. Inst. Japan, No. 107, 108, 109.*
22. *Livesley, R. K.* Analysis of Rigidly-Jointed Frames by an Electronic Digital Computer, *Engineering, Lond., 176, 230, Aug. 1953.*
23. *Livesley, R. K., and Chandler, D. B.* Stability Functions for Structural Frameworks, *Manchester, Manchester University Press, 1956.*
24. *Lovie, P. M., Rice, S. F., Schofield, K., and Somerset, F. E.* Method of Creating Metal Structures and of Jointing Devices for Use Therein, *British Patent Application No. 2681, Filed 29.6.64; completed 28.6.65.*
25. *Majid, K. I.* A Study of the Analysis and Design of Space Structures with or without Plate Components, *Research Report No. 3, Engineering Department, Manchester University, 1966.*
26. *Makowski, Z. S., and Gogate, M. N.* An Analysis of Open Grid Frameworks, *Civil and Structural Engineers Review, July 1957, pp. 341–347.*
27. *Makowski, Z. S., and Benjamin, B. S.* Analytical and Experimental Investigation of Stress Distributions in Steel Space Frames, *Proceedings of the Steel Congress, 1964, Progress in Steel Construction Work, pp. 581–606, published 1965 by the High Authority of the European Coal and Steel Community, Luxembourg.*
28. *Makowski, Z. S., and Mukhopadhyay, P.* Analytical and Experimental Stress Analysis of Double-Layer Grid Frame-works, *Technika Nauka, No. 13, December 1961, pp. 43–68.*
29. *Makowski, Z. S., and Mukhopadhyay, P.* Constructions Spatiales en Acier, *published by Centre Belgo-Luxembourgeois d'information de l'acier, Brussels, 1964.*
30. *Makowski, Z. S., and Pippard, A. J. S.* Experimental Analysis of Space Structures with Particular Reference to Braced Domes; with a Note on Stresses in Supporting

Ringgirders, *Part III, Proc. of Institution of Civil Engineers, Dec. 1952, pp. 420–441.*
31. *Makowski, Z. S., and Palmer, D.* Influence on the Torsional Rigidity upon Stress Distribution in Grid Frameworks Analysed by the Orthotropic Plate Analogy, *Biuletyn Inzynieryjno-Budowlany, Institution of Polish Engineers in Great Britain, September 1956, series 11/5, pp. 78–92.*
32. *Makowski, Z. S., and Mukhopadhyay, P.* Modern Architecture and Space Structure (*in Italian*), *published by the University of Naples, Faculty of Engineering, Napoli, 1962, Instituto di Architettura e Composizione Architettònica, Seminaro di Studi-Architettura e Struttura.*
33. *Makowski, Z. S., and Palmer, D.* Practical Simplifications in Stress Analysis of Open Grid Frameworks, *Biuletyn Inzynieryjno-Budowlany, Institution of Polish Engineers in Great Britain, March 1956, pp. 23–30.*
34. *Makowski, Z. S., and Mukhopadhyay, P.* Space Structures (*in English and Japanese*), *Column No. 5, January 1963, pp. 21–58, published by Yawata Iron and Steel Co., Ltd., Tokyo.*
35. *Makowski, Z. S., and Benjamin, B. S.* Space Structures and the Electronic Computer, *Architectural Design, pp. 8–9, January 1966.*
36. *Makowski, Z. S., and Rivera, A. R.* Space Structures—Modern Trends and Recent Developments, *The School of Architecture, The Polytechnic, London, 1961.*
37. *Makowski, Z. S.* Steel Space Structures, *Michael Joseph, 1965, London.*
38. *Makowski, Z. S., and Mukhopadhyay, P.* Structures Spatiales Theorie et Applications, *l'Architecture d'aujourd'hui, No. 99, January 1962, pp. 78–80.*
39. *Makowski, Z. S.* A Survey of Recent Three Dimensional Structures, *Archit. Design, January 1966.*
40. *Makowski, Z. S., and Mukhopadhyay, P.* Use of Prefabricated Steel Space Frames, *Civil Engineering, A Financial Times Survey, November 16th, 1964, pp. 28–29.*
41. *Means, R. S.* Building Construction Cost Data—1967, *Robert Snow Means Co., Inc., Engineers and Estimators.*
42. *Michalos, J., and Grossfield, B.* Analysis of Interconnected Space Frames, *Preliminary Publication, Seventh Congress, International Assn. for Bridge and Structural Engineering, Rio de Janeiro, 1964, pp. 275–283.*
43. *Michalos, J.* Numerical Analysis of Continuous Frames in Space, *Trans. Am. Soc. Civ. Engrs. 119, pp. 565–582, 1965.*
44. *Millar, M. A., Brotton, D. M., and Merchant, W. A.* A Computer Method for the Analysis of Non-Linear Elastic Plane Frameworks, *The Use of Electronic Digital Computers in Structural Engineering, University of New Castle upon Tyne, International Symposium, 1966.*
45. *Naslund, K. C.* Model Tests Predict Space Frame Behavior, *Architectural Record, December 1961.*
46. *Naslund, Kenneth C.* Space Frame Structures, *Space Forms in Steel, Selected Articles from AISC Engineering Journal, 1965, New York, pp. 42–47.*
47. *Nilson, Arthur H.* Shear Diaphragms of Light Gauge Steel, *Journal of the Structural Division, Proceedings of the ASCE, Vol. 86, No. ST11, Nov. 1960, pp. 111–139.*
48. *Ozaki, M.* Research on Space Structures.
49. *Rapp, Robert E.* Space Structures in Steel, *Architectural Record, Nov. 1961.*
50. *Renton, J. D.* On the Gridwork Analogy for Plates, *J. Mech. Phys. Solids, 13, pp. 413–420, 1965.*
51. *Renton, J. D.* On the Stability Analysis of Symmetrical Frameworks, *Q. J. Mech. Appl. Math., 17, Part 2, pp. 175–197, May 1964.*
52. *Renton, J. D.* Stability of Space Frames by Computer Analysis, *J. Struct. Div. Am. Soc. Civ. Engrs., No. 3237 (ST4), August 1962, pp. 81–103.*
53. *Saafan, S. S., and Brotton, D. M.* Elastic Finite Deflection Analysis of Rigid Frameworks by Digital Computer, *Laboratorio Nacional de Engenharia Civil Symposium on the Use of Computers in Civil Engineering, Lisbon, 1962, pp. 6.1–6.22.*
54. *Salvadori, M. G., and Levy, M.* Structural Design in Architecture, *Prentice-Hall, Englewood Cliffs, N. J., 1967.*
55. *Sbarounis, J. A., and Gaus, M. P.* Analysis of Two Way Truss System, *ASCE—Proc. V 85, (J. Structural Div.), N. St2, Feb. 1959 paper in 1940, pp. 45–69.*
56. *Shaw, F. S.* Limit Analysis of Grid Frameworks, *J. Structural Div. Am. Soc. Civ. Engrs., 89St5, pp. 1–14, 1963.*
57. *Solomen, E. W.* Space Truss Analysis by Relaxation, *London, Engineering Computations, 1966.*
58. *Tezcan, S. S.* Computer Analysis of Plane and Space Structures, *J. Struct. Div. Am. Soc. Civ. Engrs., No. 4780 ST2, April 1966, 143–173.*
59. *Williams, F. W.* An Approach to the Non-Linear Behavior of the Members of a Rigid Jointed Plane Framework with Finite Deflections, *Q. J. Mech. Appl. Math., XVII, 451–469, Nov. 1964.*
60. *Wright, D. T.* Static Test of Triodetic Structural Connection" (*Publication pending.*)
61. The Past Inspires Future Exhibitions Hall, *Engineering News-Record, Sept. 22, 1966, V. 177, pp. 36.*
62. Space Frame Cost Less Than $4 a Square Foot, *Architectural Record, March 1966, pp. 181–184.*
63. Space Frames Span Sports Arena, *Engineering News-Record, V. 172, No. 12, March 19, 1964, pp. 56.*
64. Steel Frames for Circular Fieldhouse, *Steel Construction Digest, Vol. XII, No. 4, 4th Quarter 1955.*
65. Manual of Steel Construction. *6th Edition, American Institute of Steel Construction, Inc., New York, N. Y., 1963.*